The 2013 CSI International symposium on
Computer Networks and Distributed Systems (CNDS)

Weighted Consensus-Based Cooperative Spectrum Sensing with Learning Automata (LA) in Cognitive Radio Networks

Milad Soltani and Abdorasoul Ghasemi

Faculty of Electrical and Computer Eng.
K.N. Toosi University of Technology, Tehran, Iran
msoltani@ee.kntu.ac.ir
arghasemi@eetd.kntu.ac.ir

Abstract. In traditional cooperative spectrum sensing such as OR-rule or AND-rule, secondary user (SU) must maintain coordination based on a fusion center. In this paper, we propose the weighted average consensus, based on Learning Automata(LA), for fully distributed cooperative spectrum sensing without fusion center. At the first stage of the proposed scheme, each SU makes measurement about presence of primary user (PU) at the beginning of each time slot, then communicates with local neighbors to exchange information to make the final decision and update its weight using a LA based algorithm. Simulation results show that the proposed scheme has better performance than the non weighted consensus and existing weighted consensus scheme. Also, the convergence time of the proposed scheme is less than the existing weighted consensus and almost equal to non weighted consensus scheme.

Keywords: Cognitive radio, Cooperative spectrum sensing, Consensus algorithms, Learning automata

1 Introduction

Cognitive radio (CR) [1], allows unlicensed users to operate in licensed spectrum bands [2]. Due to scarcity of available spectrum and notice that the assigned spectrum remains under utilized in wireless communication [2], CR technology can help to overcome this problem. In this technology, to protect the licensed PUs, each SU is equipped with a spectrum sensing module which enables it to detect the presence of PUs. Recent research shows cooperative spectrum sensing in which a group of SUs execute spectrum sensing by cooperation, has many advantages [3]-[5]. By cooperative sensing, SUs share their sensing information for making common decisions which is more precise than the individual decisions [6]. Most studies on cooperative spectrum sensing use centralized approaches such as AND-rule, OR-rule or K-out-of-N. However, according to [7] deploying the fusion center could be very difficult and not scalable in practical situation. Thus, consensus-based scheme is proposed in [2] for cooperative spectrum sensing to be distributed, scalable and more implementable. However, average consensus converges to the average of the initial measurement of the SUs which is less effective under heavy noise and fading channel. Weighted average consensus is introduced in [8] to solve these disadvantages. In [8], SUs exchange their decisions with their neighbors and weight the information with estimated average signal-to-noise ratio (SNR). However, in [8] we need prior knowledge of the fading channel model, which it means higher complexity and is not suitable for general situation. Moreover, the SUs' weights don't have any upper bound and we'll show this reduces the speed of convergence.

In this paper, we propose weighted gain combining based on LA (WGCLA), without prior knowledge of primary system and channel model. SUs compare individual decision with consensus decision to understand if individual decision is correct or not. Then they update their weights. If the i^{th} SU has higher confidence about its own measurement, increase its weight, which makes the rely less on the other SUs. On the other hand, a SU with lower confidence about its own measurement, decrease its weight and relies more on the information from the network. The numerical results show improvement compared to the equal gain combining [2] (EGC), and weighted gain combining [8] (WGC).

The rest of this paper is organized as follows. Section II gives a system model. In section III, the proposed weighted consensus is explained. Section IV explains the numerical results of the proposed scheme and make comparison with the existing approach. Finally, we conclude this study in Section V.

2 System Model

In this paper, we consider the energy detector model which is simple to implement and doesn't need prior knowledge of the primary system. For each SU, in the consensus-based spectrum sensing, received signal is modeled as

$$\mathbf{x}(t) = \begin{cases} n(t) & \mathbf{H}_0 \\ h.\mathbf{s}(t) + n(t) & \mathbf{H}_1 \end{cases} \tag{1}$$

$\mathbf{s}$(t) is the signal from PU, h is the channel gain from the PU's transmitter to SU's receiver, $n(t)$ is the additive white gaussian noise and two hypothesis $\mathbf{H}_1$ and $\mathbf{H}_0$ represent the presence and absence of the PU respectively. According to the work of [2], the output of the energy detector, Y has the following form:

$$\mathbf{Y} = \begin{cases} \chi^2_{2TW} & \mathbf{H}_0 \\ \chi^2_{2TW-2} + Y_e & \mathbf{H}_1 \end{cases} \tag{2}$$

Where χ^2_{2TW} and χ^2_{2TW-2} denote random quantities with central and non-central chi-square distributions, respectively, each with 2TW degrees of freedom and a non-centrality parameter of Y_e has an exponential distribution with parameter $2(\gamma+1)$. γ represents the average SNR of the fading channel. For simplicity we assume that the time-bandwidth product, TW, is an integer number, which is denoted by m.

