



Tyn Myint-U
Lokenath Debnath



$$\nabla^2 u = 0$$

Linear Partial Differential Equations

for Scientists
and Engineers

Fourth Edition



Birkhäuser

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Differential Equations
for Scientists and Engineers

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Tyn Myint-U
5 Sue Terrace
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USA

Lokenath Debnath
Department of Mathematics
University of Texas-Pan American
1201 W. University Drive
Edinburgh, TX 78539
USA

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Contents

	Preface to the Fourth Edition	xv
	Preface to the Third Edition	xix
1	Introduction	1
1.1	Brief Historical Comments	1
1.2	Basic Concepts and Definitions	12
1.3	Mathematical Problems	15
1.4	Linear Operators	16
1.5	Superposition Principle	20
1.6	Exercises	22
2	First-Order, Quasi-Linear Equations and Method of Characteristics	27
2.1	Introduction	27
2.2	Classification of First-Order Equations	27
2.3	Construction of a First-Order Equation	29
2.4	Geometrical Interpretation of a First-Order Equation	33
2.5	Method of Characteristics and General Solutions	35
2.6	Canonical Forms of First-Order Linear Equations	49
2.7	Method of Separation of Variables	51
2.8	Exercises	55
3	Mathematical Models	63
3.1	Classical Equations	63
3.2	The Vibrating String	65
3.3	The Vibrating Membrane	67
3.4	Waves in an Elastic Medium	69
3.5	Conduction of Heat in Solids	75
3.6	The Gravitational Potential	76
3.7	Conservation Laws and The Burgers Equation	79
3.8	The Schrödinger and the Korteweg–de Vries Equations	81
3.9	Exercises	83
4	Classification of Second-Order Linear Equations	91
4.1	Second-Order Equations in Two Independent Variables	91

4.2	Canonical Forms	93
4.3	Equations with Constant Coefficients	99
4.4	General Solutions	107
4.5	Summary and Further Simplification	111
4.6	Exercises	113
5	The Cauchy Problem and Wave Equations	117
5.1	The Cauchy Problem	117
5.2	The Cauchy–Kowalewskaya Theorem	120
5.3	Homogeneous Wave Equations	121
5.4	Initial Boundary-Value Problems	130
5.5	Equations with Nonhomogeneous Boundary Conditions .	134
5.6	Vibration of Finite String with Fixed Ends	136
5.7	Nonhomogeneous Wave Equations	139
5.8	The Riemann Method	142
5.9	Solution of the Goursat Problem	149
5.10	Spherical Wave Equation	153
5.11	Cylindrical Wave Equation	155
5.12	Exercises	158
6	Fourier Series and Integrals with Applications	167
6.1	Introduction	167
6.2	Piecewise Continuous Functions and Periodic Functions .	168
6.3	Systems of Orthogonal Functions	170
6.4	Fourier Series	171
6.5	Convergence of Fourier Series	173
6.6	Examples and Applications of Fourier Series	177
6.7	Examples and Applications of Cosine and Sine Fourier Series	183
6.8	Complex Fourier Series	194
6.9	Fourier Series on an Arbitrary Interval	196
6.10	The Riemann–Lebesgue Lemma and Pointwise Convergence Theorem	201
6.11	Uniform Convergence, Differentiation, and Integration .	208
6.12	Double Fourier Series	212
6.13	Fourier Integrals	214
6.14	Exercises	220
7	Method of Separation of Variables	231
7.1	Introduction	231
7.2	Separation of Variables	232
7.3	The Vibrating String Problem	235
7.4	Existence and Uniqueness of Solution of the Vibrating String Problem	243
7.5	The Heat Conduction Problem	248

7.6	Existence and Uniqueness of Solution of the Heat Conduction Problem	251
7.7	The Laplace and Beam Equations	254
7.8	Nonhomogeneous Problems	258
7.9	Exercises	265
8	Eigenvalue Problems and Special Functions	273
8.1	Sturm–Liouville Systems	273
8.2	Eigenvalues and Eigenfunctions	277
8.3	Eigenfunction Expansions	283
8.4	Convergence in the Mean	284
8.5	Completeness and Parseval’s Equality	286
8.6	Bessel’s Equation and Bessel’s Function	289
8.7	Adjoint Forms and Lagrange Identity	295
8.8	Singular Sturm–Liouville Systems	297
8.9	Legendre’s Equation and Legendre’s Function	302
8.10	Boundary-Value Problems Involving Ordinary Differential Equations	308
8.11	Green’s Functions for Ordinary Differential Equations . .	310
8.12	Construction of Green’s Functions	315
8.13	The Schrödinger Equation and Linear Harmonic Oscillator	317
8.14	Exercises	321
9	Boundary-Value Problems and Applications	329
9.1	Boundary-Value Problems	329
9.2	Maximum and Minimum Principles	332
9.3	Uniqueness and Continuity Theorems	333
9.4	Dirichlet Problem for a Circle	334
9.5	Dirichlet Problem for a Circular Annulus	340
9.6	Neumann Problem for a Circle	341
9.7	Dirichlet Problem for a Rectangle	343
9.8	Dirichlet Problem Involving the Poisson Equation	346
9.9	The Neumann Problem for a Rectangle	348
9.10	Exercises	351
10	Higher-Dimensional Boundary-Value Problems	361
10.1	Introduction	361
10.2	Dirichlet Problem for a Cube	361
10.3	Dirichlet Problem for a Cylinder	363
10.4	Dirichlet Problem for a Sphere	367
10.5	Three-Dimensional Wave and Heat Equations	372
10.6	Vibrating Membrane	372
10.7	Heat Flow in a Rectangular Plate	375
10.8	Waves in Three Dimensions	379

