

$$\tilde{x}(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-jk \frac{2\pi}{N} n} = \frac{1}{4} \sum_{n=0}^3 x(n) e^{-jk \frac{2\pi}{4} n}$$

$$= \frac{1}{4} (1 - e^{-jk \frac{2\pi}{4}} + e^{-jk\pi})$$

$$\tilde{x}(0) = \frac{1}{4} (1 - 1 + 1) = \frac{1}{4}$$

$$\tilde{x}(1) = \frac{1}{4} (1 - e^{-j \frac{2\pi}{4}} + e^{-j\pi}) = \frac{1}{4} (-e^{-j \frac{\pi}{2}}) = \frac{1}{4} (i)$$

$$\tilde{x}(2) = \frac{1}{4} (1 - e^{-j\pi} + e^{-j2\pi}) = \frac{1}{4} (1 - (-1) + 1) = \frac{3}{4}$$

$$\tilde{x}(3) = \frac{1}{4} (1 - e^{-j \frac{3 \cdot 2\pi}{4}} + e^{-j3\pi}) = \frac{1}{4} (-e^{-j \frac{3\pi}{2}}) = \frac{1}{4} (-i)$$

$$\frac{Y(z)}{z} = H(z)X(z) = \frac{(z-2)}{(z-1)^2} \cdot 1 = \frac{A}{(z-1)} + \frac{B}{(z-1)^2}$$

$$A = \frac{1}{(2-1)!} \lim_{z \rightarrow 1} \frac{d}{dz} \left[(z-1)^2 \frac{(z-2)}{(z-1)^2} \right] = 1$$

$$B = \lim_{z \rightarrow 1} (z-1)^2 \frac{(z-2)}{(z-1)^2} = -1$$

$$Y(z) = \frac{z}{(z-1)} + \frac{-z}{(z-1)^2} = \frac{1}{1-z^{-1}} + \frac{-z^{-1}}{(1-z^{-1})^2}$$

$$\rightarrow y(n) = u(n) - nu(n)$$

$$h(n) \rightarrow H(z)$$

$$h(n-2) \rightarrow z^{-2}H(z)$$

$$z^{-2}H(z) = \sum_{-\infty}^{\infty} h(n-2)z^{-n}$$

$$(-1)^{-2}H(-1) = \sum_{-\infty}^{\infty} h(n-2)(-1)^{-n} = \sum_{-\infty}^{\infty} h(n-2)(-1)^n$$

$$= H(-1) = \frac{-1(-1-2)}{(-1-1)^2} = \frac{3}{4}$$

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$$y(n) - 2y(n-1) = 3x(n) + x(n-1)$$

$$x(n) = \delta(n)$$

$$X(z) = 1$$

$$Y(z) - 2z^{-1}Y(z) - 2y(-1) = 3X(z) + z^{-1}X(z) + x(-1)$$

$$Y(z)(1 - 2z^{-1}) = 4 + 3 + z^{-1} + 0$$

$$Y(z) = \frac{7 + z^{-1}}{1 - 2z^{-1}} = \frac{7z^2 + z}{z^2 - 2z}$$

$$\frac{Y(z)}{z} = \frac{7z + 1}{(z-2)z} = \frac{A}{z} + \frac{B}{z-2} = \frac{-1}{z} + \frac{15}{z-2}$$

$$Y(z) = \frac{-1}{z} + \frac{15}{z-2} = \frac{-1}{z} + \frac{15}{1-2z^{-1}}$$

$$\rightarrow y(n) = \frac{-1}{2}\delta(n) + \frac{15}{2}(2)^n u(n)$$