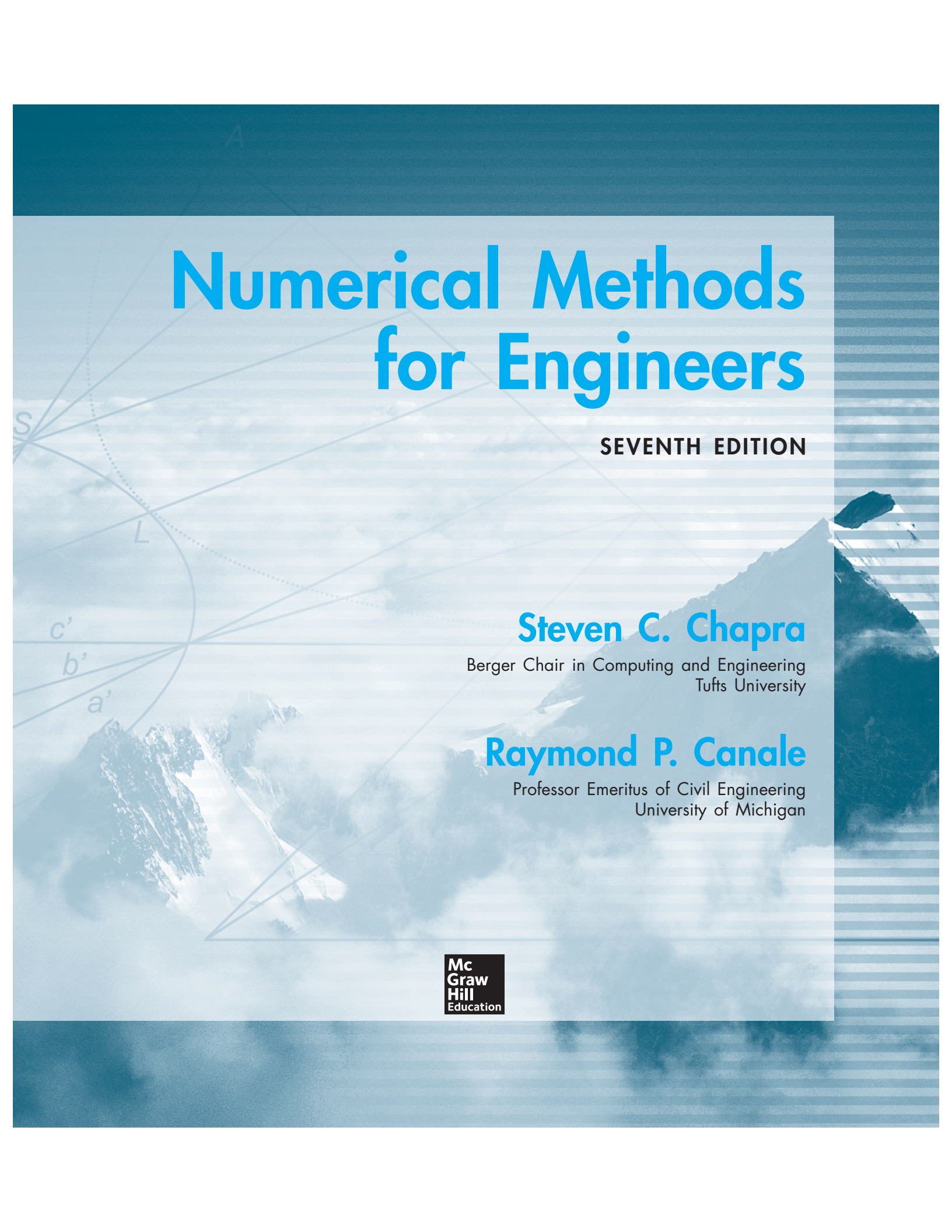


The background of the cover is a photograph of two snowboarders in mid-air against a blue sky. The snowboarder in the foreground is wearing a bright red suit and is captured in a dynamic pose with one arm raised. Below them, another snowboarder in a white suit is also in mid-air, performing a different trick. The lower portion of the image shows a majestic, snow-capped mountain peak rising above a thick layer of white clouds. The overall scene conveys a sense of adventure and high performance.

Seventh Edition

Numerical Methods *for Engineers*

Steven C. Chapra
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Numerical Methods for Engineers

SEVENTH EDITION

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