

1.

$$F(s) = \frac{s+4}{s(s-1)(s-5)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-5} = \frac{\frac{4}{5}}{s} + \frac{\frac{5}{-4}}{s-1} + \frac{\frac{9}{20}}{s-5}$$

$$A = \lim_{s \rightarrow 0} sF(s) = \frac{4}{(-1)(-5)} = \frac{4}{5}$$

$$B = \lim_{s \rightarrow 1} (s-1)F(s) = \frac{5}{(1)(-4)} = -\frac{5}{4}$$

$$C = \lim_{s \rightarrow 5} (s-5)F(s) = \frac{9}{(5)(4)} = \frac{9}{20}$$

$$\text{Re}(s) > 5 \rightarrow f(t) = \frac{4}{5}u(t) + \frac{5}{-4}e^t u(t) + \frac{9}{20}e^{5t} u(t)$$

$$\text{Re}(s) < 0 \rightarrow f(t) = -\frac{4}{5}u(-t) + \frac{5}{4}e^t u(-t) - \frac{9}{20}e^{5t} u(-t)$$

$$0 < \text{Re}(s) < 1 \rightarrow f(t) = \frac{4}{5}u(t) + \frac{5}{4}e^t u(-t) - \frac{9}{20}e^{5t} u(-t)$$

$$1 < \text{Re}(s) < 5 \rightarrow f(t) = \frac{4}{5}u(t) + \frac{5}{-4}e^t u(t) - \frac{9}{20}e^{5t} u(-t)$$

2.

$$x(t) \rightarrow X(\omega)$$

$$e^{2jt} x(t) \rightarrow X(\omega - 2)$$

$$e^{2jt} x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega - 2) e^{j\omega t} d\omega$$

$$t = \frac{1}{2} \rightarrow e^j x\left(\frac{1}{2}\right) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega - 2) e^{\frac{j\omega}{2}} d\omega$$

$$\int_{-\infty}^{\infty} X(\omega - 2) e^{\frac{j\omega}{2}} d\omega = 2\pi e^j x\left(\frac{1}{2}\right) = 3\pi e^j$$

3.

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$$X(s) = \frac{1}{s-3}, \text{Re}(s) > 3$$

$$Y(s) = H(s)X(s), R_1 \cap R_2$$

$$Y(s) = \frac{s+8}{(s-4)(s-2)(s-3)}, 3 < \text{Re}(s) < 4$$

$$Y(s) = \frac{A}{s-4} + \frac{B}{s-2} + \frac{C}{s-3} = \frac{6}{s-4} + \frac{5}{s-2} - \frac{11}{s-3}$$

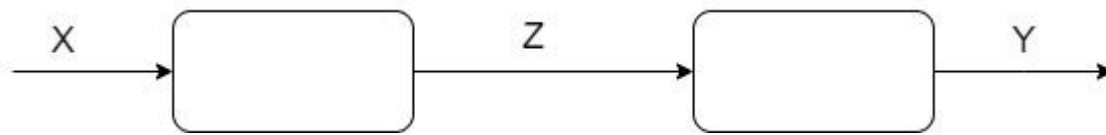
$$A = \lim_{s \rightarrow 4} (s-4)Y(s) = \frac{(12)}{(2)(1)} = 6$$

$$B = \lim_{s \rightarrow 2} (s-2)Y(s) = \frac{(10)}{(-2)(-1)} = 5$$

$$C = \lim_{s \rightarrow 3} (s-3)Y(s) = \frac{(11)}{(-1)(1)} = -11$$

$$y(t) = -6e^{4t}u(-t) + 5e^{2t}u(t) - 11e^{3t}u(t)$$

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$$H(s) = \frac{s+8}{(s-4)(s-2)}$$

$$Z = \frac{s+8}{(s-4)}X \rightarrow sZ - 4Z = sX + 8X \rightarrow \frac{dz}{dt} - 4z = \frac{dx}{dt} + 8x$$

$$Y = \frac{1}{(s-2)}Z \rightarrow sY - 2Y = Z \rightarrow \frac{dy}{dt} - 2y = z$$