SU network described by a standard graph model. G(V,E)[9] denotes an undirected graph of SUs(nodes), where set of nodes V={1,2,...,n} and edges

$E \subseteq V \times V$. The neighbors of node i are denoted by $N_i = \{j \in V : (i, j) \in E\}$. Notice the graph G must be connected which means any two different nodes in G are connected by a path.

The i^{th} SU's measurement in k+1 iteration is given by [9]:

$$x_i(k+1) = x_i(k) + \epsilon \sum_{j \in N_i} (x_j(k) - x_i(k)). \tag{3}$$

Which is re-written (2) in recursive form for convenience by [9]:

$$x(k+1) = Px(k). \tag{4}$$

P is the so-called perron matrix [2], defined as $P = I - \epsilon L$. L is the laplacian of the graph G and defined as $L = (l_{ij})_{n \times n}$[2]

$$l_{ij} = \begin{cases} |N_i| & if \quad j = i \\ -1 & if \quad j \in N_i \\ 0 & otherwize \end{cases} \tag{5}$$

In WGC the recursive scheme is given by [8]:

$$x_i(k+1) = x_i(k) + \frac{\epsilon}{\sigma_i} \sum_{j \in N_i} (x_j(k) - x_i(k)). \tag{6}$$

$\sigma_i \geq 1$ is the weighting ratio. ϵ is the step size and for peremptory convergence should be selected based on following lemmas. Each SU can only choose self weight. Whatever σ coefficient is much larger, the local measurement become more important. In (6), perron matrix defined as $P = I - \epsilon(diag(\Sigma))^{-1}L$[8], wherein $\Sigma = \{\sigma_1, \ldots, \sigma_n\}$.

Lemma 1: If G be a undirected graph with n SUs and $\Delta = \max_i |N_i|$. Then, the perron matrix with $0 \leq \epsilon \leq \frac{1}{\triangle}$ is a row stochastic and nonnegative matrix. Proof:

$$P = I - \epsilon(diag(\Sigma))^{-1}L \rightarrow P\mathbf{1} = I\mathbf{1} - \epsilon(diag(\Sigma))^{-1}L\mathbf{1} \tag{7}$$

According to the definition of graph Laplacian in (5), L always has a zero eigenvalue $\lambda_1 = 0$. This zero eigenvalues corresponds to the eigenvector $\mathbf{1} = \{1, ..., 1\}^T$. So:

$$L\mathbf{1} = 0 \times \mathbf{1} = \mathbf{0} \rightarrow P\mathbf{1} = \mathbf{1} \tag{8}$$

Which means the row sums of P is 1 and P is a row stochastic.

For convenience, the graph Laplacian is defined as $L = D - A$. Where $D = (d_{ij}) = diag(|N|_i)$ is the degree matrix of G and $A = (a_{ij})$ defined as follows:

$$a_{ij} = \begin{cases} 1, & j \epsilon N_i \\ 0, & otherwise \end{cases} \tag{9}$$

So, we have:

$$P = I - \epsilon(diag(\Sigma))^{-1}L \rightarrow P = I - \epsilon(diag(\Sigma))^{-1}D + \epsilon(diag(\Sigma))^{-1}A \qquad (10)$$

The $\epsilon(diag(\Sigma))^{-1}A$ term is nonnegative. The $I - \epsilon(diag(\Sigma))^{-1}D$ is diagonal matrix with elements $1 - \frac{\epsilon}{\sigma_i}d_i$ for $i = 1, ..., n$. So:

$$1 - \frac{\epsilon}{\sigma_i}d_i \geq 1 - \frac{\epsilon}{\sigma_i}\Delta \geq 0 \qquad (11)$$

So P is nonnegative. According to [11], as long as the diagonal entries of P are strictly smaller than one, nonzero and at least one diagonal entry is strictly positive, then P will be primitive. So, with add $\epsilon \neq 0$ and $\epsilon \neq \frac{1}{\Delta}$ limitation, P is a primitive matrix.

Lemma 2: If G be a undirected graph with n SUs, $\max_i(|N_i|) = \Delta$ and SUs follow (6) with $0 < \epsilon < \frac{1}{\triangle}$. So $x(k)_{max}/x(k)_{min}$ is subtractive function, which means, the convergence of (6), independent of the initial values.

