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Pricing-based Stackelberg game for spectrum trading in self-organised heterogeneous networks

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Abstract: In this study, the spectrum trading problem in a self-organised and two-tier heterogeneous cellular network is studied and the bandwidth sharing problem incorporated with cell range expansion technique is formulated as a Stackelberg game. Maximising the revenue of macro eNodeB (MeNB), affording minimum required bandwidth for each home eNodeB (HeNB), enhancing per femto-user throughput, and providing better quality of service for macro-users nearby each femto-cell are the main objectives of this investigation. The authors propose an iterative scheme in which MeNB and HeNBs make decisions autonomously. The MeNB as the spectrum provider decides on the unit-bandwidth price with the objective to maximise its revenue. Each HeNB, on the other side, decides on the amount of requested bandwidth as well as the number of possible nearby macro-users which will support to take advantage of MeNB discounting. The designed discounting strategy is applied for the extra bandwidth request of HeNBs to encourage them in supporting nearby macro-users. It is shown analytically that the proposed scheme converges to a unique Nash equilibrium which is Pareto optimal. Evaluating the authors' game through different simulations demonstrates that it is all beneficial for both MeNB and HeNBs and the social welfare is maximised in the network.

1 Introduction

1.1 Background

Heterogeneous network known as HetNet is a promising architecture for fifth generation (5G) mobile networks topology in International Mobile Telecommunications (IMT) advanced framework to satisfy the growing demands for various kinds of services [1, 2]. A HetNet employs different types of cells with different size to cope with massive traffic demand for diverse applications. In such networks in addition to macro-cells which provide general coverage, some low power, short range, and low cost small cells, femto-cells, are distributed inside the macro-cells to improve the spectrum utilisation and ensure reliable coverage.

Self-organising is the salient feature of 5G HetNets. Indeed, small-cells will be placed unplanned and expected to be self-configured, self-healed, and self-optimised [3]. Consequently, by eliminating the centralised role of macro eNodeB (MeNB), lower-power providers have their own authority domains and make all of their decisions autonomously.

Subscribers of each macro and femto cells get their services from a centric MeNB and home eNodeB (HeNB), respectively, which can use the same frequency bands in co-channel or partial co-channel deployment [4, 5]. In partial co-channel deployment case which is also adopted in this paper, a certain part of bandwidth is shared between the femto-cells and the corresponding macro-cell. This scheme has the advantage of compromising between the spectrum usage efficiency and the co-channel interference [5]. However, co-channel deployment of small cells has its own challenges. These small cells may use to serve their own private users in a closed access mode using the leased spectrum from the macro-cell as the spectrum owner. Correspondingly, the strong interference from HeNBs may degrade the quality of services (QoS) of macro-users especially located in the vicinity of small cells which are called as victim macro-users in Release 8 [6]. Due to necessity of mitigating the observed interference by such macro-users along with high data-rate requirement of femto-users, resource allocation problem especially in a distributed manner plays a major role in HetNets. There are three approaches in dealing with resource allocation problems: frequency assignment, power control, and joint frequency and power allocation. In this paper, we focus our attention in frequency domain approach in which the problem is that a limited bandwidth should be allocated to HeNBs who try to enhance the spectral efficiency of their users. MeNBs, on the other hand, seek to maximise its revenue from leasing spectrum to HeNBs while protecting macro-users especially victim ones from harmful interference of HeNBs.

Mitigation and coordination of the inter-tier interference between MeNBs and HeNBs in a 5G multi-tier HetNet is the key challenge which is widely investigated recently. Generally, efficient resource allocation in order to cope with cross-tier interference can be done by using different approaches and techniques. Fractional frequency reused scheme between MeNB and HeNBs is the base of many studies in this literature [7, 8]. Although, this solution can properly control the inter-tier interference by avoiding the HeNB to use the frequency of nearby macro-users, it may degrade the capacity and spectral efficiency of both femto-cell and macro-cell. Cell range expansion (CRE) technique which is introduced in the field of HetNet load-balancing in Release 11 [9], is the fundamental solution for supporting the victim macro-users with better QoS. The CRE enables HeNBs to offload the traffic for MeNB. Applying CRE not only helps the MeNB by load-balancing, but also it can help the HeNB to improve its spectral efficiency [10] or enhance femto-users' data-rate [11]. Victim macro-users can be motivated to get their services from lower-power HeNB who applies CRE by biasing technique [12].

Spectrum trading is a mechanism which solves bandwidth sharing problem according to the supply and demand rule by pricing method. Namely, MeNBs as the spectrum providers try to maximise their revenues from leasing spectrum to HeNBs who accept to pay more to get larger bandwidth. Naturally, by proper spectrum pricing and leasing between macro and femto cells in each area, efficient utilisation of spectrum becomes achievable. Discount pricing strategy is a popular method for sellers in economic problems to encourage customers to buy more good. In this paper, MeNB motivates HeNBs to support more victim macro-users by this practical strategy. A comprehensive description of key challenges and objectives of spectrum trading is provided in [13].

Despite optimality of centralised techniques in pricing-based resource allocation or interference management, finding the analytical formulation and in turn the truly optimum point is prohibitively difficult due to the non-convexity and NP-hardness of the underlying constrained optimisation problem [14]. Game theory is a powerful tool to provide tractable approach to model spectrum trading problem especially in self-organised networks in which players are rational entities able to observe and react. Additionally, due to autonomous behaviour of small-cells, decision making in the process of designing the spectrum trading mechanism in self-organised HetNets should be done hierarchically. Stackelberg is a kind of sequential game that incorporates dynamic and bargaining games with the ability to better exploit the hierarchical pricing-based interactions between users and providers in variety of scenarios. Spectrum trading problem with the aim of social welfare maximisation in a closed access and self-organised HetNet is formulated as a Stackelberg game in this paper. Namely, MeNB as a leader and HeNBs as followers negotiate with each other in a multi-stage market model according to their own satisfactions in the hope of finding a Pareto-efficient outcome. As the result of this negotiation, they reach to an agreement on the amount of shared bandwidth, the number of supported victim macro-users, and the imposed unit-bandwidth price as the operating parameters.

1.2 Related works and our contributions

Spectrum trading and bandwidth sharing problems are investigated in a lot of work using game theory. In this literature, many studies address the bandwidth sharing issue between some primary users and secondary ones in a hierarchical spectrum access-based cognitive radio networks [15–17]. On the contrary to this approach of spectrum trading, there are less attempts for spectrum trading in multi-tier HetNets which has distinct features including the multi-provider scenarios, deployment of self-organised small-cells, victim macro-users problem, and offloading concept.

More specifically, mechanism design for spectrum trading in multi-tier HetNets which consists of macro-cells and multiple femto-cells in each macro-cell area depends on the spectrum access mode. The spectrum access in these networks could be closed access, open access, and hybrid access. Chen *et al.* [18] and Tehrani and Uysal [19] deal with the spectrum trading problem in open and hybrid access mode of HetNets in which femto-cells support macro-users for interference management through the network. The problem of spectrum trading in closed access HetNets in which each femto-cell is managed by an independent authority and is not ought to support macro-users is a newly popular challenge. The reason is that macro-users near the femto eNB would suffer from sever interference.

