MAT 2377D

Midterm

21 October 2015
Time: 70 minutes

Professor: Ali Karimnezhad

Student Number: ________________

Family Name: ________________

First Name: ________________

This is a closed book examination.
Only non-programmable and non-graphic calculators are permitted.
Record your answer to each question in the table below.
Your package includes the title page, two pages with questions, a formula sheet and statistical tables.
Number of questions: **10**.
NOTE: At the end of the examination, hand in only this page. You may keep the questionnaire.

Question	Answer
1	
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4	
5	
6	
7	
8	
9	
10	

GOOD LUCK !!!

Q1. A company's warranty document states that the probability that a new swimming pool requires some repairs within the first year is 30%. What is the probability, that the fourth sold pool is the first one which requires some repairs within the first year?

(a) 0.6068 (b) 0.3932 (c) 0.6617

(d) 0.0655 (e) none of the preceding

Solution to Q1:

Define X: Number of sold pools until observing the first pool requiring repair. Thus $X \sim Ge(p)$ with $p = 0.3$.

$$P(X = 4) = 0.3 * (0.7)^{4-1} = 0.1029$$

Answer **e**.

Q2. Let X denote a number of failures of a particular machine within a month. Its probability mass function is given by

x	0	1	2	3	4	5
$P(X = x)$	0.17	0.23	0.19	0.13	0.08	0.2

- The probability that there are more than 3 failures within a month, and
- the expected number of failures within a month

are, respectively

(a) 0.28; 2.50 (b) 0.1; 2.32 (c) 0.28; 2.32

(d) 0.1; 2.50 (e) none of the preceding

Solution to Q2:

Define X: Number of failures within a month. Then,

$$P(X > 3) = P(X = 4) + P(X = 5) = 0.28$$

$$E(X) = \sum_{x=0}^{5} x f(x) = 0 \times 0.17 + \ldots + 5 \times 0.2 = 2.32$$

Answer **c**.

Q3. Two companies A and B consider making an offer for road construction. The company A makes the submission. The probability that B submits the proposal is $3/4$. If B does not submit the proposal, the probability that A gets the job is $4/5$. If B submits the proposal, the probability that A gets the job is $1/4$. What is the probability that A will not get the job?

(a) 0.6667 (b) 0.5111 (c) 0.3875

(d) 0.45 (e) none of the preceding

Solution to Q3:

Denote the events: A - the company A gets the job; B - the company B submits the proposal. We have $P(B) = 3/4$; $P(A|B') = 4/5$, $P(A|B) = 1/4$. Form this we calculate $P(B') = 1/4$. We calculate first, using the total probability rule, $P(A) = PA|B)P(B) + P(A|B')P(B') = 1/4 * 3/4 + 4/5 * 1/4 = 0.3875$. Hence $P(A') = 1 - 0.3875 = 0.6125$.
Answer **e**.

Q4. In a box of 40 fuses there are 8 defective ones. We choose 5 fuses randomly (without replacement). What is the probability that all 5 fuses are not defective?

(a) 0.4015 (b) 0.84 (c) 0.3725

(d) 0.3060 (e) none of the preceding

Solution to Q4:

Let X be the number of defective fuses. We calculate

$$P(X=0)=\frac{\binom{8}{0}\binom{32}{5}}{\binom{40}{5}}=0.3060$$

Answer **d**.

Q5. Consider a random variable X with the following probability density function:

$$f(x)=\begin{cases}0 & \text{if } x\le -1\\ \frac{3}{4}(1-x^2) & \text{if } -1<x<1\\ 0 & \text{if } x\ge 1\end{cases}$$

The value of $P(X>0.5)$ is

(a) 11/32 (b) 27/32 (c) 5/32

(d) 1 (e) none of the preceding

Solution to Q5:

To compute:

$$P(X>0.5)=\int_{0.5}^{1}\frac{3}{4}(1-x^2)\,dx=\frac{3}{4}x|_{0.5}^{1}-\frac{1}{4}x^3|_{0.5}^{1}=5/32.$$

Answer **c**.

Q6. A receptionist receives on average 5 phone calls per minute. If the number of calls follows a Poisson process, what is the probability that the waiting time for call will be greater than 1 minute?

(a) e^{-5} (b) $e^{-2/5}$ (c) $e^{-1/5}$ (d) $e^{-5/2}$ (e) none of the preceding

Solution to Q6:

We have Poisson process with $\lambda=5$. Now, waiting time in Poisson process is exponential. Let X be an exponential random variable with the parameter $\lambda=\frac{1}{5}$. To compute: $P(X>1)=\exp(-\frac{1}{1/5}\times 1)=\exp(-5)$.