Proof: To show the convergence of (6) scheme, we have:

$$x(k+1) = (I - \epsilon(diag(\Sigma))^{-1}L)x(k) \rightarrow \qquad (12)$$

$$\begin{pmatrix} x_1(k+1) \\ \vdots \\ x_n(k+1) \end{pmatrix} = \left(\begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \ldots & 0 & 1 \end{pmatrix} - \epsilon \begin{pmatrix} 1/\sigma_1 & \cdots & 0 \\ \vdots & 1/\sigma_m & \vdots \\ 0 & \cdots & 1/\sigma_n \end{pmatrix} \begin{pmatrix} d_1 & 0or-1 & \cdots \\ \vdots & d_2 & 0or-1 \\ 0or-1 & \cdots & d_n \end{pmatrix} \right) \begin{pmatrix} x_1(k) \\ \vdots \\ x_n(k) \end{pmatrix}$$

Assuming x_i and x_j are minimum and maximum measurement of SUs respectively, so:

$$\begin{cases} x_i(k+1) = x_i(k) + \frac{\epsilon}{\sigma_i}(x_p(k) + \cdots + x_q(k) - d_i x_i(k)) \\ x_j(k+1) = x_j(k) + \frac{\epsilon}{\sigma_j}(x_w(k) + \cdots + x_z(k) - d_j x_j(k)) \end{cases} \qquad (13)$$

Where the number of $x_p(k) + \cdots + x_q(k)$ and $x_w(k) + \cdots + x_z(k)$ are d_i and d_j respectively, so:

$$x_p(k) + \cdots + x_q(k) - d_i x_i(k) \begin{cases} = 0, & x_i(k) = x_p(k) = \ldots = x_q(k) \\ > 0, & otherwise \end{cases} \qquad (14)$$

$$x_i(k+1) \begin{cases} = x_i(k), & x_i(k) = x_p(k) = \ldots = x_q(k) \\ > x_i(k), & otherwise \end{cases} \qquad (15)$$

$$x_w(k) + \cdots + x_z(k) - d_j x_j(k) \begin{cases} = 0, & x_j(k) = x_w(k) = \ldots = x_z(k) \\ < 0, & otherwise \end{cases} \qquad (16)$$

$$x_j(k+1) \begin{cases} = x_j(k), & x_j(k) = x_w(k) = \ldots = x_z(k) \\ < x_j(k), & otherwise, \end{cases} \qquad (17)$$

According to (15) and (17), x_j/x_i function is strictly descending unless all of SUs measurement are same and in this case, convergence has been achieved. If the SUs graph are jointly connected, all SUs converge to[8]:

$$x_i(k) \rightarrow x^* = \frac{\sum\limits_{i=1}^{n} \sigma_i x_i(0)}{\sum\limits_{i=1}^{n} \sigma_i} \;\; as \;\; k \rightarrow \infty. \tag{18}$$

The speed of reaching a consensus is very important too. Based on lemma 3, if σ coefficient is larger, the speed of convergence is decreases.

Lemma 3: Let G be a undirected graph with n SUs and follow (6) scheme, with increase of σ, the speed of reaching a consensus is decreases.

Proof: Let assume $\Delta x_i = x_i(k+1) - x_i(k)$. Based on (13):

$$|\Delta x_i| = \frac{\epsilon}{\sigma_i}(|x_p(k)+...+x_q(k)-d_i x_i(k)|) \rightarrow \frac{d|\Delta x_i|}{d\sigma_i} = -\frac{\epsilon}{\sigma_i^2}(|x_p(k)+...+x_q(k)-d_i x_i(k)|) \tag{19}$$

$$\frac{d|\Delta x_i|}{d\sigma_i} \begin{cases} = 0, & x_j(k) = x_w(k) = \ldots = x_z(k) \\ < 0, & otherwise, \end{cases} \tag{20}$$

In this paper, we propose to set σ_i with LA, which means every SU measures a good estimate in last iteration make σ_i larger, On the other hand, a SU with worse estimate in last iteration, makes σ_i smaller, which is explain as follows.

3 Weighted Consensus Based on LA

In proposed scheme which is shown in fig. 1, we have two steps. In the first step, each SU makes measurement about presence of PU at the start of each time slot. According to the energy detector module, SUs select absence or presence of PU. In the second step, each SU communicates with neighbors to exchange its sensing information, after the convergence of the consensus algorithm, SUs compare individual decision with consensus decision to understand individual decision is correct or not. Therefore, the automation has two actions equivalent to choices that SUs selected, i.e., $a = \{a_1, a_2\}$, a_1 for correct individual decision and a_2 for incorrect. When the SU selects the correct individual decision, then the environment response is a reward, $X = 0$, and when the selected individual decision is incorrect, the response is a penalty, $X = 1$, so responses of the environment is $X \in \{0, 1\}$.

