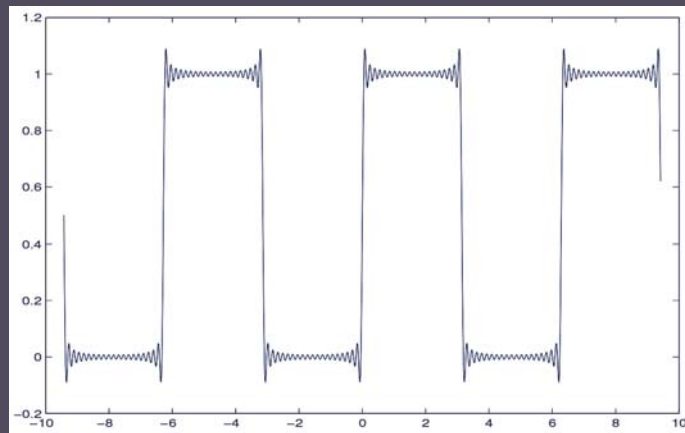


CHAPMAN & HALL/CRC APPLIED MATHEMATICS
AND NONLINEAR SCIENCE SERIES

An Introduction to
Partial Differential
Equations with
MATLAB[®]
Second Edition



Matthew P. Coleman

 CRC Press
Taylor & Francis Group

A CHAPMAN & HALL BOOK

CHAPMAN & HALL/CRC APPLIED MATHEMATICS
AND NONLINEAR SCIENCE SERIES

An Introduction to
Partial Differential
Equations with
MATLAB[®]
Second Edition

Matthew P. Coleman

Fairfield University
Connecticut, USA



CRC Press

Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **informa** business
A CHAPMAN & HALL BOOK

MATLAB® is a trademark of The MathWorks, Inc. and is used with permission. The MathWorks does not warrant the accuracy of the text or exercises in this book. This book's use or discussion of MATLAB® software or related products does not constitute endorsement or sponsorship by The MathWorks of a particular pedagogical approach or particular use of the MATLAB® software.

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2013 by Taylor & Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works
Version Date: 20130327

International Standard Book Number-13: 978-1-4398-9847-5 (eBook - PDF)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (<http://www.copyright.com/>) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Visit the Taylor & Francis Web site at
<http://www.taylorandfrancis.com>

and the CRC Press Web site at
<http://www.crcpress.com>

Contents

Preface	xi
Prelude to Chapter 1	1
1 Introduction	3
1.1 What are Partial Differential Equations?	3
1.2 PDEs We Can Already Solve	6
1.3 Initial and Boundary Conditions	10
1.4 Linear PDEs—Definitions	12
1.5 Linear PDEs—The Principle of Superposition	16
1.6 Separation of Variables for Linear, Homogeneous PDEs	19
1.7 Eigenvalue Problems	25
Prelude to Chapter 2	41
2 The Big Three PDEs	43
2.1 Second-Order, Linear, Homogeneous PDEs with Constant Coefficients	43
2.2 The Heat Equation and Diffusion	44
2.3 The Wave Equation and the Vibrating String	54
2.4 Initial and Boundary Conditions for the Heat and Wave Equations	59
2.5 Laplace’s Equation—The Potential Equation	66
2.6 Using Separation of Variables to Solve the Big Three PDEs	71
Prelude to Chapter 3	77
3 Fourier Series	79
3.1 Introduction	79
3.2 Properties of Sine and Cosine	80
3.3 The Fourier Series	89
3.4 The Fourier Series, Continued	95
3.5 The Fourier Series—Proof of Pointwise Convergence	104
3.6 Fourier Sine and Cosine Series	117
3.7 Completeness	124

Prelude to Chapter 4	127
4 Solving the Big Three PDEs on Finite Domains	129
4.1 Solving the Homogeneous Heat Equation for a Finite Rod . . .	129
4.2 Solving the Homogeneous Wave Equation for a Finite String .	138
4.3 Solving the Homogeneous Laplace's Equation on a Rectangular Domain	147
4.4 Nonhomogeneous Problems	153
Prelude to Chapter 5	161
5 Characteristics	163
5.1 First-Order PDEs with Constant Coefficients	163
5.2 First-Order PDEs with Variable Coefficients	174
5.3 The Infinite String	180
5.4 Characteristics for Semi-Infinite and Finite String Problems .	192
5.5 General Second-Order Linear PDEs and Characteristics	201
Prelude to Chapter 6	211
6 Integral Transforms	213
6.1 The Laplace Transform for PDEs	213
6.2 Fourier Sine and Cosine Transforms	220
6.3 The Fourier Transform	230
6.4 The Infinite and Semi-Infinite Heat Equations	242
6.5 Distributions, the Dirac Delta Function and Generalized Fourier Transforms	254
6.6 Proof of the Fourier Integral Formula	266
Prelude to Chapter 7	275
7 Special Functions and Orthogonal Polynomials	277
7.1 The Special Functions and Their Differential Equations	277
7.2 Ordinary Points and Power Series Solutions; Chebyshev, Her- mite and Legendre Polynomials	285
7.3 The Method of Frobenius; Laguerre Polynomials	292
7.4 Interlude: The Gamma Function	300
7.5 Bessel Functions	305
7.6 Recap: A List of Properties of Bessel Functions and Orthogonal Polynomials	317
Prelude to Chapter 8	327
8 Sturm–Liouville Theory and Generalized Fourier Series	329
8.1 Sturm–Liouville Problems	329
8.2 Regular and Periodic Sturm–Liouville Problems	337

8.3	Singular Sturm–Liouville Problems; Self-Adjoint Problems . . .	345
8.4	The Mean-Square or L^2 Norm and Convergence in the Mean . . .	354
8.5	Generalized Fourier Series; Parseval’s Equality and Completeness	361
Prelude to Chapter 9		373
9	PDEs in Higher Dimensions	375
9.1	PDEs in Higher Dimensions: Examples and Derivations	375
9.2	The Heat and Wave Equations on a Rectangle; Multiple Fourier Series	386
9.3	Laplace’s Equation in Polar Coordinates: Poisson’s Integral Formula	402
9.4	The Wave and Heat Equations in Polar Coordinates	414
9.5	Problems in Spherical Coordinates	425
9.6	The Infinite Wave Equation and Multiple Fourier Transforms	439
9.7	Postlude: Eigenvalues and Eigenfunctions of the Laplace Operator; Green’s Identities for the Laplacian	456
Prelude to Chapter 10		463
10	Nonhomogeneous Problems and Green’s Functions	465
10.1	Green’s Functions for ODEs	465
10.2	Green’s Function and the Dirac Delta Function	484
10.3	Green’s Functions for Elliptic PDEs (I): Poisson’s Equation in Two Dimensions	500
10.4	Green’s Functions for Elliptic PDEs (II): Poisson’s Equation in Three Dimensions; the Helmholtz Equation	516
10.5	Green’s Functions for Equations of Evolution	525
Prelude to Chapter 11		537
11	Numerical Methods	539
11.1	Finite Difference Approximations for ODEs	539
11.2	Finite Difference Approximations for PDEs	551
11.3	Spectral Methods and the Finite Element Method	565
A	Uniform Convergence; Differentiation and Integration of Fourier Series	579
B	Other Important Theorems	585
C	Existence and Uniqueness Theorems	591
D	A Menagerie of PDEs	601
E	MATLAB Code for Figures and Exercises	613

x

Contents

F Answers to Selected Exercises

627

References

647

Index

655