The objectives of many previous works in closed access HetNets are optimising the revenue of spectrum provider by proper pricing of the interference from spectrum leasers [20–22]. Besides, some other investigations combine CRE technique and discount strategy with spectrum sharing problem to maximise the data-rate of users belonging to small-cells, while protecting victim macro-users from strong interference of near lower-power eNBs [11, 23]. Beyond these approaches, the goal we try to achieve in this paper is maximising the social welfare in a self-organised and closed access HetNet taking into account the challenge of victim macro-users. Namely, in the side of MeNB as the spectrum provider, we attempt to maximise the revenue while the number of unattached victims are minimised. From the HeNB point of view as the spectrum leaser, the objective is maximising the average data-rate per femto-user by paying the least cost. Furthermore, given the tradeoff between entities' satisfactions, the one who values the spectrum the most, benefit the most as well. Meeting these goals will maximise social welfare in the network [24]. Additionally, due to autonomous behaviour of small-cells in self-organised HetNets, all decisions of each femto-cell are made by HeNB itself individually during our mechanism design. Hence, in contrast with [11] HeNB is the one who decides about the number of supported victim macro-users instead of MeNB. Another novelty of our scheme is that MeNB is able to balance the discount price to compromise between its income and the number of its offloaded victim users. Meanwhile, it discounts the price of spectrum for HeNBs proportional to the ratio of their supported victim macro-users to the number of their private ones. As a result, in contrast with previous works, the amount of bandwidth or implicitly the QoS provided for each victim is also considered in discount factor of each HeNB. As another difference, each HeNB is suggested to report its minimum spectrum demand to MeNB in order to meet minimum requirements of its local users.

Briefly, in this paper, we develop a spectrum bargaining Stackelberg game (SBSG) in a self-organised HetNet which is implemented in a distributed way, provides a high data-rate service per femto-user, maximises the revenue of spectrum provider, compensates the QoS impairment of victim macro-users, guarantees the minimum bandwidth demand of each HeNB, and exploits the advantages of bargaining in trading mechanism. Besides, two pricing policies are considered for MeNB as bandwidth provider, the formal spectrum pricing for affording deterministic minimum demand and discount spectrum pricing for extra allocated bandwidth which in practice, complies exactly with the economic issues.

Our simulation results show that all MeNB and HeNBs gain from the proposed game. Additionally, the HeNB who needs more bandwidth, benefit the most from the proposed algorithm provided that it serves more victim macro-users. Therefore, according to the presented economic objectives in [13] and definition of utilitarian social welfare function in [24], our game maximises the social welfare too. Furthermore, our game is shown to converge to a unique Nash equilibrium (NE) point that is proved to be analytically Pareto optimal. The outline of the paper is organised as follows: after describing our system model in Section 2, Section 3 introduces the proposed spectrum sharing Stackelberg game. The important features of this game are proved in Section 4. Our simulation results are presented in Section 5 and finally Section 6 concludes the paper.

2 System model and problem statement

We consider a two-tier HetNet similar to what is shown in Fig. 1*a* in which F self-organised and closed access femto-cells with radius of $R_{\rm F}$ randomly distributed in the area of one macro-cell with radius of $R_{\rm M}$ ($R_{\rm M} \gg R_{\rm F}$). Let I_i and M_i denote the number of femto-users served by a centric HeNB using OFDMA technology in downlink and the number of victim macro-users located in the vicinity of each femto-cell, respectively. One high power MeNB located at the centre of macro-cell provides services for macro-users and shares a limited amount of bandwidth B with coordinated HeNBs. Furthermore, in our system model, we ignore the inter-femto-cell interference by distributing the available bandwidth among HeNBs in non-overlapping way. Due to pathloss effect, macro and femto users are assumed to experience the interference signal from 7 neighbour macro-cells. Meanwhile, since our focus is on frequency domain approach, MeNB and HeNBs are supposed to send their signals with their maximum transmitting power. Generally, the required parameters for problem formulation are described in Table 1.

According to aforementioned assumptions, the signal to interference plus noise ratio (SINR) of fth femto-user belonging to HeNB i will be determined as

$$\gamma_i^{\rm f} = \frac{p^{\rm F} h_i^{\rm f}}{\sum_{m=1}^{7} p^{\rm M} h_m^{i,{\rm f}} + \sigma_0^2} \tag{1}$$

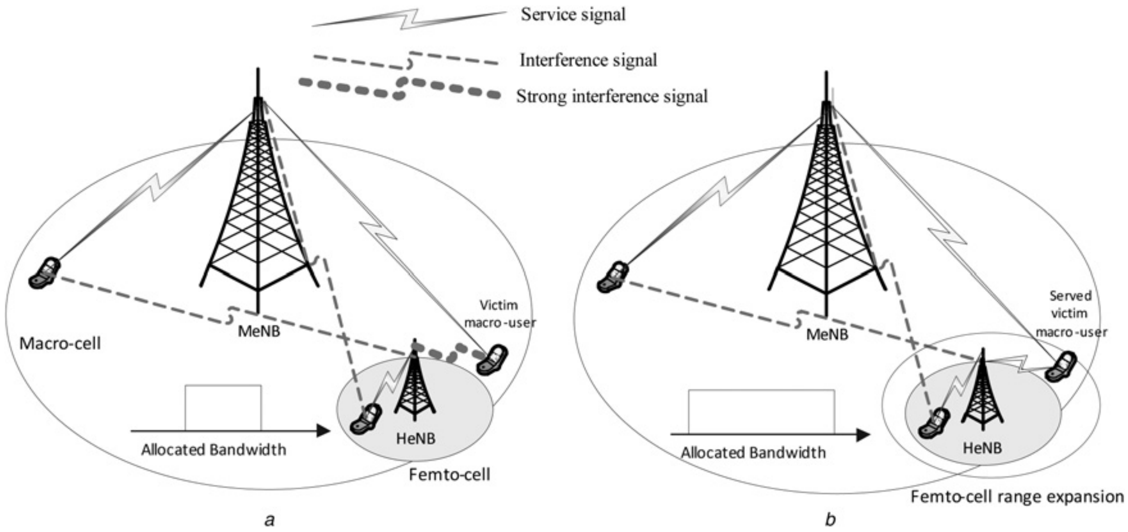

Fig. 1 *System model definition*

a Victim macro-user experiences strong interference and its QoS is degraded
b HeNB applies CRE to support victim macro-user in lieu of sharing more bandwidth with MeNB

where $\sum_{m=1}^{7} p^{\mathrm{M}} h_m^{i,\mathrm{f}}$ shows the total interference caused by assumed 7 neighbour macro-cells. So, the spectral efficiency of such user will be

$$r_i^{\mathrm{f}} = \log_2 (1 + \gamma_i^{\mathrm{f}}) \tag{2}$$

Therefore, the total spectral efficiency of HeNB i amounts to

$$r_i^{\mathrm{tot}} = \sum_{f=1}^{I_i} r_i^{\mathrm{f}} = \sum_{f=1}^{I_i} \log_2 (1 + \gamma_i^{\mathrm{f}}) \tag{3}$$

On the other hand, the SINR of each nearby macro-user k around the ith HeNB will be given by the following equation

$$\gamma_k = \frac{p^{\mathrm{M}} h_k}{\sum_{i=1}^{\mathrm{F}} p^{\mathrm{F}} h_k^i + \sum_{m=1}^{7} p^{\mathrm{M}} h_k^m + \sigma_0^2} \tag{4}$$

Similarly, the spectral efficiency of this macro-user is defined as

$$r_k = \log_2 (1 + \gamma_k) \tag{5}$$

In our spectrum trading mechanism, MeNB leases spectrum to HeNBs to serve femto-users. As a result, the macro-users nearby these autonomous HeNBs may become victims by experiencing strong interference. To cope with this QoS degradation, it is assumed that each HeNB is able to expand its coverage by applying CRE technique, as shown in Fig. 1*b*. Consequently, it can provide service for some macro-users in its vicinity. That is MeNB who tries to balance its load should encourage corresponding HeNBs by allowing them to share more bandwidth with MeNB or noticeably discounting their bandwidth expenditure. Consequently, HeNBs can provide higher data-rate for their own users by means of CRE. Since MeNB and HeNBs have conflicting objectives, they bargain to find an agreement. Meanwhile, we intend to allow the ith HeNB to determine D_i as its minimum required bandwidth corresponds to its users' services. Seeking to enhance data-rate per femto-user and also discount strategy of MeNB motivate each HeNB to serve some victim macro-users and share bandwidth with MeNB more than its minimum demand.

