K.N.TOOSI UNIVERSITY OF TECHNOLOGY

POSITIONING WITH GPS

Written By:
Dr. Hamid Ebadi

Aug 2000

CONCEPTS AND MODELS OF GPS POSITIONING

Global Positioning System (GPS) concepts and the mathematical models used in kinematic positioning are outlined in this lecture note. The basic concept of GPS, the GPS observables, and their associated mathematical models are described. The various errors affecting GPS positioning and the remedies to reduce or eliminate them are also explained.

1. 1. Global Positioning System (GPS)

GPS is a satellite-based radio positioning system developed by the U.S. Department of Defense (DOD) for accurate positioning and navigation. Radio signals are used from a constellation of earth-orbiting satellites to determine the 3D position of a receiver. The system consists of 21 satellites and three spare satellites orbiting approximately 20,000 km above the earth's surface in six orbital planes, having a period of 12 hours. GPS is an all-weather positioning system providing 24 hour world-wide coverage with at least four satellites in view at any time (Milliken et al., 1990 and Wells et al., 1986). The system has been fully operational since 1993.

GPS has three main components; the satellite system, the control system, and the users. The control system is operated by the U.S. Air Force for the Joint Program Office (JPO) of the DOD. The system consists of five monitoring stations distributed around the world. The role of these stations is to monitor the health of the satellites. These tracking stations receive signals from the satellites and transmit the collected data to the master station where new ephemerides are computed and the navigation messages are prepared for uploading to the satellites.

1. 1. 1. User Segment

Users are the third component of GPS. Civilian users wish to determine their positions using GPS signals. There are mainly three observables which have been implemented in most GPS receivers:

- Pseudorange
- Carrier beat phase
- The rate of phase change

Both position and velocity of a moving platform can be calculated by measuring signals from different GPS satellites (Wells et al., 1986).

1. 2. GPS Signals

The GPS signals are transmitted autonomously from all GPS satellites on two carrier frequencies; L1 frequency at 1575.42 MHz and L2 frequency at 1227.60 MHz. C/A code of 1.023 MHz is modulated on the L1 carrier and P code of 10.23 MHz is modulated on both L1 and L2 carriers. A satellite message containing the satellites' ephemeris is also modulated on both carriers. A summary of the signal components is given in Table 1. 1.

Table 1. 1. GPS Signal Components (from Erickson, 1992)

Carrier	Frequency	Wavelength	Modulation	Frequency	Chip length
L1	1575.42 MHz	19 cm	C/A code P code Message	1.023 MHz 10.23 MHz 50 MHz	293 m 29.3 m
L2	1227.60 MHz	24 cm	P code Message	10.23 MHz 50 MHz	29.3 m

There are two types of receivers; Single Frequency (receiving only L1 signal) and Dual Frequency (receiving both L1 and L2 signals). Most C/A code receivers correlate the incoming signal from a satellite with a replica of the code generated in the receiver. The dual frequency receivers provide access to P code data through code correlation resulting in a full L2 wavelength of 24 cm. Due to a high absolute accuracy available using P codes, Selective Availability (SA) is turned on to deteriorate the positioning accuracy.

The type of data that a receiver collects has a direct impact on both achievable accuracy and its price. The C/A code receivers are the least expensive receivers on the market which determine real time positions with horizontal accuracy of 100 m and vertical accuracy of 156 m (Lachapelle, 1993). P code receivers provide accuracies at the level of 25 m (horizontal) and 30 m (vertical) in real time mode. Access to P code is limited to U.S. and NATO military users.

Receivers which compute their positions based on carrier phase observations are more accurate because of the much finer resolution of the 19 cm and 24 cm carrier wavelengths. The most sophisticated and expensive receivers are dual frequency P code receivers that provide accuracy ranging from a part per million to a few part per billion.

Between theses two extreme cases, one can find a wide range of receivers which meet the users' accuracy requirements.

