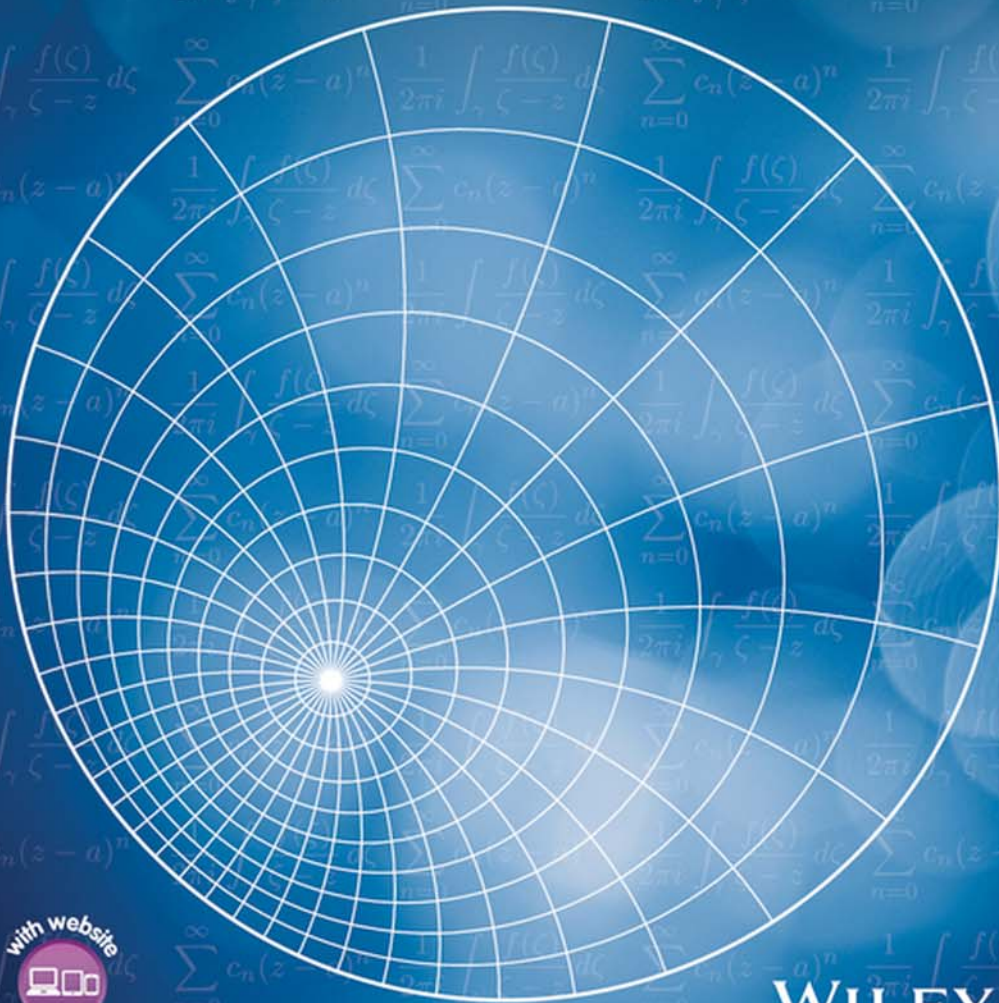


# COMPLEX ANALYSIS

A MODERN FIRST COURSE  
IN FUNCTION THEORY

JERRY R. MUIR, JR.



WILEY

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## **A Modern First Course in Function Theory**

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**Jerry R. Muir, Jr.**  
The University of Scranton

**WILEY**

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