Table 1 System parameters

Parameter	Definition
F	number of deployed femto-cells
I_i	number of users located in femto-cell i
M_i	number of victim macro-users located in vicinity of femto-cell i
σ_0^2	additive noise power
h_i^{f}	channel gain between femto-user f of femto-cell i and its HeNB
$h_m^{i,\mathrm{f}}$	channel gain between femto-user f of femto- cell i and mth neighbour macro-cell
h_k	channel gain between victim macro-user k and MeNB
h_k^i	channel gain between victim macro-user k and HeNB i
h_k^m	channel gain between victim macro-user k and mth neighbour macro-cell
p^{M}	maximum affordable transmitting power of MeNB
p^{F}	maximum affordable transmitting power of HeNB

In the procedure of our game-theoretic algorithm, MeNB imposes the unit-bandwidth price P_i to ith HeNB which maximises its revenue. Sequentially, each HeNB who is interested in sharing more bandwidth with MeNB to provide higher throughput per femto-user, offers its proposed bandwidth, b_i, together with the number of victim macro-users ready to serve, N_i, corresponding to its minimum demand and also its willingness to pay. In addition, the price of $(b_i - D_i)$ which illustrates the extra bandwidth allocated to HeNB i would be considerably discounted proportional to N_i by MeNB. As a result of negotiation between these entities in terms of bargaining, HeNBs as leasers and MeNB as provider of spectrum agree on the price, the allocated bandwidth to each HeNB, and the number of offloaded victim macro-users by each HeNB at the end of the game.

In fact, MeNB and HeNBs should bargain until they reach to an agreement which occurs in algorithm convergence. The cooperation between macro-cells and small-cells to coordinate inter-cell interference can be done through direct air interface [5].

3 Spectrum trading Stackelberg game

3.1 Game definition

Generally, a Stackelberg game is a strategic game with two types of players: leaders which choose their strategies first and followers which make decision sequentially knowing leaders' strategies. In this section, we are interested in formulating our model described in Section 2 as a SBSG. For this purpose, we define a normal game as $G = (\mathcal{N}, \mathcal{A}, \mathcal{U})$ where

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• $\mathcal{N}$ denotes the set of players or decision makers. Due to our Stackelberg game model, this set consists of HeNBs as followers and MeNB as leader.

• $\mathcal{A}$ which is called the strategy space determines the set of possible actions that each player can choose as its strategy. In our model, MeNB determines its strategy, $\underline{P} = [P_1, P_2, \ldots, P_\mathrm{F}]$, where $P_i \in [0\ \infty)$, $i = 1, \ldots, F$ is the imposed price to HeNB i and HeNB i chooses a pair of (b_i, N_i) from a two-dimensional (2D) strategy space of $[0\ B] \times [0\ M_i]$.

• $\mathcal{U}$ is the set of utility functions of players defined related to their preferences. Precisely, the utility function of each player is its benefit minus its cost. In our model, the utility of MeNB is its income, while, HeNB's utility function is defined according to its objectives of sharing at least its minimum required bandwidth with MeNB and providing high throughput per femto-user with the least cost.

Consequently, taking attention to aforementioned goals, the utility function of HeNB i can be defined as

$$U^i_{\mathrm{HeNB}}(b_i, N_i) = \left(r_i D_i + \sqrt{b_i - D_i} - P_i D_i\right) + \left(r_i \frac{I_i}{I_i + N_i}(b_i - D_i) - \frac{P_i(b_i - D_i)}{\delta\left(1 + (N_i/I_i)\right)^2}\right) \quad (6)$$

In which, r_i is the average spectral efficiency per each femto-user. That means each HeNB as a decision maker should decide about the amount of required spectrum and the number of victims to support, taking into account its local femto-users, the advertised price of the unit-bandwidth, and the discounting strategy of macro-cell for supporting victim macro-users. More exactly, the proposed objective function shown in (6) is composed of two different parts. First part is related to bandwidth demand of each HeNB. In this term, $r_i D_i$ is the throughput per femto-user of HeNB i obtained from sharing its minimum required bandwidth with MeNB. The barrier function $\sqrt{b_i - D_i}$ is also applied to ensure the minimum supply for this HeNB. Finally, $P_i D_i$ expresses the imposed cost from MeNB to it in lieu of providing the guaranteed amount of spectrum. So, the more the bandwidth demand of each HeNB is, the higher cost should be paid to MeNB. Additionally, as long as MeNB provides the deterministic target bandwidth of each HeNB, there is no chance for HeNB to apply CRE to benefit from spectrum discounting. However, the second term is related to the possible extra bandwidth allocation. The reason behind this definition is that once a victim macro-user is attached to nearby HeNB, it is treated as a femto-user and exploits the same share of spectrum as local ones. So, $(I_i/I_i + N_i)(b_i - D_i)$ is the fraction of extra bandwidth could be allocated to private femto-users. Correspondingly, the more number of victim users served by HeNB i, the more discount is applied by MeNB and also the less throughput per local femto-user could be obtained. Meanwhile, in the proposed SBSG the price of extra allocated bandwidth is discounted by factor $\delta(1 + (N_i/I_i))^2$ for HeNB i which depends on the ratio of the number of supported victims, N_i, to the number of its private users, I_i. Therefore, from the MeNB point of view, as the number of former femto-users of each HeNB increases, the amount of bandwidth allocated to the supported victim will be decreased which leads to a reduction in its achieved throughput. Consequently, using this factor we apply more discount in the price of unit-bandwidth for HeNB with less private subscribers who is able to afford better service for MeNB's victims. Thus, the QoS of victim macro-user is also implicitly considered in the proposed discounting strategy. Furthermore, δ is the constant that should be defined well by MeNB to compromise its revenue and the number of supported victims by HeNBs. Obviously, MeNB can apply more discount in the price of bandwidth by determining higher value for δ to encourage HeNBs to offload more victims at the price of revenue reduction. It should be noted that for each HeNB who supports no victims, the value of δ is equal to 1.

On the other side, from the perspective of MeNB, the objective function of MeNB for revenue maximisation can be formulated as

$$U_{\mathrm{MeNB}}(\underline{P}, \underline{b}, \underline{N}) = \sum_{i=1}^{\mathrm{F}} \left(P_i D_i + \frac{P_i(b_i - D_i)}{\delta\left(1 + (N_i/I_i)\right)^2}\right) \quad (7)$$

In this utility function, the first term denotes the receiving income of MeNB in lieu of providing minimum bandwidth target of each HeNB, while the second part is the discounted revenue obtained from associating extra bandwidth to each HeNB taking into account its offloading activity. In fact, since the minimum required bandwidth of HeNBs are deterministic, applying discount pricing strategy by MeNB would be irrational. In contrast, MeNB can use discount technique to make a stronger competition between HeNBs in obtaining more extra bandwidth by motivating them to open their resource to nearby victim macro-users. Furthermore, according to partial co-channel deployment, MeNB can share only the limited amount of bandwidths, B, with HeNBs. Therefore, according to non-overlapping bandwidth assignment to HeNBs, the optimisation problem at the side of MeNB is given by the following equation

$$\begin{aligned} \max_{\underline{P}} \quad & U_{\mathrm{MeNB}} \\ \text{s.t.} \quad & \sum_{i=1}^{\mathrm{F}} b_i \le B \end{aligned} \quad (8)$$

3.2 Players' best responses

Best response function of a player denotes its best reaction to any strategy profile chosen by other players in which its utility is maximised. In the following, we calculate the best response of each player in the proposed hierarchical game. Then, in the next section we show that making decision according to the best responses converges to the NE point which is also unique. That is a fixed point in which there is no incentive for MeNB and HeNBs to deviate from their strategies to have extra profits [25].