1. 3. GPS Observables

A pseudorange (code observation) is the difference between the transmission time at the satellite and the reception time at the receiver (Erickson, 1992). Pseudorange between the satellite and the receiver is obtained by scaling it using the speed of light. The observation equation for a pseudorange is given as (Wells et al., 1986);

$$p = \rho + c(dt - dT) + d_{\rho} + d_{ion} + d_{trop} + \varepsilon_p \qquad 1.\ 1$$

where

p is the observed pseuodorange,

ρ is the unknown satellite-receiver range,

c is the speed of light,

dt is the satellite clock error,

dT is the receiver clock error,

d_{ρ} is the orbital error,

d_{ion} is the ionospheric error,

d_{trop} is the tropospheric error,

ε_p is the code measurement noise and multipath.

The code measurement noise, ε_p, is a function of the code receiver noise, ε_{prx}, and multipath, ε_{mult}, (Lachapelle, 1991).

The satellite-receiver range, ρ, has the form of:

$$\rho = \sqrt{\left(X^s - X_r\right) + \left(Y^s - Y_r\right) + \left(Z^s - Z_r\right)} \qquad 1.\ 2$$

where

$\left(X^s, Y^s, Z^s\right)$ are satellite coordinates computed using broadcast ephemeris,

$\left(X_r, Y_r, Z_r\right)$ are the unknown receiver coordinates.

For single point positioning, the number of unknowns are four $\left(X_r, Y_r, Z_r, dT\right)$, therefore, a minimum of four satellites are required to solve for a solution at a single epoch.

The carrier phase observation is a measure of the misalignment between an incoming signal and replica of it generated by the receiver oscillator when a satellite is locked on. If a continuous lock is assumed, this measurement is a sum of the initial phase misalignment at epoch t_0 and the number of integer cycles from epoch t_0 to the current epoch t. The measured carrier phase can be written as (Erickson, 1992):

$$\Phi_{measured} = \text{fraction}(\Phi) + \text{integer}(\Phi, t_0, t) \qquad 1.3$$

Carrier phase measurements are converted from cycles to units of lengths by their wavelengths. An ambiguity term (the unknown number of integer cycles between the satellite and receiver at starting epoch t_0)should be added to carrier phase measurement in order to represent a satellite-receiver range. The carrier phase observation equation is written as (Lachapelle, 1993):

$$\Phi = \rho + c(dt - dT) + \lambda N + d_\rho - d_{ion} + d_{trop} + \varepsilon_\Phi \qquad 1.4$$

where

Φ is the observed carrier phase,

ρ is the unknown satellite-receiver range,

c is the speed of light,

dt is the satellite clock error,

dT is the receiver clock error,

λ is the carrier wavelength,

N is the unknown integer cycle ambiguity,

d_ρ is the orbital error,

d_{ion} is the ionospheric error,

d_{trop} is the tropospheric error,

ε_Φ is the carrier phase measurement noise and multipath.

The differences between pseudorange and carrier phase observation equations are the addition of ambiguity term, λN, for carrier phase observations and the reversal of signs for the ionospheric correction term d_{ion} due to the phase advance, while code is delayed.

Doppler frequency is the third fundamental GPS observation which is the first derivative of the carrier phase with respect to time. The Doppler frequency is measured on the pseudorange. The observation equation for GPS Doppler frequency can be written as (Liu, 1993):

$$\dot{\Phi} = \dot{\rho} + c(\dot{d}t - \dot{d}T) + \dot{d}_\rho - \dot{d}_{ion} + \dot{d}_{trop} + \varepsilon_{\dot{\Phi}} \qquad 1.5$$

where (·) denotes a time derivative. As seen in the above equation, this measurement is not a function of the carrier phase ambiguity, therefore, it is free from cycle slips and can be used to determine the receiver velocity.

1. 4. GPS Error Sources

The GPS errors consist of orbital errors, satellite and receiver clock errors, tropospheric and ionospheric delays, receiver noise, and multipath. They are explained in the following sections.

2. 4. 1 Orbital Error

Orbital error initiates from the uncertainties of the predicted ephemerides and Selective Availability (SA). An estimation of the broadcast ephemerides error is about 20 m. If post-mission ephemerides are used, then the precise orbits are accurate to 1 m. SA is implemented by both satellite clock dithering and degrading satellite orbital information to prevent unauthorized real-time use of full GPS position and velocity accuracy.

