

پاسخ کویز دوم

۱- یک پریود از این سیگنال برابر است با: $x_1(n) = \delta(n) - \delta(n-3)$

$$X_1(\Omega) = 1 - e^{-j3\Omega} \rightarrow C_k = \frac{1}{\pi} X_1(\Omega) \Big|_{\Omega=k\frac{\pi}{\tau}} = \frac{1}{\pi} (1 - e^{-jk\frac{\pi}{\tau}}) \rightarrow C_0 = 0, C_1 = \frac{1}{\pi} (1 - j)$$

$$C_r = \frac{1}{\pi} (\tau) = 0.5, C_{-r} = \frac{1}{\pi} (1 + j)$$

$$X(z) = \frac{z^{-1} - 3z^{-2}}{(1-z^{-1})(1-2z^{-1})} = \frac{z-3}{(z-1)(z-2)} \rightarrow X_1(z) = \frac{X(z)}{z} = \frac{z-3}{z(z-1)(z-2)} = \frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2}$$

ناحیه همگرایی برابر است با: $|z| < 1$

$$A = \lim_{z \rightarrow 0} zX_1(z) = \lim_{z \rightarrow 0} \frac{z-3}{(z-1)(z-2)} = -1/5, B = \lim_{z \rightarrow 1} (z-1)X_1(z) = \lim_{z \rightarrow 1} \frac{z-3}{z(z-2)} = 2$$

$$C = \lim_{z \rightarrow 2} (z-2)X_1(z) = \lim_{z \rightarrow 2} \frac{z-3}{z(z-1)} = -1/5 \rightarrow X(z) = -1/5 + \frac{2}{1-z^{-1}} - \frac{1/5}{1-2z^{-1}}$$

$$x(n) = -1/5\delta(n) - 2u(-n-1) + 1/5(2)^n u(-n-1)$$

$$X(z) = \frac{z^{-1} - 3z^{-2}}{(1-z^{-1})(1-2z^{-1})} = \frac{z^{-1} - 3z^{-2}}{1 - 3z^{-1} + 2z^{-2}} \quad \text{روش دوم:}$$

$$\begin{array}{l} -3z^{-2} + z^{-1} \quad | \quad 2z^{-2} - 3z^{-1} + 1 \\ -3z^{-2} + 6z^{-1} - 1/5 \quad | \quad -1/5 - 1/75z - 1/175z^2 - 1/9375z^3 \\ \hline -3/5z^{-1} + 1/5 \\ -3/5z^{-1} + 6/5 - 1/75z \\ \hline -3/5 + 1/75z \\ -3/5 + 6/5 - 1/75z - 1/175z^2 \\ \hline -3/5 + 1/75z - 1/175z^2 \\ -3/5 + 6/5 - 1/75z - 1/175z^2 - 1/9375z^3 \\ \hline -3/5 + 1/75z - 1/175z^2 + 1/9375z^3 \end{array}$$

پس: $x(0) = -1/5, x(-1) = -1/75, x(-2) = -1/175, x(-3) = -1/9375$

$$H(z) = \frac{1}{1-z^{-1}} = \frac{Y(z)}{X(z)} \rightarrow y(n) - y(n-1) = x(n) \rightarrow Y(z) - z^{-1}Y(z) - y(-1) = X(z) \quad 4$$

$$Y(z) - z^{-1}Y(z) - (-1) = \frac{1}{1-z^{-1}} \rightarrow (1-z^{-1})Y(z) = \frac{1}{1-z^{-1}} - 1 = \frac{z^{-1}}{1-z^{-1}} \rightarrow Y(z) = \frac{z^{-1}}{(1-z^{-1})^2}$$

$$u(n) \rightarrow \frac{1}{1-z^{-1}} \Rightarrow nu(n) \rightarrow -z \frac{d}{dz} \left(\frac{1}{1-z^{-1}} \right) = -z \frac{-z^{-2}}{(1-z^{-1})^2} = -\frac{-z^{-1}}{(1-z^{-1})^2} \Rightarrow u(n) = -nu(n)$$