We can find the best responses of players using backward induction. Namely, MeNB first chooses a strategy according to the operator policy. Sequentially, HeNB chooses the anticipated response to the observed strategy of the leader. In other words, in the procedure of the proposed SBSG, given the initial selection of MeNB for unit-bandwidth price, each HeNB chooses its best decision (b_i^*, N_i^*) from its 2D strategy space such that its utility function is maximised. Then, MeNB performs backward induction according to the reactions of HeNBs along with the limited shared bandwidth constraint and imposes new unit-bandwidth price P_i^* to HeNB i that maximises its revenue. This bargaining procedure continues until equilibrium achievement. Mathematically, an action profile should be found that satisfy (9)

$$\begin{aligned} \underline{P}^* &= \mathrm{BR}_{\mathrm{MeNB}}(\underline{b}^*, \underline{N}^*), \\ (b_i^*, N_i^*) &= \mathrm{BR}_{\mathrm{HeNB}_i}(P_i^*), \quad i = 1, 2, \ldots, F. \end{aligned} \quad (9)$$

In which, $\underline{P}^* = [P_1^*, P_2^*, \ldots, P_\mathrm{F}^*]$, $\underline{b}^* = [b_1^*, b_2^*, \ldots, b_\mathrm{F}^*]$, and similarly $\underline{N}^* = [N_1^*, N_2^*, \ldots, N_\mathrm{F}^*]$. Also $\mathrm{BR}_{\mathrm{HeNB}_i}$ is the best response of HeNB i given that the strategy P_i^* is made by MeNB. Similarly, best response of MeNB knowing the chosen strategies of HeNBs is represented by $\mathrm{BR}_{\mathrm{MeNB}}$. Consequently, from the HeNB point of view, by maximising (6), and solving the following equations

$$\frac{\partial U^i_{\mathrm{HeNB}}}{\partial b_i} = r_i \frac{I_i}{I_i + N_i} + \frac{1}{2\sqrt{b_i - D_i}} - \frac{P_i}{\delta\left(1 + (N_i/I_i)\right)^2} = 0 \quad (10)$$

$$\frac{\partial U^i_{\mathrm{HeNB}}}{\partial N_i} = -r_i \frac{I_i}{(I_i + N_i)^2} b_i + \frac{2 P_i b_i}{\delta\left(1 + (N_i/I_i)\right)^3} = 0 \quad (11)$$

The best response of HeNB i is given by the following equation

$$b_i^* = \left(\frac{\delta(I_i + N_i)^2}{2I_i(P_i I_i - r_i \delta(I_i + N_i))} \right)^2 + D_i \tag{12}$$

$$N_i^* = \frac{2P_i I_i}{\delta r_i} - I_i \tag{13}$$

Also from the MeNB point of view, applying the backward induction, we substitute (12) and (13) in MeNB objective function shown in (7). Then, the best response of MeNB is deduced by solving the optimisation problem shown in (8) by applying Karush–Kuhn–Tucker (KKT) conditions. The Lagrangian function of (8) is

$$U_{\mathrm{MeNB}}(\underline{P}, \underline{b}^*, \underline{N}^*) = \sum_{i=1}^{\mathrm{F}} \left(P_i D_i + \frac{P_i(b_i^* - D_i)}{\delta(1 + (N_i^*/I_i))^2} \right) - \lambda \left(\sum_{i=1}^{\mathrm{F}} b_i^* - B \right) \tag{14}$$

where λ is the Lagrange multiplier for bandwidth constraint in (8). Therefore, the best unit-bandwidth price imposed to ith HeNB is obtained as

$$P_i^* = \frac{\delta^2 r_i^4 (D_i + (1/\delta r_i^2))}{8\lambda} \tag{15}$$

In which, the Lagrange multiplier is calculated using the sub-gradient method [26]. So, the proposed bandwidth sharing game can be designed as Algorithm 1 (see Fig. 2) which iteratively continues until convergence.

Algorithm 1

Require: α: the step size

λ^0: the initial Lagrange multiplier

b_i^0: initial bandwidth allocated to each HeNB

δ: discount rate

1: **Decision making at MeNB**

Given: $\underline{b}^k, \underline{N}^k$

use the sub-gradient method to update Lagrange multiplier by:

$$\lambda^{k+1} = \lambda^k - \alpha \left(B - \sum_{i=1}^{F} b_i^k \right)$$

update imposed price to HeNB i using:

$$P_i^{k+1} = \frac{\delta^2 r_i^4 \left(D_i + \frac{1}{\delta r_i^2} \right)}{8\lambda^{k+1}}$$

2: **Decision making at each HeNB**

Given: P_i^t

update the number of victim macro-users to serve using:

$$N_i^{t+1} = \max \left(0, \frac{2P_i^t I_i}{\delta r_i} - I_i \right)$$

update the bandwidth request using:

$$b_i^{t+1} = \max \left(0, \left(\frac{\delta(I_i + N_i^{t+1})^2}{2I_i \left(p_i^t I_i - \delta r_i (I_i + N_i^{t+1}) \right)} \right)^2 + D_i \right)$$

Fig. 2 *Iterative decision making in MeNB and each HeNB using the proposed SBSG*

4 Analysis of the proposed SBSG

The proposed SBSG has important features which are discussed in this section. We first show that a unique NE exists for the proposed game and its convergence is guaranteed by making decisions according to the best response strategies calculated in last section. Then, the Pareto optimality of this NE is proved.

4.1 Existence and uniqueness of NE

The existence of an NE for a normal Stackelberg game depends on appropriate definition of the payoff functions of the sub-game. Mathematically, in our game a strategy set $\{\underline{P}^*, (\underline{b}^*, \underline{N}^*)\}$ is defined as a NE if we have

$$\begin{aligned} & U_{\mathrm{MeNB}}(\underline{P}^*, \underline{b}^*, \underline{N}^*) \geq U_{\mathrm{MeNB}}(\underline{P}, \underline{b}^*, \underline{N}^*) \quad \forall \underline{P} \\ & U_{\mathrm{HeNB}}^i(P_i^*, b_i^*, N_i^*) \geq U_{\mathrm{HeNB}}^i(P_i^*, b_i, N_i) \quad \forall (b_i, N_i), i = 1, \ldots, F \end{aligned} \tag{16}$$

Moreover, according to Kakutani fixed point theorem, a game should meet two following conditions to have a pure NE [25]:

(i) The strategy space of each player should be non-empty, closed, bounded, and also a convex set.

The utility function of each player should be not only continuous in strategy space, but also should be a quasi-concave function of player's strategy.

Proposition 1: The proposed SBSG has unique NE.

