۱-

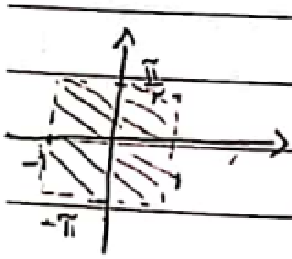
آ۲ $-1<x<1$ و $-\pi<y<\frac{\pi}{2}$

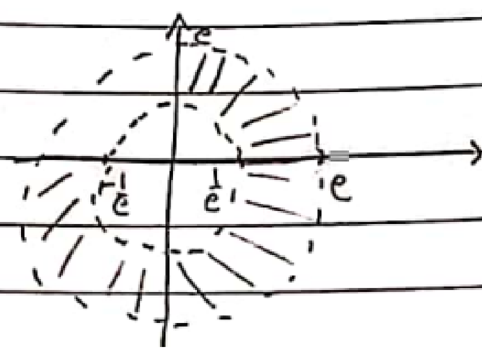
ملنا

$e^{z}=e^{x+iy}$

$\begin{cases}\rho=e^{x}\\ \varphi=y\end{cases}$

$\begin{cases}-1<x<1 \to e^{-1}<\rho<e^{1}\\ -\pi<y<\frac{\pi}{2} \quad -\pi<\varphi<\frac{\pi}{2}\end{cases}$





$$\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = -\frac{2}{i} \int_{|z|=1} \frac{dz}{z^2-2\sqrt{2}\,z+1}$$

تابع انتگران دارای دو قطب ساده $z=\sqrt{2}\pm 1$ است که تنها قطب $z=\sqrt{2}-1$ در داخل دایره‌یکه است. بنابراین

$$\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = -2\pi i \times \frac{2}{i} \operatorname*{Res}_{z=\sqrt{2}-1} \frac{1}{z^2-2\sqrt{2}\,z+1} = -\frac{1}{2}(-4\pi) = 2\pi$$

$x^2 u_{xy} + 3y^2 u = 0$ $\qquad u(x,y) = F(x)\,G(y)$

$u_x = F'(x)G(y)$

$u_{xy} = F'(x)G'(y)$

$x^2 F'(x)G'(y) + 3y^2 F(x)G(y) = 0 \Rightarrow$

$x^2 F'(x)G'(y) = -3y^2 F(x)G(y) \Rightarrow \dfrac{x^2 F'(x)}{F(x)} = \dfrac{-3y^2 G(y)}{G'(y)} = K$

$x^2 \dfrac{F'(x)}{F(x)} = K \Rightarrow \dfrac{F'(x)}{F(x)} = \dfrac{K}{x^2} \Rightarrow \int \dfrac{F'(x)}{F(x)} = \int \dfrac{K}{x^2}\,dx \Rightarrow$

$\ln F(x) = \dfrac{-K}{x} + C \Rightarrow F(x) = c_1 e^{-\frac{K}{x}}$

$\dfrac{G'(y)}{G(y)} = \dfrac{-3y^2}{K} \overset{\int}{\Rightarrow} \ln G(y) = \dfrac{-y^3}{K} + c \Rightarrow G(y) = c_2 e^{-\frac{y^3}{K}}$

$u_{(x,y)} = \underbrace{c_1 c_2}_{c_3} e^{-\frac{K}{x}} e^{-\frac{y^3}{K}} \Rightarrow u_{(x,y)} = c_3 e^{-\frac{K}{x}} e^{-\frac{y^3}{K}}$