



AN INTRODUCTION TO
**NUMERICAL
METHODS AND
ANALYSIS**

Second Edition

JAMES F. EPPERSON

WILEY

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