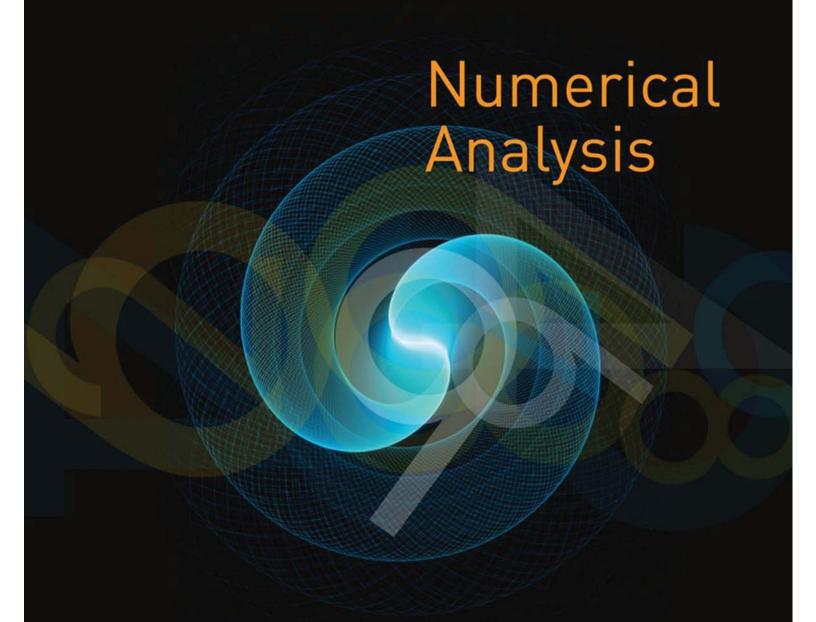
Richard L. Burden J. Douglas Faires



Ninth Edition



Numerical Analysis

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Youngstown State University

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