1. 4. 2. Satellite and Receiver Clock Errors

The satellite clock error is defined as the difference between satellite clock time and true GPS time. The functional relationship between these two times is given as (Wells et al., 1986):

$$\Delta t_{sv} = a_0 + a_1(t - t_0) + a_2(t - t_0)^2 \qquad 1.\ 6$$

where

Δt_{sv} is the difference between satellite clock and GPS time,

t is the measurement transmission time,

t_0 is the reference time,

a_0 is the satellite clock time offset,

a_1 is the frequency offset,

a_2 is the frequency drift.

GPS satellites use atomic clocks which maintain a highly accurate GPS time. However, the accuracy is deteriorated by SA. Single differencing between two receivers removes the satellite clock error.

Receiver clock error is defined as the offset of the receiver clock time with respect to GPS time. Geodetic receivers are generally synchronized with GPS time before observation sessions but this synchronization accuracy is at the millisecond level. The receiver clock may also drift after synchronization. The receiver clock error depends on receiver hardware and can be estimated as an unknown parameter or eliminated by differencing from one receiver to two satellites.

1. 4. 3. Tropospheric and Ionospheric Delays

The tropospheric delay is caused by the refractions of a GPS signal in the lower atmosphere (the layer from the earth surface to approximately 60 km). The magnitude of this error is influenced by a number of parameters such as the temperature, humidity, pressure, and the type of the terrain below the signal path. A number of studies have been performed to create tropospheric models (Hopfield, 1969, Saastamoinen, 1973, Black, 1978). A thorough analysis of these models can be found in (Hoffmann et al., 1992).

The ionospheric layer is roughly from 50 to 1000 km above the earth surface. GPS signals traveling through the ionosphere are affected by both refraction and dispersion. The refractive group index of the ionosphere is greater than 1, meaning that the group velocity of radio waves is less than the speed of light in vacuum. The refractive phase index of ionosphere is less than 1, therefore, the phase velocity of radio waves is greater than the speed of light in vacuum. This causes an advance on the measured carrier phase and delay on the measured pseudorange. The ionospheric delay is directly affected by the Total Electron Content (TEC) along the propagation path (Klobuchar, 1983). The ionospheric error may range from 5 m (at night, the satellite at the zenith) to 150 m (at midday and the satellite at low elevation)(Wells et al. 1986).

Ionospheric effect can be assessed by taking dual frequency measurements and using the dispersive nature of the ionosphere. The techniques based on dual frequency correction can remove most of the ionospheric error. However, during high solar activity cycle and mid afternoons this technique may not be adequate for certain applications (Well et al., 1986). Another way to reduce ionospheric effect is to use differencing observations from one satellite between two stations due to the spatial correlation between the stations. The third method is to apply the broadcast model for reducing the ionospheric error (Klobuchar, 1983).

1. 4. 4. Receiver Noise

Receiver measurement noise includes the thermal noise intercepted by the antenna or generated by the internal components of the receiver (Martin, 1980). It is affected by signal to noise density, the tracking bandwidth, and code tracking mechanization parameters. The noise levels for C/A code pseudorange is 1m, for P code pseudorange is 10 cm and for carrier phase is 5 mm. The new narrow correlator C/A code receivers can achieve 10 cm accuracy for C/A code pseudorange.

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1. 4. 5 Multipath

Multipath is the phenomena where the reception of signals is reflected by objects and surfaces in the environment around the antenna (Liu, 1993). Pseudorange multipath can reach up to one chip length of the PRN codes (e.g. 293 m for C/A code and 29.3 m for P code) while carrier phase multipath is less than 25% of the carrier phase wavelength (e.g. 5 cm for L1)(Georgiadou and Kleusberg, 1989). In an airborne GPS environment, multipath error signatures are generally randomized due to the aircraft motion and flexing (Shi, 1994).

1. 5. Differential GPS

In order to achieve high accuracy for geodetic positioning, differential GPS techniques are used to eliminate or reduce several GPS error sources.

