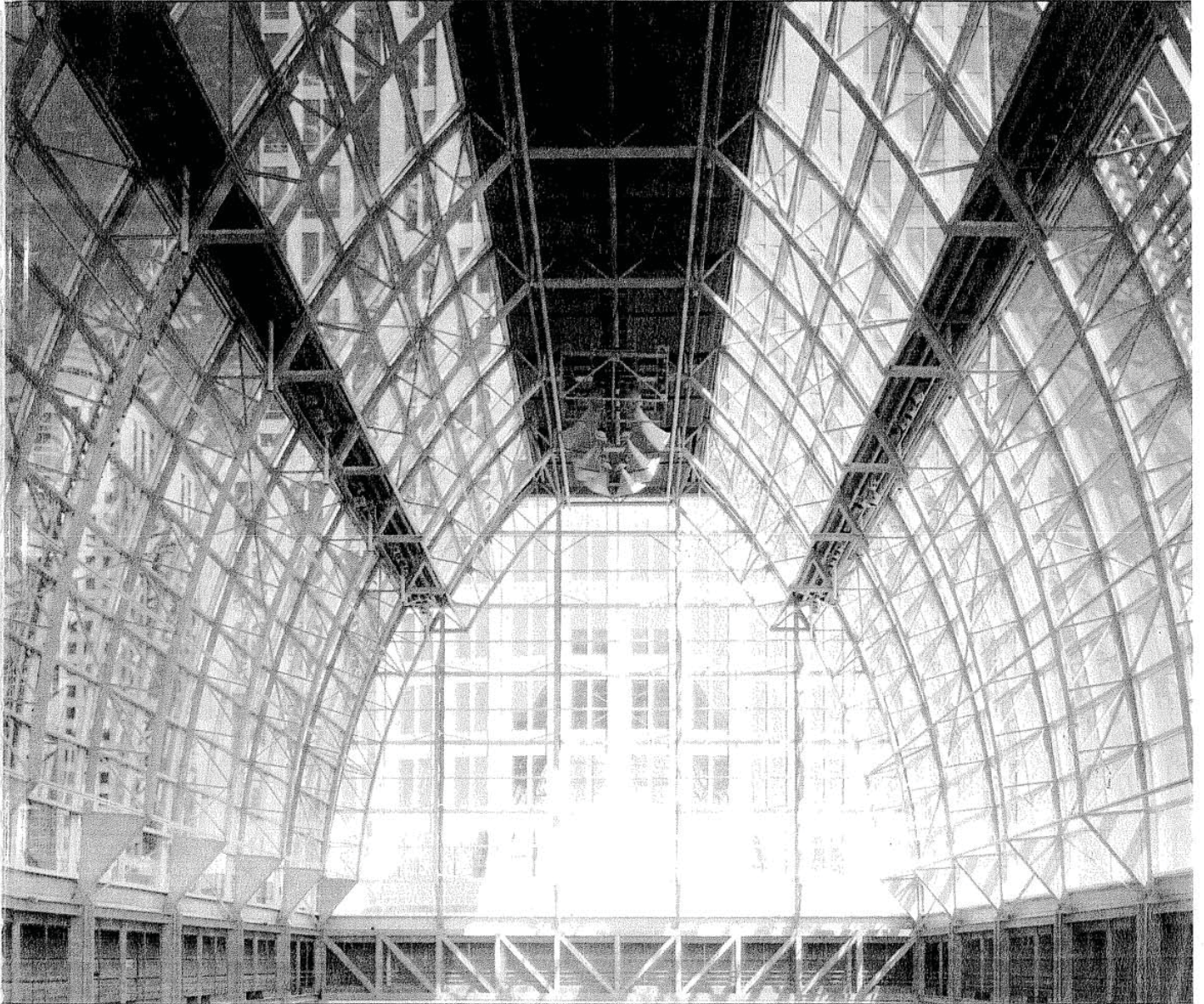


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ADVANCED ENGINEERING MATHEMATICS



Michael D. Greenberg

Advanced Engineering Mathematics

SECOND EDITION

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