The learning automation of the SU selects the individual decision in time slot t, based on energy detector. After receiving the response from the environment, the learning automation uses a reinforcement scheme to update the probability vector $\mathbf{P}(t) = (P_1(t), P_2(t))$, which $P_1(t)$ is the probability of correct individual decision and $P_2(t)$ is the probability of incorrect individual decision. Then we use (15) to update the weighting factor σ_i as follow

$$\sigma_i = \begin{cases} 2P_1(t), & if\ P_1(t) \geq 0.5 \\ 1, & if\ P_1(t) < 0.5 \end{cases} \tag{21}$$

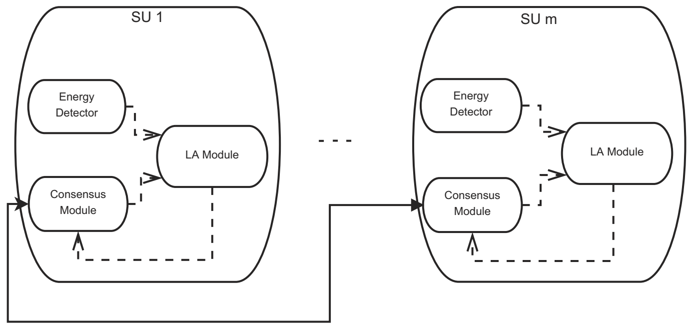

Fig. 1. Block diagram of WGCLA

Where, the saturation effect is required to ensure that $\sigma_i \geq 1$ for all i. In the proposed method, to update SUs weight in each time slot, we use a linear scheme which uses equations (22) and (23) for updating probability vector $\mathbf{P}(t+1)$. Where the SU selects decision i

$$\begin{aligned} P_j(t+1) &= P_j(t) - g_j(\mathbf{P}(t)) \text{ individual decision } i \text{ is correct at time slot } t \text{ , for all } j \neq i \\ P_j(t+1) &= P_j(t) + h_j(\mathbf{P}(t)) \text{ individual decision } i \text{ is incorrect at time slot } t \text{ , for all } j \neq i \end{aligned} \tag{22}$$

For preserving probability measure, we should have $\sum_{j=1}^{M} P_j(t) = 1$, so that

$$\begin{aligned} P_i(t+1) &= P_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{r} g_j(\mathbf{P}(t)) \text{ when individual decision } i \text{ is correct at time slot } t, \\ P_i(t+1) &= P_i(t) - \sum_{\substack{j=1 \\ j \neq i}}^{r} h_j(\mathbf{P}(t)) \text{ when individual decision } i \text{ is incorrect at time slot } t \end{aligned} \tag{23}$$

Where, $g_j(.)$ and $h_j(.)$ are the reward and penalty functions respectively which are continuous and nonnegative, satisfying (24) [10].

$$0 < g_j(P) < P_j,\ 0 < \sum_{\substack{j=1 \\ j \neq i}}^{M} [P_j + h_j(P)] < 1, i = 1, \ldots, M \tag{24}$$

Algorithm 1 Weighted consensus based on LA

Initialization:
Select α and β according to the LA scheme
$P_m(1) = \frac{1}{2}, m = 1, 2$
for $t = 1$ to T **do**
  $i =$ The selected decision based on energy detector
  if individual decision equal with consensus decision **then**
    $P_i(t+1) = P_i(t) + \alpha[1 - P_i(t)]$
    $P_j(t+1) = P_j(t) - \alpha P_j(t),\ j \neq i$
  else
    $P_i(t+1) = (1-\beta).P_i(t)$
    $P_j(t+1) = \frac{\beta}{M-1} + (1-\beta).P_j(t),\ j \neq i$
  end if
end for

This assumption ensures that all the components of $\mathbf{P}(t+1)$ remain in (0,1). In linear reinforcement schemes the reward and penalty functions are given by (25).

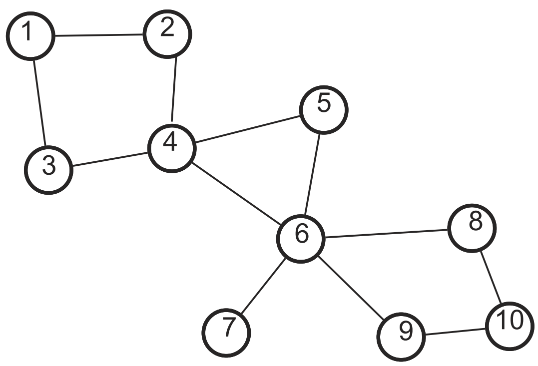

Fig. 2. Communication network of 10 SUs [2]

$$g_j(\mathbf{P}(t)) = \alpha P_j(t),\ \ h_j(\mathbf{P}(t)) = \frac{\beta}{M-1} P_j(t) \tag{25}$$

Where α and β are reward and penalty parameters and $0 < \alpha < 1, 0 \leq \beta < 1$ [10]. The pseudo code of the proposed scheme is presented in Algorithm 1.

In the initialization phase, we set the probability of two decisions for the first time slot to 0.5. This is because at this time slot, the automaton does not have any information about the accuracy of measurements in SUs. It will attain information about these accuracies of measurement in the consecutive time slots by interacting with the environment and updating vector $\mathbf{P}(t)$.

