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$$\int_0^{\infty} \frac{x^{\nu}}{x^{\nu}+1} dx = \frac{1}{\nu} \int_{-\infty}^{\infty} \frac{x^{\nu}}{x^{\nu}+1} dx = \pi i \sum R_j : f(z) = \frac{z^{\nu}}{z^{\nu}+1}, \lim_{z \rightarrow \infty} z f(z) = 0 \quad -1$$

$$16+z^{\nu}=0 \rightarrow z^{\nu}=-16 \Rightarrow z=2e^{i\frac{\pi}{\nu}}, 2e^{i\frac{3\pi}{\nu}}, 2e^{i\frac{5\pi}{\nu}}, 2e^{i\frac{7\pi}{\nu}}$$

قط اولین و دومین ریشه بالای صفحه است. $z_1 = 2e^{i\frac{\pi}{\nu}}, z_2 = 2e^{i\frac{3\pi}{\nu}}$

$$R_1 = \lim_{z \rightarrow z_1} \frac{z^{\nu}}{\nu z^{\nu-1}} = \frac{1}{\nu} \lim_{z \rightarrow z_1} \frac{1}{z} = \frac{1}{\nu} e^{-i\frac{\pi}{\nu}}, \quad R_2 = \lim_{z \rightarrow z_2} \frac{z^{\nu}}{\nu z^{\nu-1}} = \frac{1}{\nu} \lim_{z \rightarrow z_2} \frac{1}{z} = \frac{1}{\nu} e^{-i\frac{3\pi}{\nu}}$$

$$R_1 + R_2 = \frac{1}{\nu} \left[\frac{1}{\sqrt{\nu}} - \frac{i}{\sqrt{\nu}} - \frac{1}{\sqrt{\nu}} - \frac{i}{\sqrt{\nu}} \right] = \frac{-i}{\nu \sqrt{\nu}} \rightarrow \int_0^{\infty} \frac{x^{\nu}}{x^{\nu}+1} dx = \frac{\pi}{\nu \sqrt{\nu}} \quad -2$$

$$\frac{1-(-\infty)}{1-z} \div \frac{-1-(-\infty)}{-1-z} = \frac{-1-0}{-1-w} \div \frac{1-0}{1-w} \rightarrow \frac{1-(-\infty)}{1-z} \frac{-1-z}{-1-(-\infty)} = \frac{-1}{-1-w} \frac{1-w}{1} \rightarrow \frac{1+z}{z-1} = \frac{1-w}{1+w} \rightarrow w = -\frac{1}{z} \quad -2$$

$$w = \sin(z) = \sin(x+iy) = \sin(x)\cos(iy) + \cos(x)\sin(iy) = \sin(x)\cosh(y) + i\cos(x)\sinh(y) \quad -3$$

$$\begin{cases} u = \sin(x)\cosh(y) = \frac{1}{\sqrt{\nu}} \cosh(y) \\ v = \cos(x)\sinh(y) = \frac{1}{\sqrt{\nu}} \sinh(y) \end{cases} \rightarrow 2u^{\nu} - 2v^{\nu} = 1 \rightarrow u^{\nu} - v^{\nu} = 0.5 \quad -4$$

$$yz \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = -2xy \rightarrow \frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{-2xy} \rightarrow \frac{xdx}{xyz} = \frac{ydy}{yxz} = \frac{zdz}{-2xyz} \quad -4$$

$$xdx - ydy = 0 \rightarrow x^{\nu} - y^{\nu} = c_1, \quad 2ydy + zdz = 0 \rightarrow y^{\nu} + 0.5z^{\nu} = c_2 \rightarrow y^{\nu} + 0.5z^{\nu} = f(x^{\nu} - y^{\nu}) \quad -5$$

$$x^{\nu} + y^{\nu} = 16 \rightarrow x^{\nu} = 16 - y^{\nu}, \quad z = 3 \rightarrow y^{\nu} + 4/5 = f(16 - 2y^{\nu}), \quad w = 16 - 2y^{\nu} \rightarrow$$

$$f(w) = 8 - 0.5w + 4/5 = 12/5 - 0.5w \rightarrow y^{\nu} + 0.5z^{\nu} = 12/5 - 0.5(x^{\nu} - y^{\nu}) \rightarrow x^{\nu} + y^{\nu} + z^{\nu} = 25$$

$$u(x, t) = \frac{1}{\nu} f(x - at) + \frac{1}{\nu} f(x + at) = 2 \cos(2x - 2at) + 2 \cos(2x + 2at) = 4 \cos(2x) \cos(2at) \quad -5$$

$$x = 0, \quad t = 1 \rightarrow u(0, 1) = 4 \cos(a) = 2 \rightarrow \cos(2a) = 0.5 \rightarrow 2a = 2k\pi \pm \frac{\pi}{3} \rightarrow a = k\pi \pm \frac{\pi}{6}$$