

## جواب کوئیز اول

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$$\int_1^2 x^r e^{-x} dx \rightarrow f(x) = x^r e^{-x} \rightarrow f'(x) = -x^r e^{-x} + r x^{r-1} e^{-x}$$

$$f''(x) = x^r e^{-x} - r x^{r-1} e^{-x} - r x^{r-1} e^{-x} + r(r-1)x^{r-2} e^{-x} = (x^r - 2r x^{r-1} + r(r-1)x^{r-2})e^{-x}$$

$$f'''(x) = (-x^r + 2r x^{r-1} - r(r-1)x^{r-2})e^{-x} = -[(x-r)^r + r(1-x)]e^{-x} < 0 : x \in [1, 2] \rightarrow M_r = 2$$

$$n \geq \sqrt{\frac{M_r(b-a)^r}{12\Delta I}} = \sqrt{\frac{2(1-0)^r}{12(0.005)}} = 5/\sqrt{7} \rightarrow n = 6, \quad h = \frac{b-a}{n} = \frac{1}{6}$$

$$I \approx \frac{1}{12} [f(0) + 2f(\frac{1}{6}) + 2f(\frac{2}{6}) + 2f(\frac{3}{6}) + 2f(\frac{4}{6}) + 2f(\frac{5}{6}) + f(1)] =$$

$$\frac{1}{12} [0 + 0.47 + 0.159 + 0.303 + 0.456 + 0.604 + 0.368] = 0.161$$

$$E = -\frac{(b-a)^r}{12n^r} f''(\xi) = -\frac{1}{432} f''(\xi) \rightarrow |E| \leq \frac{1}{432} M_r \rightarrow |E| \leq \frac{1}{216} \approx 0.0046$$

$$CT_n(f) = T_n(f) - \frac{h^r}{12} [f'(b) - f'(a)] \rightarrow CT_r(f) = 0.161 - \frac{1}{432} [e^{-1} - 0] = 0.160$$

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$$\begin{cases} 10x_1 - x_2 + 2x_3 = 6 \\ -x_1 + 11x_2 - x_3 + 3x_4 = 25 \\ 2x_1 - x_2 + 10x_3 - x_4 = -11 \\ 3x_2 - x_3 + 8x_4 = 15 \end{cases} \rightarrow \begin{bmatrix} 10 & -1 & 2 & 0 & 6 \\ -1 & 11 & -1 & 3 & 25 \\ 2 & -1 & 10 & -1 & -11 \\ 0 & 3 & -1 & 8 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -0.1 & 0.2 & 0 & 0.6 \\ -1 & 11 & -1 & 3 & 25 \\ 2 & -1 & 10 & -1 & -11 \\ 0 & 3 & -1 & 8 & 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -0.1 & 0.2 & 0 & 0.6 \\ 0 & 10.9 & -0.8 & 3 & 24.4 \\ 0 & -0.8 & 9.6 & -1 & -11.2 \\ 0 & 3 & -1 & 8 & 15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -0.1 & 0.2 & 0 & 0.6 \\ 0 & 1 & -0.07 & 0.28 & 2.25 \\ 0 & -0.8 & 9.6 & -1 & -11.2 \\ 0 & 3 & -1 & 8 & 15 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0.19 & 0.3 & 0.84 \\ 0 & 1 & -0.07 & 0.28 & 2.25 \\ 0 & 0 & 9.54 & -0.78 & -10.32 \\ 0 & 0 & -0.79 & 7.16 & 7.95 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0.19 & 0.3 & 0.84 \\ 0 & 1 & -0.07 & 0.28 & 2.25 \\ 0 & 0 & 1 & -0.08 & -1.08 \\ 0 & 0 & -0.79 & 7.16 & 7.95 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0.5 & 1.05 \\ 0 & 1 & 0 & 0.27 & 2.27 \\ 0 & 0 & 1 & -0.08 & -1.08 \\ 0 & 0 & 0 & 7.10 & 7.10 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$