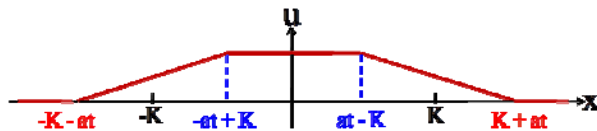


$$\left\{ \begin{array}{l} -K - at < x < -at + K < 0 \Rightarrow x + at > -K, x + at < K \\ -2at - K < x - at < -2at + K < -K \\ u(x, t) = \frac{1}{2a} \int_{-K}^{x+at} 1 ds = \frac{x + at + K}{2a} \\ \\ 0 < -K + at < x < at + K \Rightarrow x - at > -K, x - at < K \\ K < 2at - K < x + at < 2at + K \\ u(x, t) = \frac{1}{2a} \int_{x-at}^K 1 ds = \frac{-x + at + K}{2a} \\ \\ -at + K < x < at - K \Rightarrow x + at > K, x - at < -K \\ u(x, t) = \frac{1}{2a} \int_{-K}^K 1 ds = \frac{K}{a} \end{array} \right.$$



معادله موج

مثال ۱: ارتعاش در سیم دو سو نامحدود

$$g(x) = \begin{cases} 1 & : |x| < K \\ 0 & : |x| > K \end{cases}$$

$$u(x, t) = \frac{1}{2a} \int_{x-at}^{x+at} g(s) ds \quad (1)$$

$$g(s) = \begin{cases} 1 & : |s| < K \\ 0 & : |s| > K \end{cases}$$

حالت ۱: $t < \frac{K}{a}$ ($at < K$)

$$x < -K - at \rightarrow x + at < -K \Rightarrow u = 0$$

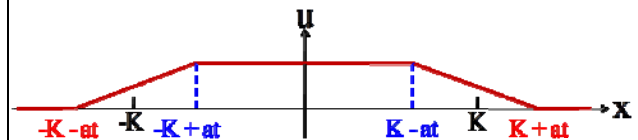
$$x > K + at \rightarrow x - at > K \Rightarrow u = 0$$

در این حالت عبارت داخل انتگرال رابطه ۱ صفر است.

$$\left\{ \begin{array}{l} -K - at < x < -K + at < 0 \Rightarrow x + at > -K, x - at < -K \\ u(x, t) = \frac{1}{2a} \int_{-K}^{x+at} 1 ds = \frac{x + at + K}{2a} \end{array} \right.$$

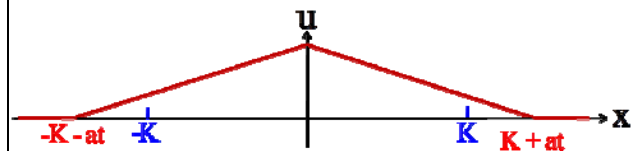
$$\left\{ \begin{array}{l} 0 < -at + K < x < K + at \Rightarrow x + at > K, x - at < K \\ u(x, t) = \frac{1}{2a} \int_{x-at}^K 1 ds = \frac{-x + at + K}{2a} \end{array} \right.$$

$$\left\{ \begin{array}{l} -K + at < x < K - at \Rightarrow x + at < K, x - at > -K \\ u(x, t) = \frac{1}{2a} \int_{x-at}^{x+at} 1 ds = t \end{array} \right.$$



حالت ۲: $t = \frac{K}{a}$

فقط فاصله $-K + at < x < K - at$ به نقطه $x = 0$ تبدیل می شود.



حالت ۳: $t > \frac{K}{a}$ ($at > K$)

$$x < -K - at \rightarrow x + at < -K \Rightarrow u = 0$$

$$x > K + at \rightarrow x - at > K \Rightarrow u = 0$$

در این حالت عبارت داخل انتگرال رابطه ۱ صفر است.