

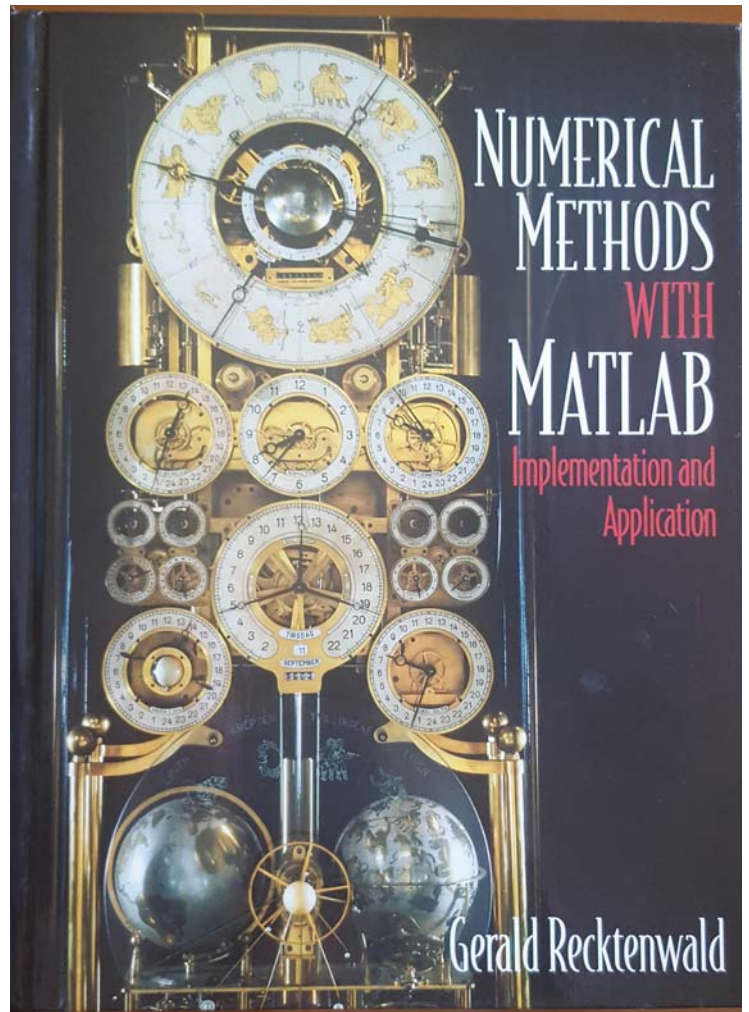
# Numerical Methods with MATLAB

Implementations and Applications

Gerald W. Recktenwald

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