1. 5. 1 Single Differencing

The observation equations for pseudorange, carrier phase, and Doppler frequency contain bias terms such as satellite and receiver clock errors, orbital errors, and atmospheric effects. Many of these errors are spatially correlated to some extent between the receivers tracking simultaneous satellites. Some errors are satellite dependent (orbital, atmospheric, and satellite clock errors) and some errors are receiver dependent (receiver clock error). Single differencing (between satellites or between receivers) and double differencing (between receivers and between satellites) of GPS observations can be applied to eliminate or effectively reduce the common errors. The single " between receivers" and " between satellites" differences are shown in Figures 1.1 and 1.2.

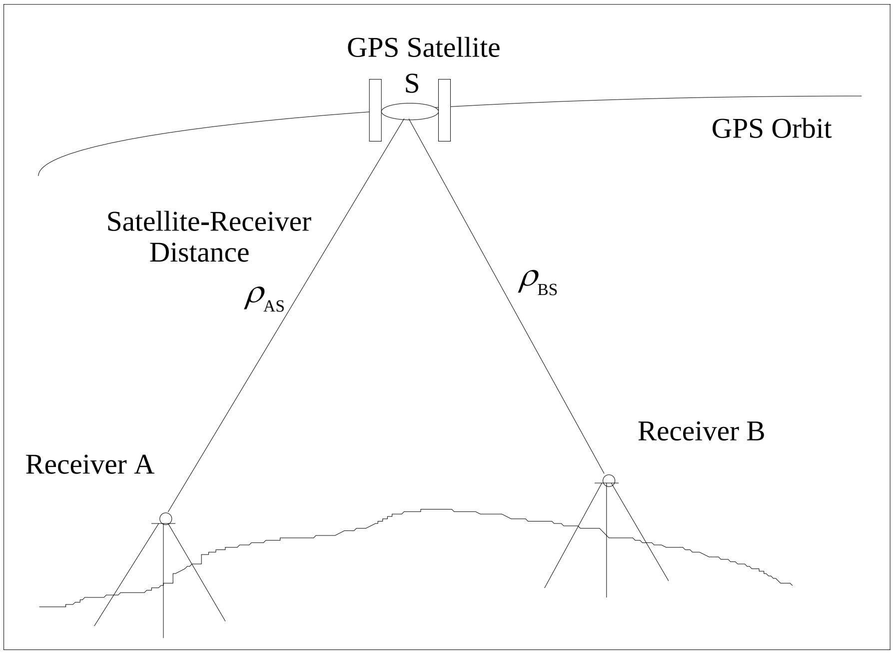

Figure 1.1. Single Differencing Between Receivers

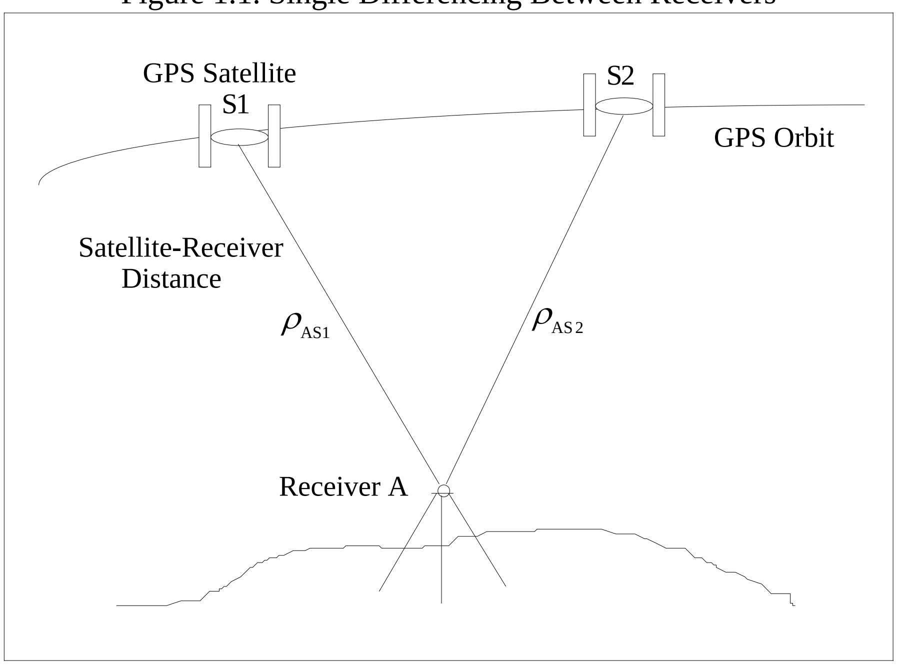

Figure 1.2. Single Differencing Between Satellites

The single difference equations for the pseudorange, carrier phase, and Doppler frequency are (Lachapelle, 1991):