Proof: Since the strategy space of each HeNB is a non-empty 2D subset of $[0\ B] \times [0\ M_i]$ and the strategy space of MeNB is all of the possible positive prices, the first condition is obviously satisfied. Also, the utility functions are continuous in strategy space. So, existence of the NE depends on the quasi-concavity of utility functions with respect to their own strategies. By calculating the Hessian of the payoff function in (6), we obtain

$$\nabla^2_{(b_i, N_i)} U_{\mathrm{HeNB}}^i = \begin{bmatrix} -\dfrac{1}{4\sqrt{(b_i - D_i)^3}} & A \\ A & -\dfrac{2P_i I_i^2 (b_i - D_i)}{(I_i + N_i)^4} \end{bmatrix} \tag{17}$$

where $A = -r_i(I_i/(I_i + N_i)^2) + (2P_i I_i/(I_i + N_i)^3)$. According to (11), $A = 0$ due to following the best response at each side. Therefore, since $(\partial^2 U_{\mathrm{HeNB}}^i / \partial b_i^2) < 0$ and $|\nabla^2 U_{\mathrm{HeNB}}^i| > 0$, the Hessian is negative definite and the proposed utility function is quasi-concave in its 2D strategy space. On the other hand, by calculating the second-order partial derivative of (7) with respect to P_i, we obtain

$$\frac{\partial^2 U_{\mathrm{MeNB}}}{\partial P_i^2} = -\frac{8\lambda}{r_i^4} < 0 \tag{18}$$

So, the payoff function of MeNB is strictly concave. Satisfying the aforementioned conditions indicates that not only the proposed game has at least one pure NE but it is also unique due to its strict concavity. □

In the next section, we analytically prove that the iterative decision making at each side according to obtained best strategy profile converges to this unique NE.

4.2 Algorithm convergence

So far, we have proved that there exists a unique NE for the proposed SBSG. However, since the decisions in the MeNB and HeNBs are made in a distributed manner and asynchronously in our algorithm, an important question which is raised is about the convergence behaviour of the proposed iterative scheme to the obtained NE. Using asynchronous convergence theorem established in [27], Yate in [28] shows that if the utility-based iterative algorithm determined as $\underline{X}^{(k+1)} = f(\underline{X}^{(k)})$ in which $\boldsymbol{X}$ is the decision vector, has a fixed point and also the updating function f satisfies the following conditions, function f will be a standard function and converges to the unique fixed point.

- Positivity, i.e. $f(\underline{X}^{(k)}) \geq 0$.
- Monotonicity which means $\underline{X}_1^{(k)} \geq \underline{X}_2^{(k)} \Rightarrow f(\underline{X}_1^{(k)}) \geq f(\underline{X}_2^{(k)})$.
- Scalability in the sense that $\rho f(\underline{X}^{(k)}) \geq f(\rho \underline{X}^{(k)})\ \forall \rho \geq 1$.

Proposition 2: The proposed SBSG asynchronously converges to the unique NE.

Proof: Considering the proposed game and iterative bandwidth allocation process shown in Algorithm 1 (Fig. 2), the first condition for convergence of our algorithm is apparently satisfied. By considering the updating functions of b_i^*, N_i^*, and P_i^* as

$$\begin{aligned} b_i^{(k+1)} &= f_i(b_i^{(k)}), \\ N_i^{(k+1)} &= g_i(N_i^{(k)}), \\ P_i^{(k+1)} &= h_i(P_i^{(k)}). \end{aligned} \tag{19}$$

Calculated updating functions have monotonicity decreasing property if their first derivative respect to Lagrange multiplier become negative. Since $(\partial h_i/\partial \lambda) < 0$, $(\partial f_i/\partial h_i) > 0$ and $(\partial g_i/\partial h_i) > 0$, easily we can conclude that the iterative proposed scheme satisfies the second necessity condition, as well.
Eventually, by multiplying the allocated bandwidth to each HeNB by factor $\rho\ \geq 1$ we have

$$\lambda(\underline{b}^{(k)}) \leq \lambda(\rho \underline{b}^{(k)}) \tag{20}$$

where $\underline{b}^{(k)} = \{b_i^{(k)} | 1 \leq i \leq F\}$. In this case, evidently we have

$$\begin{aligned} h_i(\rho \underline{b}^{(k)}) &\leq h_i(\underline{b}^{(k)}) \leq \rho h_i(\underline{b}^{(k)}), \\ f_i(\rho \underline{b}^{(k)}) &\leq f_i(\underline{b}^{(k)}) \leq \rho f_i(\underline{b}^{(k)}), \\ g_i(\rho \underline{b}^{(k)}) &\leq g_i(\underline{b}^{(k)}) \leq \rho g_i(\underline{b}^{(k)}). \end{aligned} \tag{21}$$

Conclusively, the convergence of the proposed iterative algorithm to the unique NE is guaranteed. □

4.3 Pareto optimality

Pareto optimality examines the efficiency of game theoretical algorithms. By Pareto optimality, we mean that it is impossible to improve the social benefit of the network and provide a situation for one of the players to have extra profit without any reduction in the utility of other players [29]. In the following, we prove that the proposed scheme has the Pareto-efficient outcome. That is from the HeNBs point of view, no other strategy profile can enhance the throughput per femto-user of one of them without any raise in its cost or in the number of supported victims or any reduction in the utility of other HeNBs or MeNB. From the MeNB point of view, on the other side, it is impossible to earn more income with at least the same number of supported victims, or offload more victims without any reduction in obtained revenue.

Proposition 3: The NE of the proposed SBSG is Pareto optimal.

Proof: By contradiction suppose not. In the sense that it is possible to find a better strategy profile in which without any reduction in the utility of HeNBs and MeNB, their total utility will get improved. Thus, more social benefits will be yielded. Namely, there exists a strategy profile $\{\underline{P}^\circ, (\underline{b}^\circ, \underline{N}^\circ)\}$ where $\underline{P}^\circ = [P_1^\circ, P_2^\circ, \ldots, P_F^\circ]$, $\underline{b}^\circ = [b_1^\circ, b_2^\circ \ldots, b_F^\circ]$, and $\underline{N}^\circ = [N_1^\circ, N_2^\circ, \ldots, N_F^\circ]$ which is preferred to the obtained NE, $\{\underline{P}^*, (\underline{b}^*, \underline{N}^*)\}$, by one of the corresponding entities and is not less preferred by the others. According to the objective functions defined in (6) and (7), the total utility of MeNB and HeNBs will be

$$\begin{aligned} U_{\text{total}} &= \sum_{i=1}^{\text{F}} U_{\text{HeNB}_i} + U_{\text{MeNB}} \\ &= \sum_{i=1}^{\text{F}} r_i D_i + r_i \frac{I_i}{I_i + N_i}(b_i - D_i) + \sqrt{b_i - D_i} \end{aligned} \tag{22}$$

In this case, we have

$$\begin{aligned} U_{\text{total}}(\underline{P}^\circ, \underline{b}^\circ, \underline{N}^\circ) > U_{\text{total}}&(\underline{P}^*, \underline{b}^*, \underline{N}^*) \\ \text{s.t.} \quad &\sum_{i=1}^{\text{F}} b_i^\circ \leq B, \\ &\sum_{i=1}^{\text{F}} N_i^\circ \geq \sum_{i=1}^{\text{F}} N_i^*. \end{aligned} \tag{23}$$