4 Simulation Results and Discussions

In this section, to compare simulation results with [2] and [8], the simulation results are reported.

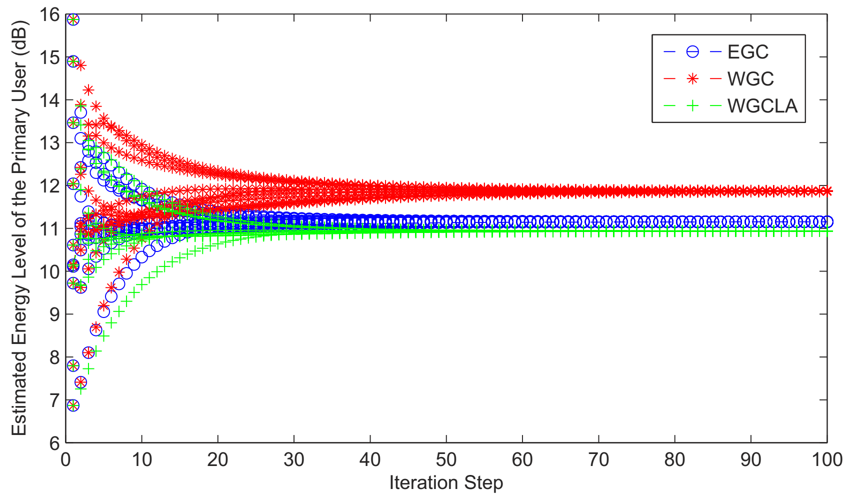

Fig. 3. Convergence of the network in idle time slot with a 10-node. $\varepsilon = 0.19$

4.1 Network Setup

We set up a similar scenario as in [2] and [8] shows in Fig. 2. In particular, 10 SUs cooperate with each other and form a communication graph as shown in Fig. 2. As in [2] we assume that all SUs are experiencing independent identically distributed (i.i.d) Rayleigh fading without spatial correlation. For PUs traffic pattern, we used poisson distribution with parameter $\lambda = 0.2$ for arrival rate and exponential distribution with parameter $\mu = 1$ for service time. In the sensing stage, we directly generate the output Y_i of every SUs energy detector individually from (2), with m $= 6$ at the selected center frequency and bandwidth of interest. Each SU sets initial value equal to the received signal and starts the measurement fusion using the proposed scheme with the step size $\epsilon = 0.19$ in (6). The final decision is made after the consensus result x^* is reached. In other words, when variation of SUs is less than a defined threshold, the final decision will be taken. We note that, in first iteration, all SUs have same weight which is the same as in [2]. After enough iterations SU's network learned which means SU has higher confidence about its own measurement, makes the iteration (6) rely less on the local information exchange.

Fig. 3 shows the convergence of the proposed algorithm, algorithm in [2] and [8] respectively in idle time slots. Because PU is absence, SUs must make a consensus in lower value. As we see the performance has been significantly improved by our propose algorithm.

Fig. 4 shows the convergence of the proposed algorithm, algorithm in [2] and [8] respectively under busy time slot and busy channel under fixed communication graph. In Fig .4, PU is present in the busy channel, as we see, because algorithm in [8] doesn't have any upper limit for its weighting factor σ^i,the convergence of the algorithm requires more than 400 iterations and algorithm in [2] converges in lower value that is not optimal for this situation.

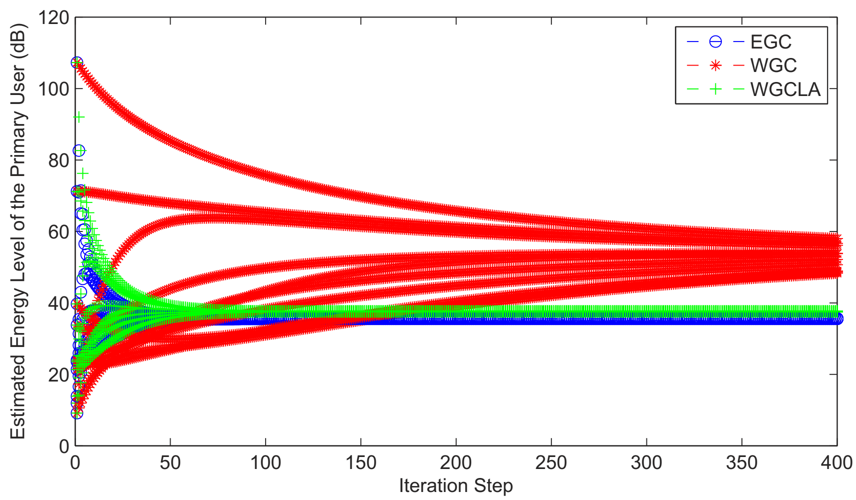

Fig. 4. Convergence of the network in busy time slot and busy channel with a 10-node. $\varepsilon = 0.19$

A high probability of missing detection (P_m), which increases the interference to primary users. On the other hand, a high probability of false alarm (P_f) will result in low spectrum utilization. In Fig. 5, for comparison with existing method, we consider P_m, P_f, and average SNR γ as metrics. We can see that the proposed algorithm has better performance than the existing EGC and WGC scheme. In Fig. 5, each secondary user has different average SNRs varying from 3 to 8 dB, We can see from this figure that the proposed scheme improved performance compared to existing method.