"between receivers"

$$\Delta\rho = \Delta\rho + \Delta d_{\rho} - c\Delta dT + \Delta d_{ion} + \Delta d_{trop} + \Delta \varepsilon_{p} \qquad 1.7$$

$$\Delta\Phi = \Delta\rho + \Delta d_{\rho} - c\Delta dT + \lambda\Delta N - \Delta d_{ion} + \Delta d_{trop} + \Delta \varepsilon_{\Phi} \qquad 1.8$$

$$\Delta\dot{\Phi} = \Delta\dot{\rho} + \Delta\dot{d}_{\rho} - c\Delta\dot{d}T - \Delta \dot{d}_{ion} + \Delta \dot{d}_{trop} + \Delta \varepsilon_{\dot{\Phi}} \qquad 1.9$$

"between satellites"

$$\nabla\rho = \nabla\rho + \nabla d_{\rho} + c\nabla dt + \nabla d_{ion} + \nabla d_{trop} + \nabla \varepsilon_{p} \qquad 1.10$$

$$\nabla\Phi = \nabla\rho + \nabla d_{\rho} + c\nabla dt + \lambda\nabla N - \nabla d_{ion} + \nabla d_{trop} + \nabla \varepsilon_{\Phi} \qquad 1.11$$

$$\nabla\dot{\Phi} = \nabla\dot{\rho} + \nabla\dot{d}_{\rho} + c\nabla\dot{d}t - \nabla \dot{d}_{ion} + \nabla \dot{d}_{trop} + \nabla \varepsilon_{\dot{\Phi}} \qquad 1.12$$

where

Δ denotes a single difference operator between receivers,

∇ denotes a single difference operator between satellites.

In the single difference observable (between receivers), the satellite clock error has been eliminated and the orbital error and atmospheric effects have been reduced and their residuals can be neglected for monitor-remote distances less than 30 km under normal atmospheric conditions. The relative receiver clock error, however, may be significant and must be estimated along with the parameters of position, velocity, and carrier phase ambiguity (Liu, 1993). Single difference observable (between satellites) eliminates receiver clock error.

1. 5. 2. Double Differencing

This technique is based on taking difference between receivers and between satellites (Figure 1.3).

