Subject:
Date

۱-

$$u = XY \longrightarrow X''Y = -XY'' \longrightarrow -\frac{X''}{X} = \frac{Y''}{Y}$$

$$\frac{X''}{X} = C \qquad \frac{Y''}{Y} = C \ \text{—} \ \text{II}$$

$$t^2 = C \longrightarrow t = \pm\sqrt{C} \qquad X = k_1 e^{\sqrt{C}x} + k_2 e^{-\sqrt{C}x}$$

$$\text{II} \longrightarrow Y = k_3 e^{\sqrt{C}y} + k_4 e^{-\sqrt{C}y} \longrightarrow u = XY$$

۲-

$$u = XY \longrightarrow X'Y + XY' = 2(x+y)XY$$

$$\longrightarrow \frac{X'}{X} + \frac{Y'}{Y} = 2x + 2y \longrightarrow \frac{X'}{X} - 2x = \frac{-Y'}{Y} + 2y$$

$$\begin{cases} \dfrac{X'}{X} = 2x + C \longrightarrow X = k_1 e^{x^2 + Cx} \\ \dfrac{Y'}{Y} = 2y - C \longrightarrow Y = k_2 e^{y^2 - Cy} \end{cases} \longrightarrow u = XY = k e^{x^2 + y^2 + C(x-y)}$$

۳-

تمدید فرد $\longrightarrow \dfrac{2}{\pi}\displaystyle\int_0^{\pi} f(x)\sin nx\, dx = \dfrac{2}{\pi}\Big[\int_0^{\frac{\pi}{4}} \dfrac{x}{10}\sin nx\, dx$

$$+ \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\frac{-x}{10} + \frac{\pi}{20}\right)\sin nx\, dx + \int_{\frac{3\pi}{4}}^{\pi} \left(\frac{x}{10} - \frac{\pi}{10}\right)\sin nx\, dx\Big]$$

$$= \frac{4}{5\pi n^2}\sin\frac{n\pi}{4} \longrightarrow u(x,t) = \sum_{n=1}^{\infty} \frac{4}{5\pi n^2}\sin\frac{n\pi}{4}\cos(cnt)(\sin nx)$$

PAPCO

$$u_y = \frac{\partial u}{\partial r} \times \frac{\partial r}{\partial x} + \frac{\partial u}{\partial z} \times \frac{\partial z}{\partial x} = x \frac{\partial u}{\partial z}$$ - ۴

$$u_{yy} = x^2 \frac{\partial^2 u}{\partial z^2}$$

$$u_{xy} = \frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial z \partial r} + y \frac{\partial^2 u}{\partial z^2}$$

$$\cancel{x \frac{\partial u}{\partial z}} + x \frac{\partial^2 u}{\partial z \partial r} + xy \frac{\partial^2 u}{\partial z^2} = x^2 y \frac{\partial^2 u}{\partial z^2} + \cancel{x \frac{\partial u}{\partial z}}$$

$$(x^2 y - xy) \frac{\partial^2 u}{\partial z^2} - x \frac{\partial^2 u}{\partial z \partial r} = 0$$

$$(x^2 y - xy) t^2 = tx \qquad t = 0 ,\ t = \frac{1}{xy - y}$$

$$u = C_1 + C_2 e^{\frac{1}{xy-y} \times r} = \boxed{C_1 + C_2 e^{\frac{x}{xy-y}}}$$

$$L = 2\pi \qquad A_n \xrightarrow{\lambda = \frac{c\pi}{2\pi} = c} \boxed{a_1 \sin x \cos(ct)}$$ - ۵

Subject:
Date

$c=2 \longrightarrow u_t = u_r r_t + u_z z_t = 2(u_r - u_z)$ — ۶

$u_{tt} = 4[u_{rr} - 2u_{rz} + u_{zz}]$

$u_{xx} = u_{rr} + 2u_{rz} + u_{zz}$

$\cancel{4}[\cancel{u_{rr}} - 2u_{rz} + \cancel{u_{zz}}] = \cancel{4}\times[\cancel{u_{rr}} + 2u_{rz} + \cancel{u_{zz}}]$

$u_{rz}=0 \longrightarrow u_r = \phi(r) \longrightarrow u = \phi(r) + \psi(z)$

$\boxed{u = \phi(x+2t) + \psi(x-2t)}$ *

II $\phi(x) + \psi(x) = 0$

I $2\phi'(x) - 2\psi'(x) = x+1$

I $\xrightarrow{\text{انتگرال}} 2\phi(x) - 2\psi(x) = \frac{x^2}{2} + x + K$

II $\longrightarrow \phi(x) = -\psi(x) \longrightarrow \phi(x) = \frac{x^2}{8} + \frac{x}{4} + K_2$

$\psi(x) = -\frac{x^2}{8} - \frac{x}{4} - K_2$

با جایگذاری بدست می‌آید. ← * در این معادله جایگذاری شود