

۱. برای تابع $f(t) = a_0 \sin(2t)$ توابع چگالی احتمال، Autocorrelation و Power Spectral Density را محاسبه و تعیین نمایید و آنها را رسم کنید.

۲. یک نویز اتفاقی $x(t)$ را در نظر بگیرید و فرض کنید که تابع Autocorrelation آن دارای رفتار نوسانی به صورت زیر باشد.

$$\phi_{xx}(\tau) = e^{-a\tau} \cos(b|\tau| + c)$$

مطلوبست محاسبه تابع طیف چگالی توان (با اثبات کامل).

3. Design a system for controlling the shaft angle of DC motor. The entire state is measured and can be used to generate the motor control voltage. The desired shaft angle is zero degrees. Note that this can be yield any desired shaft angle by the appropriate definition of reference direction for shaft angle.

A block diagram for the DC motor is given in Figure where $\theta(t)$ is the motor shaft angle, and $u(t)$ is the DC voltage applied to the motor. Design a control system to minimize the cost function,

$$J(x(t), u(t)) = \frac{1}{2} \int_0^{0.4} \left\{ \begin{bmatrix} \theta(t) & \dot{\theta}(t) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix} + 10^{-8} u^2 \right\} dt$$

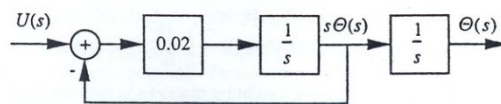


Fig. block diagram for DC motor

Simulate the closed loop system with an initial shaft angle of 10 degrees and an initial shaft velocity of zero. Using the steady-state gains, simulate the closed loop system with the same initial conditions. Plot the plant state and the control input for these two simulations and compare the results. Compute the cost for each of these two controllers and compare.

a. Repeat the above when the final time in the cost function is changed to 0.2. Comment on the result.

b. Repeat the above when the final time in the cost function is changed to 0.1. Comment on the result.