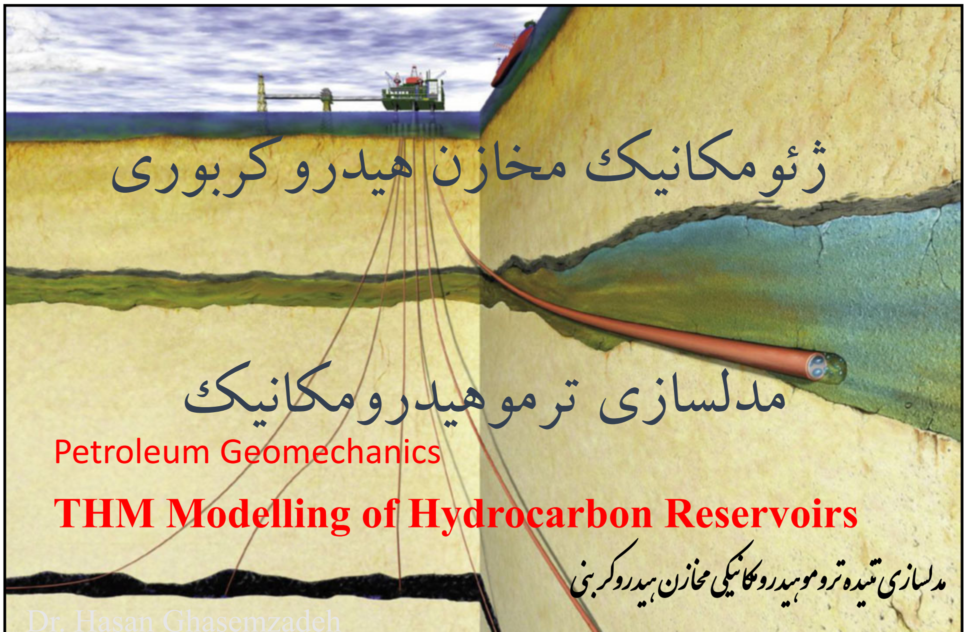


ژئومکانیک مخازن هیدرو کربوری

مدلسازی ترموهیدرومکانیک

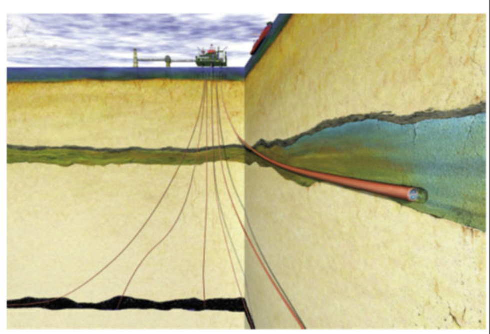
Petroleum Geomechanics

THM Modelling of Hydrocarbon Reservoirs

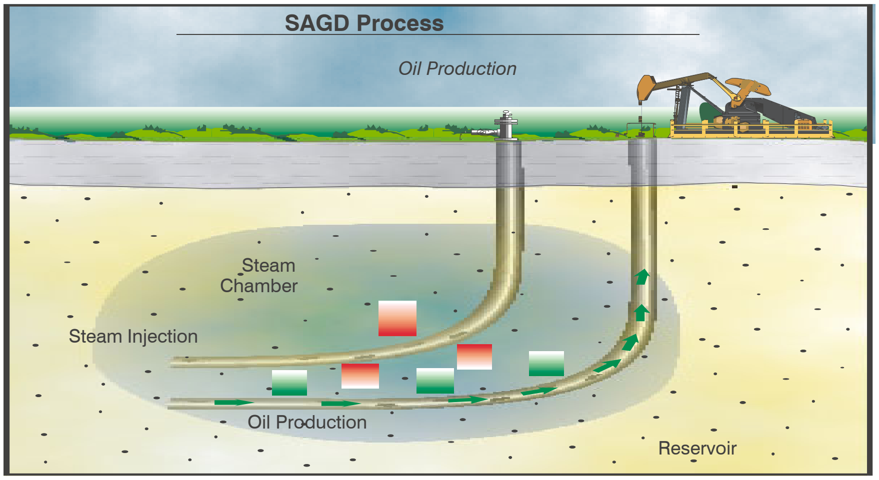
مدلسازی تنیده ترموهیدرومکانیکی مخازن هیدروکربنی

