۱ -

$$\int_0^{2\pi} \frac{\cos x}{17-8\cos x}\,dx = \oint_{|z|=1} \frac{\frac{z+\frac{1}{z}}{2}}{17-8\left(\frac{z+\frac{1}{z}}{2}\right)} = \frac{z^2+1}{-2(z-4)(4z-1)}$$ -۱

$z=\frac{1}{4} \longrightarrow$ در دایره است

$z=4 \longrightarrow$ در دایره نیست

$$a_{-1} = \lim_{z\to\frac{1}{4}} \left(z-\frac{1}{4}\right) \times \frac{z^2+1}{\frac{-2(z-4)(4z-1)}{4}} = \frac{\frac{17}{16}}{30} = \frac{17}{16\times 30}$$

$$2\pi i \times \frac{17}{16\times 30} = \boxed{\frac{17\pi i}{240}}$$

$$\int_{-\infty}^{+\infty} \frac{1}{1+x^6}\,dx \longrightarrow f(z)=\frac{1}{1+z^6} \Longrightarrow 1+z^6=0 \longrightarrow z=\sqrt[6]{-1}$$

ریشه ششم عدد ۱-

$$\operatorname{cis}\left(\frac{2k\pi-\frac{\pi}{2}}{6}\right) \longrightarrow \begin{cases} k=0 & \operatorname{cis}\left(\frac{-\pi}{12}\right) \\ k=1 & \operatorname{cis}\left(\frac{\pi}{4}\right) \\ k=2 & \operatorname{cis}\left(\frac{7\pi}{12}\right) \\ k=3 & \operatorname{cis}\left(\frac{11\pi}{12}\right) \\ k=4 & \operatorname{cis}\left(\frac{5\pi}{4}\right) \\ k=5 & \operatorname{cis}\left(\frac{19\pi}{12}\right) \end{cases}$$

$$a_{-1} = \lim_{z\to\frac{-\pi}{12}} \left(z+\frac{\pi}{12}\right) \times \frac{1}{z^6+1} \xrightarrow{HoP} \frac{1}{6\operatorname{cis}\left(\frac{-5\pi}{12}\right)}$$

⋮

۶ باقیمانده را حساب کرده باهم جمع می‌کنیم در ۲πi ضرب می‌کنیم.

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$$\int_0^{\pi} \frac{d\theta}{1+\frac{1}{3}\cos\theta} = \oint_{|z|=1} \frac{dz}{iz\left(1+\frac{1}{6}\left(z+\frac{1}{z}\right)\right)} = 6\oint_{|z|=1} \frac{dz}{(z^2+6z+1)i}$$

$$z^2+6z+1=0 \longrightarrow -3\pm\sqrt{8} \quad \begin{cases} -3+2\sqrt{2} & \checkmark \\ -3-2\sqrt{2} & \times \end{cases}$$

$$a_{-1} = \lim_{z\to -3+2\sqrt{2}} (z+3-2\sqrt{2}) \times \frac{6}{i(z^2+6z+1)} \xrightarrow{\text{Hop}} \frac{-6i}{4\sqrt{2}} = \frac{-3i}{2\sqrt{2}}$$

$$2\pi i \times \frac{-3i}{2\sqrt{2}} = \frac{3\pi}{\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{2}\pi}$$

$$\oint_{|z|=1} \frac{dz}{iz\left(\frac{z^2+2az+1}{2z}\right)} = \oint_{|z|=1} \frac{-2i\,dz}{z^2+2az+1} \qquad -2$$

$$z = -a\pm\sqrt{a^2-1} \quad \begin{cases} -a+\sqrt{a^2-1} & \checkmark \\ -a-\sqrt{a^2-1} & \times \end{cases}$$

$$a_{-1} = \lim (z+a-\sqrt{a^2-1}) \frac{d}{dz} \frac{(-2i)}{z^2+2az+1} = \text{I}$$

$$\text{I}\ \frac{(2z+2a) * 2i}{z^2+2az+1} \times (z+a-\sqrt{a^2-1}) \xrightarrow{\text{Hop}} \bigcirc \xrightarrow{\text{Hop}}$$

دوبار Hop بزنیم

$$a_{-1} = \frac{-ai}{2(a^2-1)^{\frac{3}{2}}}$$

$$2\pi i \times \frac{-ai}{2(a^2-1)^{\frac{3}{2}}} = \boxed{\frac{\pi a}{(a^2-1)^{\frac{3}{2}}}}$$

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$$\int_0^{\pi} \sin^{2n}\theta \, d\theta = \oint_{|z|=1} \left(\frac{z-\frac{1}{z}}{2i}\right)^{2n} \times \frac{dz}{iz} = \int \frac{(z^2-1)^{2n}}{z^{2n}\times(2i)^{2n}\times z} \qquad \int \frac{(z^2-1)^{2n}}{z^{2n+1}\times 2^{2n}}$$

$$z=0 \longrightarrow 2n+1 \longrightarrow \lim \frac{d^{2n-2}}{dz^{n-2}} \frac{(z^2-1)^{2n}}{2^{2n}} \longrightarrow H_oP, ! 2n-2$$

$$a_{-1} = \frac{(2n)! \times -i}{2^{2n}\times(n!)^2\times 2} \longrightarrow 2\pi i \times a_{-1} = \frac{(2n)!}{2^{2n}(n!)^2} \times \pi$$

۳-

الف)

$$u+iv = e^{x+iy} = e^{x}(\cos(y)+i\sin(y)) \longrightarrow u = e^{x}\cos y$$

$$v = e^{x}\sin y$$

$$|w| = e^{x} \longrightarrow e^{-3} < |w| < e^{-2}$$

$$\arg(w) = y \longrightarrow 0 \leq \arg(w) < \frac{\pi}{2}$$

ب)

$$e^{-1} < |w| < e^{3}$$

$$-\pi < \arg(w) < \pi$$

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$\sin z = \sin(x+iy) = \sin x \operatorname{ch} y + i \cos x \operatorname{sh}(y)$

$(\sin z)^2 = \sin^2 x \operatorname{ch}^2 y - \cos^2 x \operatorname{sh}^2 y + 2i \sin x \cos x \operatorname{sh} y \operatorname{ch} y$

$\Rightarrow v = \frac{1}{2} \sin 2x \operatorname{sh} 2y$

$0 \le x \le \frac{\pi}{2} \Rightarrow 0 \le 2x \le \pi \longrightarrow \sin 2x > 0$

$y < 0 \longrightarrow \frac{\operatorname{sh} 2y}{2} > 0 \longrightarrow \boxed{v \geq 0}$