The constraints in (23) should be satisfied to avoid any degradation in MeNB' s profit and make it at least indifferent between these strategy profiles. Equivalently, at least one of (24) and (25) should be held

$$\begin{aligned} \exists i \quad \text{s.t.} \quad & U_{\text{HeNB}_i}(P_i^\circ, b_i^\circ, N_i^\circ) > U_{\text{HeNB}_i}(P_i^*, b_i^*, N_i^*), \\ & U_{\text{HeNB}_j}(P_j^\circ, b_j^\circ, N_j^\circ) \geq U_{\text{HeNB}_j}(P_j^*, b_j^*, N_j^*) \quad \forall j \neq i, \\ & U_{\text{MeNB}}(\underline{P}^\circ, \underline{b}^\circ, \underline{N}^\circ) \geq U_{\text{MeNB}}(\underline{P}^*, \underline{b}^*, \underline{N}^*). \end{aligned} \tag{24}$$

$$\begin{aligned} & U_{\text{MeNB}}(\underline{P}^\circ, \underline{b}^\circ, \underline{N}^\circ) > U_{\text{MeNB}}(\underline{P}^*, \underline{b}^*, \underline{N}^*), \\ & U_{\text{HeNB}_i}(P_i^\circ, b_i^\circ, N_i^\circ) \geq U_{\text{HeNB}_i}(P_i^*, b_i^*, N_i^*) \quad \forall i. \end{aligned} \tag{25}$$

First of all, it should be noticed that the inequality constraint in (8) is active. The reason is that with the fixed number of supported victims and unit-bandwidth price, the utility of HeNBs are strictly increasing regards to shared bandwidth. Thus, it is impossible to have residential and not allocated supply bandwidth in equilibrium point. So we have

$$\sum_{i=1}^{F} b_i^\circ = \sum_{i=1}^{F} b_i^* = B \tag{26}$$

From the HeNBs point of view, holding (24) is only possible when HeNB i shares $b_i^\circ > b_i^*$ with MeNB and supports $N_i^\circ \leq N_i^*$, or shares equal amount of spectrum and offloads $N_i^\circ < N_i^*$ for MeNB. According to (26) a raise in the allocated bandwidth to HeNB i leads to a reduction in the allocated bandwidth to another HeNB, i.e. HeNB m. So, if $b_i^\circ = b_i^* + \beta$ then $b_m^\circ = b_m^* - \beta$. Since the utility of HeNB m should not be degraded, less victims will be attached to this HeNB and $N_m^\circ < N_m^*$. Satisfying the last constraint in (23) and making other entities at least indifferent, HeNB i has to increase its offloaded activity. Hence, contrary to the supposition, $N_i^\circ > N_i^*$. Thus, the only possible case to improve the utility of HeNB i is $b_i^\circ = b_i^*$ and $N_i^\circ < N_i^*$ which certainly leads to a reduction in the utility of at least another HeNB or the profit of MeNB. Consequently, there is no chance for none of HeNBs to

Table 2 Final results at the end of the game

	HeNB 1	HeNB 2	HeNB 3	HeNB 4
minimum bandwidth demand	5	3	4	2
allocated bandwidth with SBSG	7.1051	4.2257	5.6047	2.9545
number of served victim macro-users	4	2	3	1
discounting factor	5.44	2.78	4	1.78
unit bandwidth price	3.3821	3.0339	3.2780	2.7724

enhance its throughput per femto-user unless it supports more victims. Consequently, (24) could not be held.

From the MeNB point of view, it is obvious that a little raise in the obtained revenue is only possible when the applied discount is decreased and less victims are offloaded to HeNBs. Namely, if $\mathrm{Rev}^{\circ}_{\mathrm{MeNB}}$ and $\mathrm{Rev}^{*}_{\mathrm{MeNB}}$ denote the revenue of MeNB by making decisions according to the new strategy profile and our scheme, respectively, then $\mathrm{Rev}^{\circ}_{\mathrm{MeNB}} = \mathrm{Rev}^{*}_{\mathrm{MeNB}} + \epsilon$ inevitably leads to $\sum_{i=1}^{F} N_i^{\circ} = \sum_{i=1}^{F} N_i^{*} - \epsilon'$. Vice versa, supporting more victims by HeNBs leads to capture more discount and certainly reduces the revenue of MeNB. Therefore, there is no opportunity for MeNB to earn more income and also offload the same or more number of victims to HeNBs. Furthermore, increasing the revenue of MeNB exactly leads to higher imposed cost to HeNBs which apparently reduces their utilities. Therefore, holding (25) is also impossible. □

So, the NE of our SBSG is Pareto optimal in the sense that the total utility of players, HeNBs and MeNB, and in turn the social benefit are maximised.

5 Simulation results

In this section, we show the results obtained by simulating the proposed scheme in a HetNet in which one centre MeNB as the provider shares $B = 20$ MHz bandwidth with four HeNBs located at the middle of four autonomous femto-cells with radius of $R_{\mathrm{F}} = 10$ m. Femto-cells are assumed to be distributed uniformly in a macro-cell with radius of $R_{\mathrm{M}} = 500$ m. Also, MeNB serves $M_i = 5$ victim macro-users which are located nearby each HeNB who serves $I_i = 3$ femto-users. Meanwhile, assuming path loss model, channel gains can be defined as $h_i^m = K(d_i^m)^{-n}$ in which d_i^m is the distance between user i as receivers and eNB m. Path loss exponent n is set to 3.7 in macro-cell and 2 in femto-cell. The value of constant K is varied between 38.5 and 53.5 dB in femto-cell for femto-user and victim macro-user, respectively, while it is fixed to 137.4 dB in macro cell [30]. Furthermore, noise variance, P^{F} and P^{M} are considered to be −174 dBm, 125 mW, and 40 W [30]. The results are the average of 1000 runs where the corresponding 95% confidence intervals are also reported.

To evaluate the proposed SBSG, we consider three simulation setups. The first one concerns with the properties of the proposed SBSG including the equilibrium properties, convergence behaviour, signalling overhead, and the effect of variable parameters in system performance. In the second setup, we compare our scheme with the proposed Stackelberg game presented in [11] and also the simple scenario of proportional bandwidth allocation. In both of these simulation setups, the value of δ is considered to be 1. Finally, the simulation result is provided to show how MeNB is able to compromise its revenue and the number of its attached victims by appropriate selection of δ.

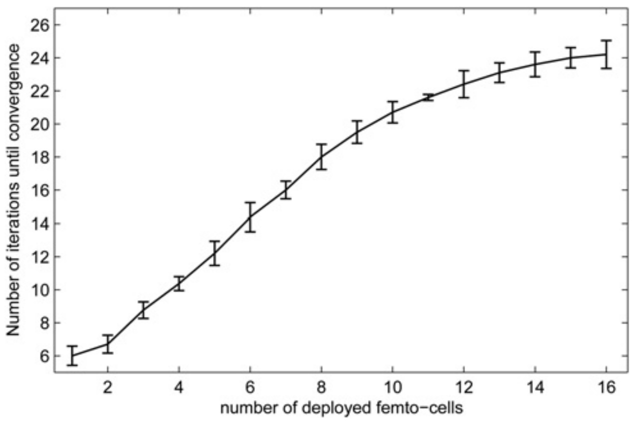

Fig. 3 *Proposed SBSG convergence behaviour*

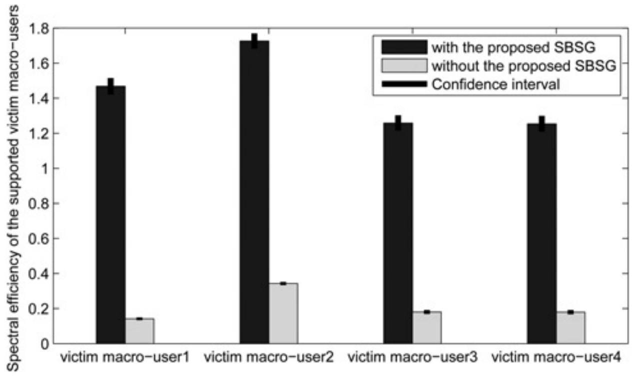

Fig. 4 *Improvement obtained by the proposed SBSG in spectral efficiency of the supported victim macro-users*