Dr. Hasan Ghasemzadeh
دکتر حسن قاسم زاده
K.N. Toosi University of Technology

Content



EOR- Steam Assisted Gravity Drainage



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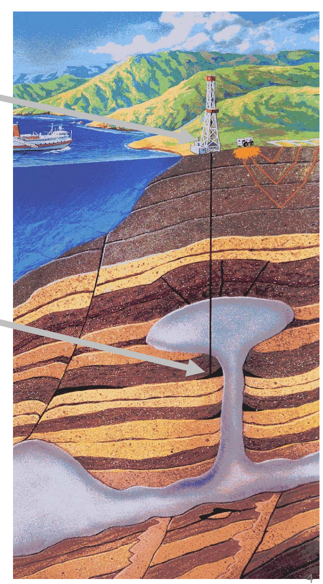
Subsidence
Equipment damage
Well damage

Reservoir deformation and
Temperature effects on flow regime
Fractured Reservoir
Sand production
Thermal Enhanced Oil Recovery



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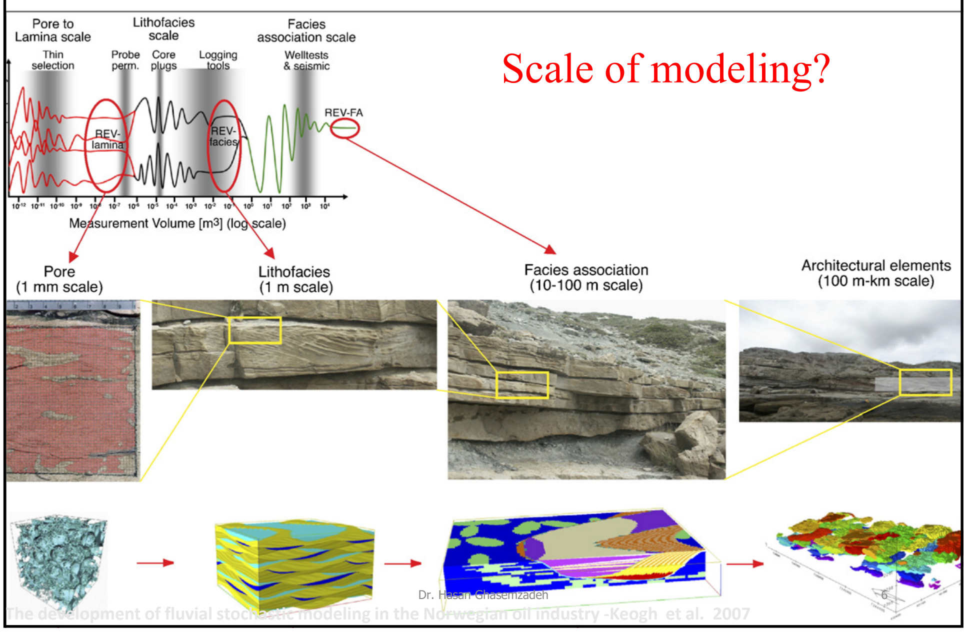
	Thermodynamic Force (Gradient of Potential)		
Fluxes J	Hydraulic head	Temperature	Stress
Fluid	Hydraulic conduction ***Darcy's law*** $v = -K\nabla p$	Thermo-osmosis Density changes	Consolidation or Swelling Fracture closure
Heat	Isothermal heat transfer	Thermal conduction ***Fourier's law*** $h = -\Lambda\nabla T$	thermoelastcity Phase change
Strain	Swelling or Consolidation Fracture opening (effective stress changing)	Thermal expansion Density changes	Constitutive law ***Hook's law*** $\sigma = E\,\varepsilon$ Elastoplastic or Elasto-Visco-Plastic

Scale of modeling?