Fig. 6 shows the detection probability versus average SNR, and the decision threshold λ is chosen to keep $P_f = 10^{-2}$. We can see that, for $p_d \geq 0.9$ our proposed scheme reach $\lambda \geq 2.8dB$ but for same probability of p_d in EGC and WGC scheme we have $\lambda \geq 3.2dB$ and $\lambda \geq 4dB$ respectively.

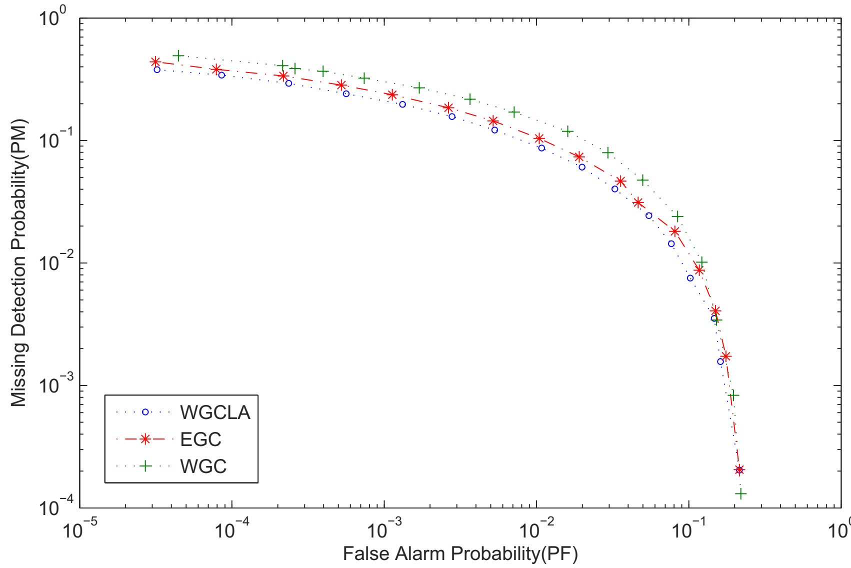

Fig. 5. Missing detection probability P_m versus false alarm probability P_f. (Each secondary user has different average SNRs varying from 3 to 8 dB.)

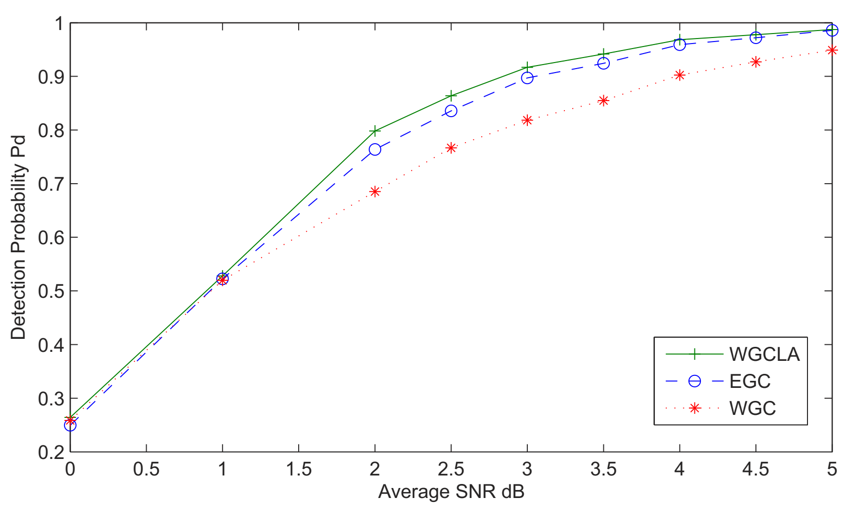

Fig. 6. Detection probability (P_d) versus average SNR ($P_f = 10^{-2}$, $TW = 5$)