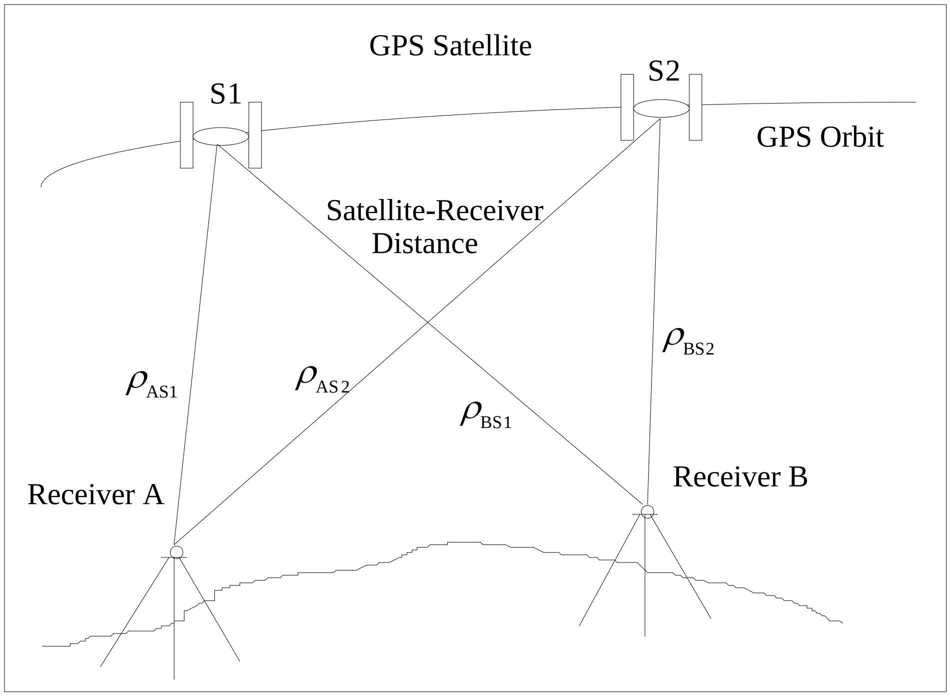

Figure 1.3. Double Differencing

Double difference observation equations are written as(Lachapelle, 1993):

$$\nabla\Delta\rho = \nabla\Delta\rho + \nabla\Delta d_{\rho} + \nabla\Delta d_{ion} + \nabla\Delta d_{trop} + \nabla\Delta \varepsilon_{p} \tag{1.13}$$

$$\nabla\Delta\Phi = \nabla\Delta\rho + \nabla\Delta d_{\rho} + \lambda\nabla\Delta N - \nabla\Delta d_{ion} + \nabla\Delta d_{trop} + \nabla\Delta \varepsilon_{\Phi} \tag{1.14}$$

$$\nabla\Delta\dot{\Phi} = \nabla\Delta\dot{\rho} + \nabla\Delta\dot{d}_{\rho} - \nabla\Delta \dot{d}_{ion} + \nabla\Delta \dot{d}_{trop} + \nabla\Delta \varepsilon_{\dot{\Phi}} \tag{1.15}$$

where $\nabla\Delta$ denotes the double difference operator between two stations and two satellites.

The advantage of using this observable is that both the receiver and satellite clock errors have been canceled out, while the disadvantage is increased noise. This method also allows to optimally exploit the integer nature of carrier phase ambiguity. Double differencing GPS positioning is considered as the best processing method (Cannon, 1987,1991, Remondi,1984). This observable still contains the double difference ambiguity term which has to be resolved before the beginning of the kinematic mission and then fixed in kinematic surveys. In airborne kinematic positioning, cycle slips may often occur in carrier phase observation due to aircraft dynamics (e.g. turning) and multipath effects. Therefore, it is mandatory to resolve ambiguity on the fly for precise GPS positioning.

1. 6. Algorithms for Kinematic GPS

There are mainly two algorithms being used in kinematic GPS; Kalman filtering and least squares (Schwarz et al., 1989, Cannon, 1987,1991, Georgiadou and Kleusberg, 1991). Under certain conditions, one algorithm is equivalent to the other one in terms of computational aspects. It is important to know about the features of the algorithms and their relationships in kinematic GPS.

1. 6. 1 Kalman Filter Algorithm

Assuming the system model and measurement model have the form of:

$$X_k = \Phi_{k,k-1} X_{k-1} + W_{k,k-1} \qquad 1.\ 16$$

$$l_k = A_k X_k + \varepsilon_k \qquad 1.\ 17$$

for the update equations:

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k\{l_k - A_k \hat{X}_k(-)\} \qquad 1.\ 18$$

$$C_k^X(+) = \{I - K_k A_k\} C_k^X(-) \qquad 1.\ 19$$

$$K_k = C_k^X(-) A_k^T \{A_k C_k^X(-) A_k^T + C_l^{-1}\}^{-1} \qquad 1.\ 20$$

and for the prediction equations:

$$\hat{X}_k(-) = \Phi_{k,k-1} \hat{X}_{k-1}(+) \qquad 1.\ 21$$

$$C_k^X(-) = \Phi_{k,k-1} C_{k-1}^X(+) \Phi_{k,k-1}^T + C_{k,k-1}^W \qquad 1.\ 22$$

where

X is the state vector,

Φ is the transition matrix,

I is the identity matrix,

W is the system process noise vector,

A is the design matrix,

ε is the measurement noise,

k is the epoch number,

C^W is the covariance matrix of W,

K is the Kalman gain matrix,

C_l is the covariance matrix of l,

C^X is the covariance matrix of X,

(-) is a predicted quantity,

(+) is an updated quantity,

(^) is an estimated quantity.

