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Introductory Methods of Numerical Analysis



S.S. Sastry

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Fifth Edition

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INTRODUCTORY METHODS OF NUMERICAL ANALYSIS, Fifth Edition
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