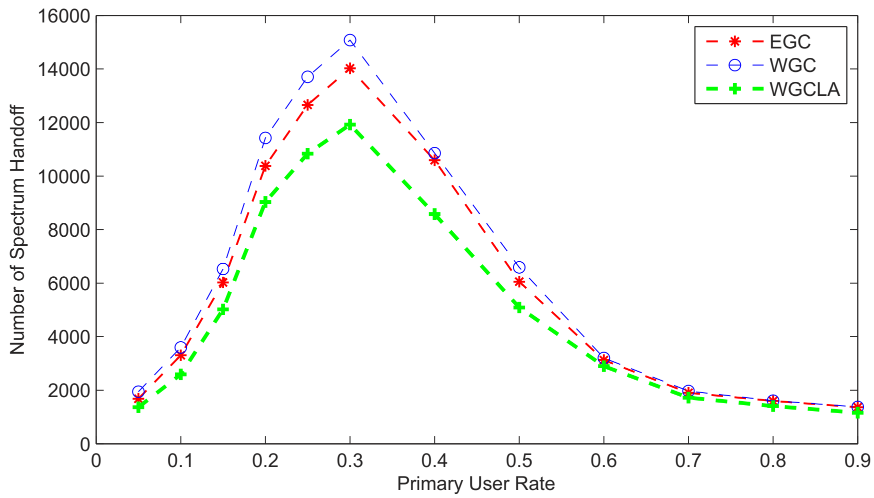

Fig. 7. The number of spectrum handoff versus PUs arrival rate in channel. ($P_m = 5 \times 10^{-2}$)

To evaluate the SUs quality of service in proposed method, we set up four channels with four PUs and 10 SUs scenario. After 10 SUs perform consensus sensing, one of them try to use idle channels. In this simulation, we use number of spectrum handoff and maximum delay as metric in SUs quality of service. In fig. 7, PUs have different arrival rate varying from 0.05 to 0.9 (pkt/sec). Fig. 7 shows the number of spectrum handoff in proposed algorithm, algorithm in [2] and [8] respectively. Compared with previous methods, the performance has been significantly improved by our propose algorithm.

Fig. 8 shows the number of spectrum handoff with respect to the SU arrivals rate varying from 0.05 to 0.9 (pkt/sec). We can see that when the SU arrivals rate is greater than 0.2 (pkt/sec), our proposed scheme achieves much better performance than the algorithm in [2] and [8].

Maximum delay is another SUs quality of service parameters. In other scenario, the maximum delay based on the number of time slot has been studied. In fig. 9, PUs have different arrival rate varying from 0.2 to 3 (pkt/sec) and in fig. 10 SU has different arrival rate varying from 0.2 to 3 (pkt/sec). As we see in fig. 9 and fig. 10, the maximum delay has been decreased by our propose algorithm.

5 Conclusion and Future Work

In this paper, we present a distributed cooperative spectrum sensing based on consensus algorithm and learning automata in cognitive radio networks. Simulation results show our approach has more accurate results than the existing

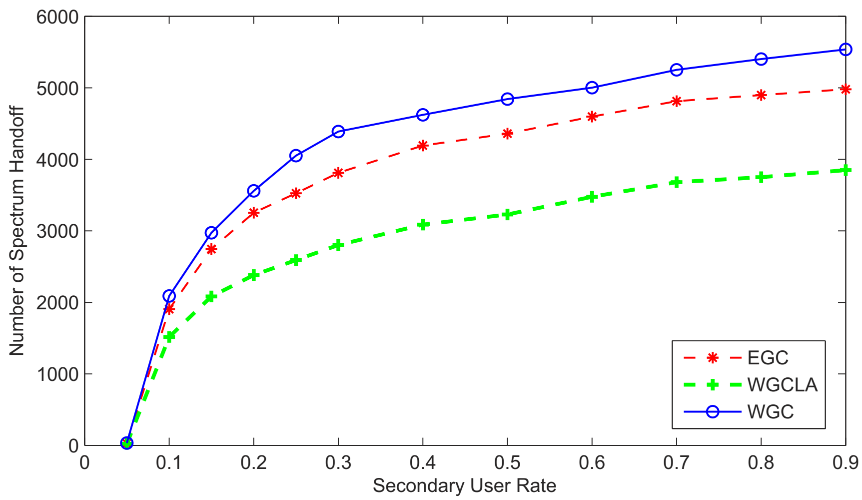

Fig. 8. The number of spectrum handoff versus SU arrival rate in channel. ($P_m = 5 \times 10^{-2}$)