5.1 Properties of the proposed SBSG

The final operating parameters on which HeNBs and MeNB agreed at equilibrium are shown in Table 2. Correspondingly, it expresses how the proposed SBSG allocates bandwidth to each HeNB related to the number of victim macro-users ready to serve, its willingness to pay, and its minimum bandwidth demand. As it is demonstrated, the more the minimum bandwidth demand is requested by one HeNB, the higher unit-bandwidth price is imposed from MeNB to maximise its revenue. However, the HeNB with greater minimum required bandwidth supports more victim macro-users and gets more discount to share more bandwidth with MeNB. So, the price of spectrum is noticeably lessened for this HeNB. Also, Table 2 shows that the difference between allocated bandwidth and the demand of the HeNB who attaches more victims is more than the others. Consequently, the HeNB with highest minimum bandwidth demand benefits the most from the proposed SBSG at the price of offloading more number of victims for MeNB. Naturally, we can say that this HeNB valuates the bandwidth the most. Therefore, according to spectrum trading concepts the proposed SBSG can properly maximise the social welfare.

To give insight on the impact of signalling overhead in system performance, we evaluate the convergence speed of our algorithm versus the number of femto-cells deployment via Fig. 3. It is first observed that the competition between MeNB and a few number of autonomous HeNBs reaches to equilibrium rapidly after <10 iterations. In addition, although the agreement between MeNB and HeNBs will be obtained slower as the number of femto-cells increases, the degree of effectiveness will be diminished in dense femto-cells deployment. For example, by deploying 16 autonomous femto-cells, the convergence will be occurred after less than only 30 iterations. Conclusively, the proposed scheme imposes only the limited amount of signalling overhead to the system. It should be noted that the total minimum demand of HeNBs is assumed to be fixed in this scenario to have fair comparison.

In addition, the improvement of system performance obtained by applying spectrum discounting proposed in SBSG is depicted in Fig. 4. In this regard, we compare the QoS of victim macro-users when they are served by MeNB itself with the scenario that HeNB attaches them by applying CRE. It is visible in Fig. 4 that discount pricing strategy of the proposed SBSG can considerably improve the system performance on serving victim macro-users by

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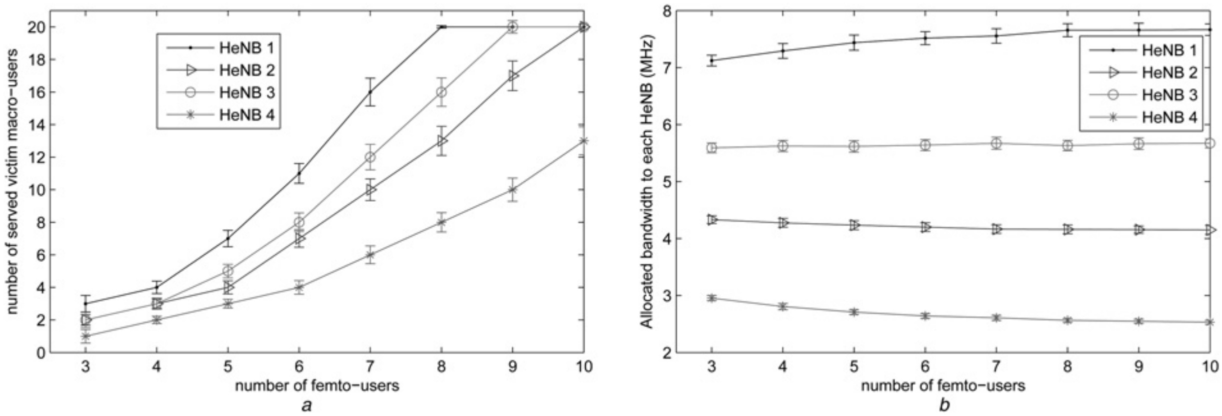

Fig. 5 *Effect of the number of femto-users on the system performance*

a Number of supported victim macro-users by HeNBs containing different number of femto-users
b Allocated bandwidth to HeNBs served different number of femto-users

load-balancing technique. On the other hand, the proposed SBSG prepares the opportunity for macro-users who experience strong interference to enhance their spectral efficiency by switching to HeNBs (probably with biasing technique) and getting their services from them instead of MeNB.

As the last result of the first simulation setup, Fig. 5 examines the effect of number of femto-users on the performance of the proposed SBSG. In this simulation it is assumed that 20 victim macro-users are located in the vicinity of each femto-cell. It is verified from Fig. 5 that increasing the number of femto-users of each HeNB leads to stronger competition between HeNBs. Therefore, each of them should serve more victim macro-users to provide better throughput per each of its private user. As it is depicted in Fig. 5, the HeNB which needs more deterministic minimum bandwidth, enables itself to obtain more discount and get larger bandwidth by accepting to afford services of more victim macro-users. However, the one with less minimum bandwidth demand has less motivation to support victims. The reason is that when the number of private users of all HeNBs is the same, the HeNB with higher minimum bandwidth demand may be supposed to provide more complicated applications for femto-users. So, the spectrum is more valuable for this HeNB and accordingly it needs to share larger bandwidth with MeNB to afford high data-rate requirements of its users.

5.2 Comparison with other mechanisms

To evaluate the amount of improvement caused by making the proposed SBSG in use against a simple proportional approach, we consider a scenario in which total available bandwidth, B, is allocated to HeNBs proportional to their minimum demands and compare its performance with our algorithm. The average throughput per femto-user of each HeNB in each of these scenarios is illustrated in Fig. 6. Since the average throughput value is directly related to the amount of shared spectrum, it can be easily deduced that in the proposed SBSG more bandwidth is assigned to the HeNB who needs it more and accepts to serve more victim macro-users. Due to the limited amount of total available bandwidth to share with HeNBs, increasing the bandwidth assigned to each one leads to inevitable decrease in bandwidth allocated to some others.

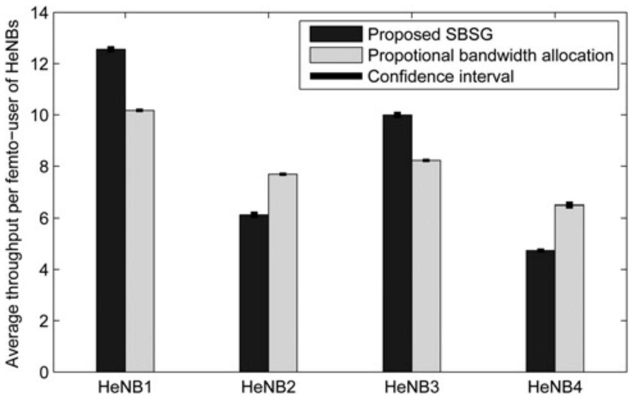

Fig. 6 *Improvement obtained by the proposed SBSG in average throughput per femto-user of the HeNBs*

As the next evaluation, we investigate the effect of the location of HeNBs on the system performance for this purpose, we determine three HeNBs with equal target bandwidth located at (1/3)R, (1/2)R, and (3/4)R. According to Fig. 7 the more the distance between HeNB and MeNB are, the more interest HeNB has to support victim macro-users in order to share more bandwidth with MeNB. The reason is that farther macro-users will harm more from strong interference of HeNB. So, their QoS can improve noticeably by switching to HeNB. In other side, the farther HeNBs can share more bandwidth by offloading more victim macro-users for MeNB. Meanwhile, in comparison with Stackelberg game presented in [11], the proposed SBSG allocates greater amount of spectrum to the farthest HeNB as it is depicted in Fig. 7*a*.