Answer **a**.

Q7. A company manufactures hockey pucks. It is known that their weight is normally distributed with mean 1 and the variance 0.01. The pucks used by NHL must weight between 0.8 and 1.2. What is the probability that a randomly chosen puck can be used by NHL?

(a) 1 (b) 0.4560 (c) 0.9545 (d) 0.9772 (e) none of the preceding

Solution to Q7:

$$P(0.8 < X < 1.2) = P\left(\frac{0.8-1.0}{0.1} < Z < \frac{1.2-1.0}{0.1}\right) = \Phi(2) - \Phi(-2) = 0.977250 - 0.022750 = 0.9545$$

Answer **c**.

Q8. If $X \sim N(10, 1)$, the value of k such that $P(X > k) = 0.30152$ is closest to

(a) 0.48 (b) 0.52 (c) 9.48

(d) 10.52 (e) none of the preceding

Solution to Q8:

$$P(X > k) = P((X-10)/1 > (k-10)/1) = 1 - P(Z \leq k - 10) = .30152.$$

Thus, $k - 10 = 0.52$ and $k = 10.52$.
Answer **d**.

Q9. A manufacturer produces computers. To check customer satisfaction with a product, 100 people have been interviewed. The results are as follows:

	Satisfied	Not Satisfied
Male	19	41
Female	12	28

Given that a randomly selected customer is ‘Satisfied’, the probability that it is male, is:

(a) 0.19 (b) 0.3166 (c) 0.3577 (d) 0.6129 (e) none of the preceding

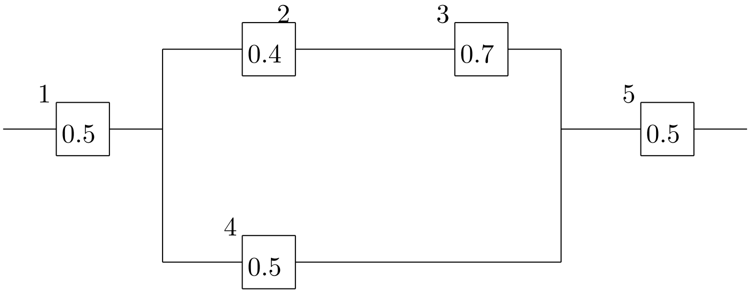
Solution to Q9:

Let M - ’Male’, F - ’Female’, S - ’Satisfied’, S^c - ’Not Satisfied’ To compute $P(M|S)$.

$$P(M|S) = \frac{P(M \cap S)}{P(S)} = \frac{19/100}{31/100} = 0.6129$$

Answer **d**.

Q10. Consider the following system with five components. We say that it is functional if there exists a path of functional components from left to right. The probability of each component functions is shown. Assume that the components function or fail independently. What is the probability that the system works?



(a) 0.84 (b) 0.16 (c) 0.035 (d) 0.50 (e) none of the preceding

Solution to Q10:

Call 'Box B' - components 2,3,4, 'Box C' - components 2,3.

$$
\begin{aligned}
P(\text{Box C operates}) &= P(\text{component 2 operates and component 3 operates}) \\
&= P(\text{component 2 operates})P(\text{component 3 operates}) = 0.4 \times 0.7 = 0.28.
\end{aligned}
$$

$$
\begin{aligned}
P(\text{Box B operates}) &= P(\text{Box C operates or component 4 operates}) \\
&= P(\text{Box C operates}) + P(\text{component 4 operates}) - \\
&\quad P(\text{Box C operates})P(\text{component 4 operates}) \\
&= 0.28 + 0.5 - 0.28 * 0.5 = 0.64.
\end{aligned}
$$

$$
\begin{aligned}
P(\text{system operates}) &= P(\text{component 1 and Box B and component 5 operate}) \\
&= P(\text{component 1 operates})P(\text{Box B operates})P(\text{component 5 operates}) \\
&= 0.5 * 0.64 * 0.5 = 0.16.
\end{aligned}
$$

Answer **b**.

This is the last question

Solutions to multiple choice questions:

Q1 $\longrightarrow$ e

Q2 $\longrightarrow$ c

Q3 $\longrightarrow$ e

Q4 $\longrightarrow$ a

Q4 $\longrightarrow$ d

Q5 $\longrightarrow$ c

Q6 $\longrightarrow$ a

Q7 $\longrightarrow$ c

Q8 $\longrightarrow$ d

Q9 $\longrightarrow$ d

Q10 $\longrightarrow$ b