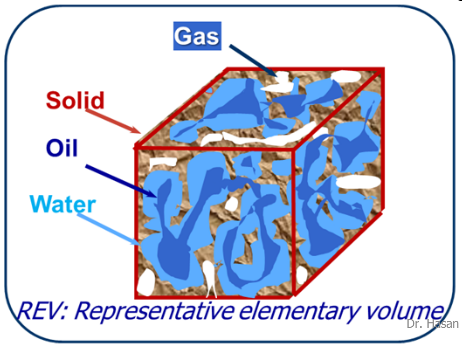


Scale: Microscopic – Macroscopic- Multiscale…

Equation base: Biot Theory- Mixed Theory,…

Solution Method: FEM-FDM-FVM-DEM, CVFEM…

Governing equations



Phases: Solid, Oil, Water and Gas are present everywhere.

Equations are writing on micro level and averaging in Macro scale based on Mixed-theory

Numerical solution is CVFEM

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- فرضیات بکار گرفته شده در مدلسازی
  - تغییر شکل محیط و گرادیان آنها بسیار کوچک فرض شده اند

$$\frac{\partial f}{\partial x} \approx \frac{\partial f}{\partial X}; \quad \boldsymbol{D} \approx -\frac{d\varepsilon}{dt}; \quad \boldsymbol{\varepsilon} = -\frac{1}{2}\left[\nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^T\right]$$

  - رابطه بین تانسور گرادیان سرعت فاز جامد و تانسور نرخ تغییر شکل

$$\nabla \boldsymbol{v}_s = \boldsymbol{D} + \boldsymbol{L}; \quad \boldsymbol{D} = \nabla \boldsymbol{v}_s + (\nabla \boldsymbol{v}_s)^T; \quad \boldsymbol{L} = \nabla \boldsymbol{v}_s - (\nabla \boldsymbol{v}_s)^T$$

  - رفتار فاز جامد بصورت رفتار حرارتی-کشسان خطی در نظر گرفته شده است
  - تمام فازهای موجود در یک نقطه بلافاصله به تعادل حرارتی می رسند
  - فاز جامد و فاز آب هیچگونه تبادل جرمی با فازهای دیگر ندارند
  - تنها تبادل جرم مجاز در مجموعه مخلوط حل شدن فاز گاز در فاز نفت می باشد
  - علائم استاندارد مکانیک خاک در معادلات استفاده شده است

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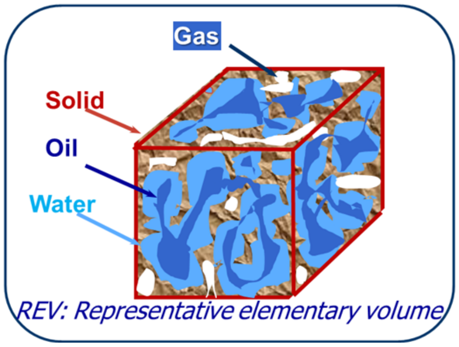

Phases: Solid, Oil, Water and Gas are present everywhere.

Equations are writing on micro level and averaging in Macro scale based on Mixed-theory

Numerical solution is CVFEM

$$\frac{Df_\alpha}{Dt} = \frac{\partial f_\alpha}{\partial t}$$: Lagrangian description for solid

$$\frac{Df_\alpha}{Dt} = \frac{\partial f_\alpha}{\partial t} + (\nabla f_\alpha).\boldsymbol{v}_\alpha$$: Eulerian description for fluids

$$\frac{D^s f_\alpha}{Dt} = \frac{\partial f_\alpha}{\partial t} + (\nabla f_\alpha).\boldsymbol{v}_s$$: Lagrangian-Eulerian description

$$\frac{Df_\alpha}{Dt} = \frac{D^s f_\alpha}{\partial t} + (\nabla f_\alpha).\boldsymbol{w}_\alpha$$

$$\boldsymbol{w}_\alpha = \boldsymbol{v}_\alpha - \boldsymbol{v}_s, \qquad \alpha \neq s$$