Different definitions of the transition matrix, Φ, and the covariance of the system process noise, C^W, can be used based on the choice of the state space model for kinematic GPS (Schwarz et al., 1989). The covariance matrix of the system process noise, C^W, is given as (Shi, 1994):

$$C^W = \int_0^{\Delta t} \Phi(z) Q(z) \Phi^T(z) dz \qquad 1.\ 23$$

where Q is the spectral density matrix. The state space model is affected by parameters such as, the system dynamics, state vector, and the assumption on the process behavior of the system (Gelb, 1974, Schwarz et al., 1989). The state space model plays an important role in improving the interpolation accuracy when the data rate is low. Schwarz et al.

(1989) have shown that with a 3 seconds data rate, positioning accuracy improves when using a constant velocity model and velocity accuracy improves when using a constant acceleration model.

The Kalman filter can be implemented with different kinematic GPS models and different measurements (Shi, 1994). The process noise is also fully used in the filter by considering the spectral density matrix, Q, which allows the system to adjust the contribution to the estimates from the observables at the measurement epoch versus a contribution before the epoch.

The Kalman filter is usually employed in kinematic GPS applications where the remote receiver is installed on a moving platform and the reference receiver is set up on the ground station.

1. 6. 2. Least Squares Algorithm

The least squares algorithm for kinematic GPS does not use dynamic information (Georgiadou and Kleusberg, 1991). In this algorithm, no assumption is made on the remote motion and no system process noise is considered. If a priori information about unknown parameter is used, the approach is called sequential least squares but if only observables at the measurement epoch are used, it is called the least squares approach.

If the measurement model is considered as:

$$l_k = A_{k-1} X_{k-1} + \varepsilon_{k-1} \tag{1. 24}$$

then the equation for the estimated vector and its covariance matrix in the sequential l least squares approach are given as (Krakiwsky, 1990):

$$\hat{X}_k(-) = \hat{X}_{k-1}(+) + \Delta X \qquad 1.\ 25$$

$$C_k^X(-) = C_{k-1}^X + C^{\Delta X} \qquad 1.\ 26$$

$$\hat{X}_k = \hat{X}_k(-) + \left[A_k^T C_l^{-1} A_k + \{C_k^X\}^{-1}\right]^{-1} A_k^T C_l^{-1}\left[l_k - A_k \hat{X}_k(-)\right] \qquad 1.\ 27$$

$$C_k^X = \left[A_k^T C_l^{-1} A_k + \{C_k^X(-)\}^{-1}\right]^{-1} \qquad 1.\ 28$$

where

ΔX is the increment vector over two successive epochs,

$C^{\Delta X}$ is the covariance matrix of ΔX,

(-) is for an estimate based on data collected before epoch k.

The equations for least squares approach are written as:

$$\hat{X}_k = \left[A_k^T C_l^{-1} A_k\right]^{-1} A_k^T C_l^{-1} l_k \qquad 2.\ 29$$

$$C_k^X = \left[A_k^T C_l^{-1} A_k\right]^{-1} \qquad 1.\ 30$$

In the least squares approach, the discrete position of the remote station is computed by using observations at one epoch, without any need of the process noise information or dynamic assumption. Therefore, the positioning solutions in successive epochs are independent. This approach can be applied to the case when the reference receiver is used either in static or in kinematic mode and a high data rate is used. Shi (1994) found that with a 2 Hz or even 1 Hz data rate, the position of an aircraft (with the speed of 80 m per second) from the Kalman filter algorithm (with a constant velocity model) are identical to those from the least squares approach. The Kalman filter algorithm can be mathematically transformed to the least squares approaches. The mathematical proofs can be found in (Shi,1994).

The advantage of using Kalman filter is that it has a general form of the equations which allows the implementation of different kinematic GPS models and measurements. In addition to this, because of its flexibility, it can meet the needs of a practical application in different dynamic environments.

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