10.9	Heat Conduction in a Rectangular Volume	381
10.10	The Schrödinger Equation and the Hydrogen Atom . . .	382
10.11	Method of Eigenfunctions and Vibration of Membrane . .	392
10.12	Time-Dependent Boundary-Value Problems	395
10.13	Exercises	398
11	Green's Functions and Boundary-Value Problems	407
11.1	Introduction	407
11.2	The Dirac Delta Function	409
11.3	Properties of Green's Functions	412
11.4	Method of Green's Functions	414
11.5	Dirichlet's Problem for the Laplace Operator	416
11.6	Dirichlet's Problem for the Helmholtz Operator	418
11.7	Method of Images	420
11.8	Method of Eigenfunctions	423
11.9	Higher-Dimensional Problems	425
11.10	Neumann Problem	430
11.11	Exercises	433
12	Integral Transform Methods with Applications	439
12.1	Introduction	439
12.2	Fourier Transforms	440
12.3	Properties of Fourier Transforms	444
12.4	Convolution Theorem of the Fourier Transform	448
12.5	The Fourier Transforms of Step and Impulse Functions .	453
12.6	Fourier Sine and Cosine Transforms	456
12.7	Asymptotic Approximation of Integrals by Stationary Phase Method	458
12.8	Laplace Transforms	460
12.9	Properties of Laplace Transforms	463
12.10	Convolution Theorem of the Laplace Transform	467
12.11	Laplace Transforms of the Heaviside and Dirac Delta Functions	470
12.12	Hankel Transforms	488
12.13	Properties of Hankel Transforms and Applications	491
12.14	Mellin Transforms and their Operational Properties . . .	495
12.15	Finite Fourier Transforms and Applications	499
12.16	Finite Hankel Transforms and Applications	504
12.17	Solution of Fractional Partial Differential Equations . . .	510
12.18	Exercises	521
13	Nonlinear Partial Differential Equations with Applications	535
13.1	Introduction	535

13.2	One-Dimensional Wave Equation and Method of Characteristics	536
13.3	Linear Dispersive Waves	540
13.4	Nonlinear Dispersive Waves and Whitham's Equations . .	545
13.5	Nonlinear Instability	548
13.6	The Traffic Flow Model	549
13.7	Flood Waves in Rivers	552
13.8	Riemann's Simple Waves of Finite Amplitude	553
13.9	Discontinuous Solutions and Shock Waves	561
13.10	Structure of Shock Waves and Burgers' Equation	563
13.11	The Korteweg–de Vries Equation and Solitons	573
13.12	The Nonlinear Schrödinger Equation and Solitary Waves.	581
13.13	The Lax Pair and the Zakharov and Shabat Scheme . . .	590
13.14	Exercises	595
14	Numerical and Approximation Methods	601
14.1	Introduction	601
14.2	Finite Difference Approximations, Convergence, and Stability	602
14.3	Lax–Wendroff Explicit Method	605
14.4	Explicit Finite Difference Methods	608
14.5	Implicit Finite Difference Methods	624
14.6	Variational Methods and the Euler–Lagrange Equations .	629
14.7	The Rayleigh–Ritz Approximation Method	647
14.8	The Galerkin Approximation Method	655
14.9	The Kantorovich Method	659
14.10	The Finite Element Method	663
14.11	Exercises	668
15	Tables of Integral Transforms	681
15.1	Fourier Transforms	681
15.2	Fourier Sine Transforms	683
15.3	Fourier Cosine Transforms	685
15.4	Laplace Transforms	687
15.5	Hankel Transforms	691
15.6	Finite Hankel Transforms	695
Answers and Hints to Selected Exercises		697
1.6	Exercises	697
2.8	Exercises	698
3.9	Exercises	704
4.6	Exercises	707
5.12	Exercises	712
6.14	Exercises	715
7.9	Exercises	724

8.14 Exercises	726
9.10 Exercises	727
10.13 Exercises	731
11.11 Exercises	739
12.18 Exercises	740
14.11 Exercises	745
Appendix: Some Special Functions and Their Properties	749
A-1 Gamma, Beta, Error, and Airy Functions	749
A-2 Hermite Polynomials and Weber–Hermite Functions . . .	757
Bibliography	761
Index	771