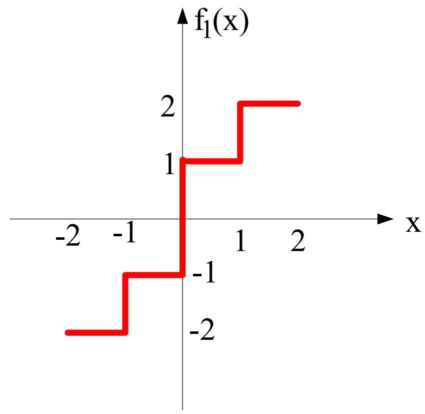
پاسخ امتحان میان ترم اول ریاضی مهندسی ۱۴۰۱/۱۲/۲۴

۱- $L=2 \to 2L=4$ تمدید فرد تابع f(x):



پس : $a_0 = a_n = 0$

$$b_n = \frac{2}{L}\int_0^2 f_1(x)\sin\left(\frac{n\pi x}{L}\right)dx = \int_0^2 f_1(x)\sin\left(\frac{n\pi x}{2}\right)dx = \int_0^1 \sin\left(\frac{n\pi x}{2}\right)dx + \int_1^2 2\sin\left(\frac{n\pi x}{2}\right)dx$$

$$b_n = \frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big|_0^1 + \frac{-4}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big|_1^2 = \frac{-2}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right)-1\right] - \frac{4}{n\pi}\left[\cos(n\pi)-\cos\left(\frac{n\pi}{2}\right)\right]$$

$$b_n = \frac{2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{2}{n\pi} - \frac{4}{n\pi}\cos(n\pi) = \begin{cases} \dfrac{6}{n\pi} & : \text{n odd} \\ -\dfrac{2}{n\pi} + \dfrac{2}{n\pi}\cos\left(\dfrac{n\pi}{2}\right) & : \text{n even} \end{cases}$$

$$f(x) = \frac{6}{\pi}\sin\left(\frac{\pi x}{2}\right) - \frac{4}{2\pi}\sin(\pi x) + \frac{6}{3\pi}\sin\left(\frac{3\pi x}{2}\right) + \frac{6}{5\pi}\sin\left(\frac{5\pi x}{2}\right) - \frac{4}{6\pi}\sin(3\pi x) + \ldots$$

$$x = 1 \to \frac{1+2}{2} = \frac{6}{\pi} - \frac{6}{3\pi} + \frac{6}{5\pi} - + \ldots \to 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + - \ldots = \frac{\pi}{4}$$

۲- می توان با تعیین $B(\omega)$, $A(\omega)$ نیز سوال را حل نمود.

$$g(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-iwx} f(x)dx = \frac{1}{\sqrt{2\pi}}\left[\int_0^1 e^{-iwx}dx + \int_1^2 2e^{-iwx}dx\right] = \frac{1}{\sqrt{2\pi}}\left[\frac{-1}{iw}e^{-iwx}\Big|_0^1 + \frac{-2}{iw}e^{-iwx}\Big|_1^2\right]$$

$$g(w) = \frac{1}{\sqrt{2\pi}}\left[\frac{-1}{iw}(e^{-iw}-1) + \frac{-2}{iw}(e^{-2iw}-e^{-iw})\right] = \frac{1}{iw\sqrt{2\pi}}(1+e^{-iw}-2e^{-2iw})$$

$$f(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{iwx}\frac{1}{iw\sqrt{2\pi}}(1+e^{-iw}-2e^{-2iw})dw \to f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{iw}(1+e^{-iw}-2e^{-2iw})e^{iwx}dw$$