

Library of Congress Cataloging in Publication Data

Daniels, Richard W., date.  
Approximation methods for electronic filter design.  
Includes bibliographies.  
1. Electric filters. 2. Approximation theory.  
I. Title. II. TK7872.F5D36 621.3815'32 74-8091  
ISBN 0-07-015308-6

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1 2 3 4 5 6 7 8 9 0 K P K P 7 9 8 7 6 5 4

The editors for this book were Tyler G. Hicks and Stanley  
E. Redka, the designer was Naomi Auerbach, and its production  
was supervised by George E. Oechsner. It was set in Modern 8A  
by Bi-Comp, Inc.  
It was printed and bound by The Kingsport Press.

TK7872  
.F5D36  
c.1

# Approximation Methods for Electronic Filter Design

With Applications to Passive, Active,  
and Digital Networks

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McGraw-Hill Book Company  
New York St. Louis San Francisco Düsseldorf Johannesburg  
Kuala Lumpur London Mexico Montreal New Delhi  
Panama Paris São Paulo Singapore  
Sydney Tokyo Toronto



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