

1. Solve the following system of nonlinear equations :

$$\begin{cases} -2x^3 + 3y^2 + 42 = 0 \\ 5x^2 + 3y^3 - 69 = 0 \end{cases}$$

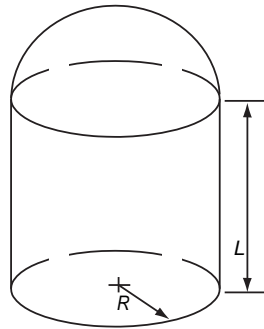
- Use Newton-Raphson Method. Start at $x=2$ & $y=1$, and carry out the first five iterations. What is the precision of solution?
 - Use the fixed-point Method. Start at $x=2$ & $y=1$, and carry out the first three iterations. What is the precision of solution?
 - Use Steepest Descent Method. Start at $x=2$ & $y=1$, and carry out the first three iterations. What is the precision of solution?
 - Use Homotopy Method (with 4-order Runge-Kutta Method). Start at $x=2$ & $y=1$, and $N=4$.
 - Compare (a) , (b) , (c) and parts.
2. Find the least squares polynomials of degrees 1,2,and 3 for the data in the following table. Compute the error (summation of squares of error) in the each case and compare cases.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

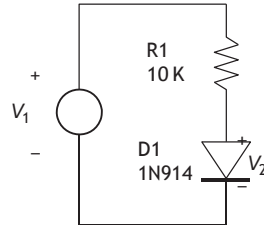
3. Construct the second-degree Legendre least squares approximation to $f(x) = \cos(\pi x)$ over the interval $[-1,1]$. Compute the error (Integral of squares of error).
4. Minimize the statement $f = x_1 + 2x_2 + x_3$ subject to

$$\begin{cases} x_1 - 2x_2 + x_3 \geq 2 \\ -x_1 + x_2 + x_3 \geq 4 \\ 2x_1 + x_3 \geq 6 \\ x_1 + x_2 + x_3 \geq 2 \end{cases}, x_j \geq 0 : \forall j$$

5. A silo is to consist of a right circular cylinder of radius R and length L , with a hemispherical roof (see the following figure). Assume that the silo is to have a specified volume $V = 8400 \text{ m}^3$. Find the dimensions R and L that make its surface area S a minimum. Assume that the silo has a floor of the same material.



6. Write a C program for gauss-newton method in nonlinear fitting and use following Example : The following Figure shows a resistor-diode circuit using a type 1N914 silicon diode (D1) and a $10 \text{ K}\Omega$ resistor (R1). The following Table shows a list laboratory measurements of V_2 for various applied voltage levels of V_1 at room temperature (300°K).



V1(volts)	0.189	0.333	0.393	0.819	1.067	1.289	1.656	1.808	2.442	4.971
V2(volts)	0.189	0.317	0.356	0.464	0.487	0.501	0.518	0.522	0.541	0.566
V1(volts)	6.005	6.933	7.934	9.014	10.040	11.009	15.045	24.64	29.79	
V2(volts)	0.579	0.595	0.602	0.607	0.613	0.619	0.634	0.647	0.657	

$$i_D = I_s e^{\frac{V_r}{V_T}}$$