The last issue to be considered in our assessments is evaluating the effect of number of HeNBs in the number of offloaded victim macro-users per HeNB which is shown in Fig. 8. It is observed that as the number of HeNBs increases, less victim macro-users will be attached by each one which is due to obtaining less possible bandwidth. Reasonably, when each HeNB can share less bandwidth with MeNB, its interest to serve victim macro-users will decrease as well. Furthermore, in Fig. 8 the number of served victim macro-users per HeNB obtained by the proposed SBSG is compared with the one proposed in [11]. In order to be fair, in this scenario equal minimum demand is considered for all HeNBs. The reason behind lower total victims that is attached to HeNBs in the proposed SBSG compared with [11] is related to the revenue priority of MeNB against its victims by setting the value of δ to 1. It should be noticed that while supporting victims is the goal for MeNB, at the same time it is the cost for HeNBs and their motivations to attach victims correspond to the amount of discount applied by MeNB. However, putting discount strategy in use, MeNB has the conflicting objectives of maximising the revenue and minimising the number of victims. In the next simulation setup, we show that how MeNB can adapt its discount strategy to manage the tradeoff between its revenue and the number of its victims.

5.3 Compromising the revenue of MeNB and the number of its supported victims

Naturally, MeNB can apply more discount to inspire more interest in HeNBs to increase their offloading activities at the price of revenue

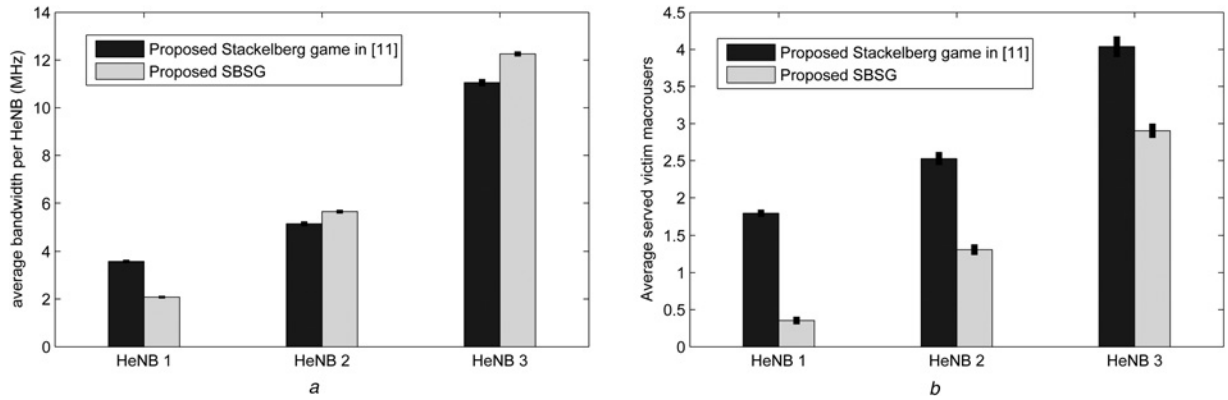

Fig. 7 *Effect of the location of HeNB on the system performance*

a Allocated bandwidth to HeNBs in different locations
b Number of victim macro-users offloaded by HeNBs in different locations

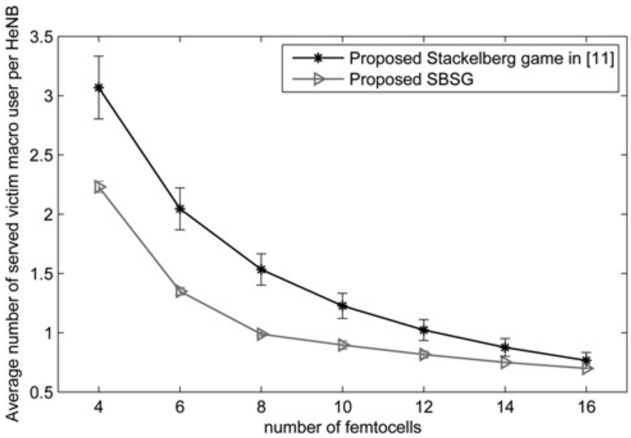

Fig. 8 *Effect of the number of deployed femto-cells on the number of offloaded victims*

reduction. Accordingly, in this section we evaluate the effect of more discounting from MeNB by setting higher value for δ on the system performance. As it is visible in Fig. 9, the well-defined variable δ provides the opportunity for MeNB to decide about its priorities and choose among Pareto optimal operating points by adapting the value of δ. On the other hand, HeNBs who need to balance the profit of attaching victims and spectrum expenditure, tend to offload more victim macro-users when higher value is applied for δ and in turn their payments are decreased. Moreover, as it is

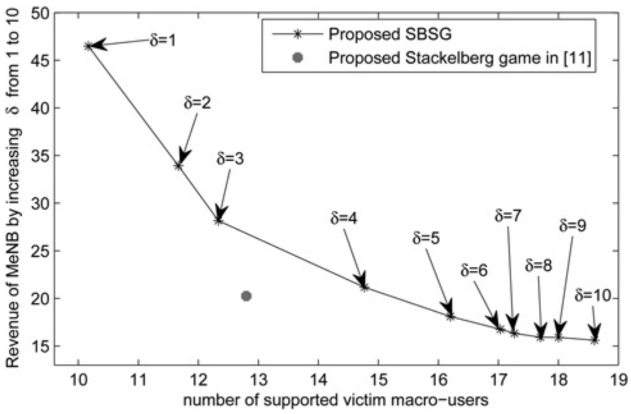

Fig. 9 *Effect of the amount of discounting on the number of supported victims*

depicted in Fig. 9, the number of supported victims proposed by Hamouda *et al.* [11] can be considered as a special case of the presented mechanism in this paper by appropriate selection of δ. In particular, with four closed femto-cells deployment and setting the value of δ between 3 and 4 in our scheme, MeNB is able to offload the equal total number of victims to HeNBs as suggested in [11], while earning more income.

6 Conclusion

The main challenge of a HetNet with closed access and autonomous femto-cells is enhancing the throughput of femto-users with no QoS degradation for macro-users located nearby femto-cells. Properly applying CRE technique and efficient spectrum trading mechanism can help a HetNet to achieve these goals. Accordingly, in this paper an SBSG incorporated with CRE technique is presented for a self-organised HetNet in which the actions of closed femto-cells are performed autonomously. Thus, in the proposed mechanism, each side of the game, MeNB and HeNB, has separate authority domain and attempts to selfishly maximise its utility function. MeNB tries to not only maximise its revenue but also offload the most number of victims by appropriate spectrum discounting. While from the HeNB point of view, sharing maximum possible bandwidth to provide higher data-rate per femto-user by paying minimum cost are the goals. Considering these conflicting objectives and exploiting suitable discounting strategy, our scheme maximises the social welfare in the whole network. Simulation results show that the proposed game is converged to the Pareto-efficient NE point. Since in this paper we focus our attention in bandwidth sharing problem, energy efficiency, and power consumption which are important factors in resource allocation problems are not taken into account in our model. Consequently, one future goal is to provide an efficient power allocation or in better case, jointly power and bandwidth allocation for HetNets.

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