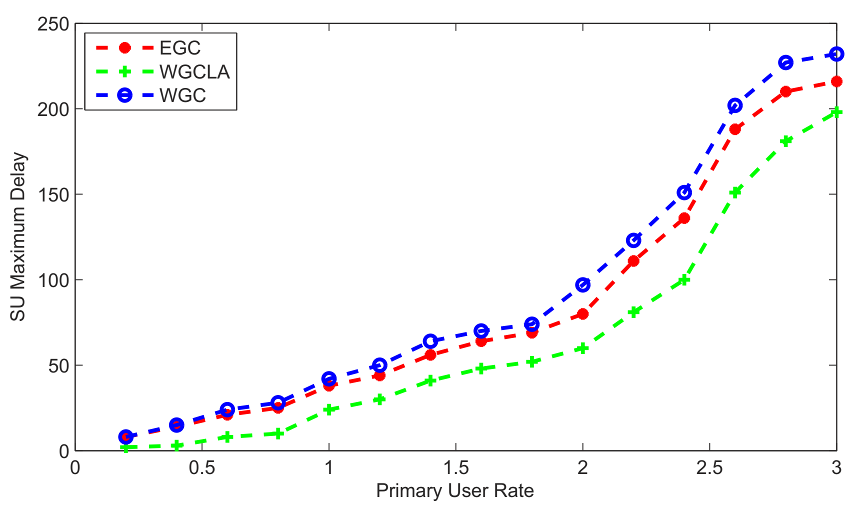

Fig. 9. The maximum delay versus PUs arrival rate in channel. ($P_m = 5 \times 10^{-2}$)

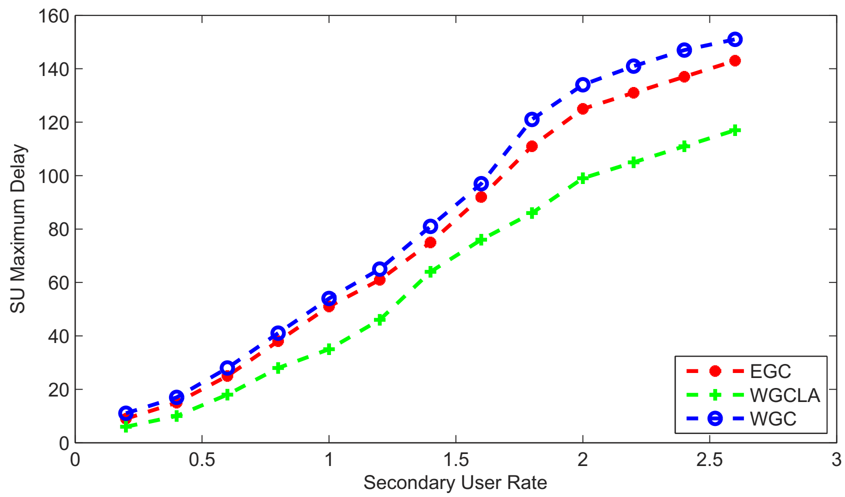

Fig. 10. The maximum delay versus SU arrival rate in channel. ($P_m = 5 \times 10^{-2}$)

scheme in practical conditions. Future work will pursue to use Multi Response Learning Automata (MRLA) to get more accurate weight for each SU.

References

1. J. Mitola, "Cognitive radio: An integrated agent architecture for software defined radio," Ph.D. dissertation, Roy. Inst. Technol. (KTH), Stockholm, Sweden, 2000.
2. Z. Li, F. R. Yu and M. Huang, "A Distributed Consensus-Based Cooperative Spectrum-Sensing Scheme in Cognitive Radios," IEEE Trans. Vehicular Technology., vol. 59, no. 1, pp. 383-393, Jan. 2010.
3. K. B. Letaief and W. Zhang, "Cooperative communications for cognitive radio networks," Proc. IEEE, vol. 97, no. 5, pp. 878893, May 2009.
4. A. Ghasemi and E. Sousa, "Collaborative spectrum sensing for opportunistic access in fading environments," in Proc. IEEE DySPAN, pp. 131136, 2005.
5. J. Unnikrishnan and V. V. Veeravalli, "Cooperative sensing for primary detection in cognitive radio," IEEE J. Sel. Topics Signal Process., vol. 2, no. 1, pp. 1827, Feb. 2008.
6. I. F. Akyildiz, B. F. Lo, and R. Balakrishnan, "Cooperative spectrum sensing in cognitive radio networks: A survey," Physical Communication, vol. 4, no. 1, pp. 40 62, 2011.

7. S. Mishra, A. Sahai, and R. Brodersen, "Cooperative sensing among cognitive radios," in Proc. IEEE ICC, pp. 16581663, 2006.
8. W. Zhang, Z. Wang, Y. Guo, H. Liu, Y. Chen and J. Mitola III, "Distributed Cooperative Spectrum Sensing based on Weighted Average Consensus," IEEE GLOBECOM, pp. 1-6, Dec 2011.
9. R. Olfati-Saber, J. Fax, and R. Murray, "Consensus and cooperation in networked multi-agent systems," Proc. IEEE, vol. 95, no. 1, pp. 215233, Jan. 2007.
10. K. S. Narendra, and M. A. L. Thathachar. "Learning Automata: An Introduction," Prentice Hall, 1989.
11. R. A. Horn and C. R. Johnson, Matrix Analysis. Cambridge, U.K.: Cambridge Univ. Press, 1987.