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- **Mass Balance**

$$\frac{D^s[n_w\rho_w]}{Dt} + n_w\rho_w\nabla.(\boldsymbol{w}_w + \boldsymbol{v}_s) + (\nabla n_w\rho_w)^T.\boldsymbol{w}_w = \dot{M}_w$$

$$\Rightarrow \frac{D^s[n_w\rho_w]}{Dt} + \nabla.(n_w\rho_w\boldsymbol{w}_w) + n_w\rho_w\nabla.\boldsymbol{v}_s = \dot{M}_w$$

✓ Water phase

$$\frac{D^s[n_o\rho_o]}{Dt} + \nabla.(n_o\rho_o\boldsymbol{w}_o) + n_o\rho_o\nabla.\boldsymbol{v}_s = e^o_{go} + \dot{M}_o$$

$$\rho_o = \frac{R_{so}\rho_{Gs} + \rho_{Os}}{B_o}; \quad \dot{M}_o = \dot{M}_{oO} + \dot{M}_{oG}$$

✓ Oil phase

$$\frac{D^s[n_g\rho_g]}{Dt} + \nabla.(n_g\rho_g\boldsymbol{w}_g) + n_g\rho_g\nabla.\boldsymbol{v}_s = e^g_{go} + \dot{M}_g$$

✓ Gas phase

$$e^g_{og} = -e^o_{og} = \dot{m}_{og}$$

✓ Mass transfer between oil and gas phases

Momentum Balance

$$(1-n)\rho_s \frac{D^s \boldsymbol{v}_s}{Dt} + \nabla.\big((1-n)\boldsymbol{t}_s\big) = \sum_{\beta \neq s} \boldsymbol{T}^s_{s\beta} + (1-n)\rho_s \boldsymbol{g}$$
Solid phase

$$n_\alpha \rho_\alpha \frac{D^s \boldsymbol{v}_\alpha}{Dt} + \nabla.(n_\alpha \boldsymbol{t}_\alpha) + n_\alpha \rho_\alpha (\nabla \boldsymbol{v}_\alpha).\boldsymbol{w}_\alpha = \sum_{\beta \neq \alpha} \boldsymbol{T}^\alpha_{\alpha\beta} + n_\alpha \rho_\alpha \boldsymbol{g}, \quad \begin{cases} \alpha = w, o, g \\ \beta = s, w, o, g \end{cases}$$
Fluid phases

$$\sum_\alpha \sum_{\beta \neq \alpha} \boldsymbol{T}^\alpha_{\alpha\beta} + \boldsymbol{v}_g e^g_{og} + \boldsymbol{v}_o e^o_{og} = 0, \qquad \alpha, \beta = s, w, o, g$$
Constraints: Momentum transfer between phases

$$\boldsymbol{T}^\alpha_{\alpha s} = -\boldsymbol{R}_\alpha n_\alpha \boldsymbol{w}_\alpha + p_\alpha \nabla n_\alpha; \quad \boldsymbol{K}_\alpha = n_\alpha (\boldsymbol{R}_\alpha)^{-1}; \quad \boldsymbol{K}_\alpha = \frac{k_{r\alpha} \boldsymbol{K}}{\mu_\alpha}$$
Momentum transfer between fluids and solid ignoring momentum transfer between fluids

$$\Rightarrow \quad \nabla.\boldsymbol{\sigma} = \rho \boldsymbol{g}$$
$$n_\alpha \boldsymbol{w}_\alpha = \boldsymbol{K}_\alpha \left[\rho_\alpha \boldsymbol{g} - \nabla p_\alpha \right]$$

$$k_{rw} = \mathbb{F}_1(n_w); \quad k_{rg} = \mathbb{F}_2(n_g); \quad k_{ro} = \mathbb{F}_3(n_w, n_g)$$
Relative Permeability

$$\boldsymbol{K}_{t+1} = \frac{n^3_{t+1}(1-n_t)^2}{n^3_t (1-n_{t+1})^2} \boldsymbol{K}_t$$
Absolute Permeability

Energy Balance

- Local equilibrium hypothesis

$$\rho C \frac{D^s T}{Dt} + (n_w \rho_w C_w \boldsymbol{w}_w + n_g \rho_g C_g \boldsymbol{w}_g + n_o \rho_o C_o \boldsymbol{w}_o).\nabla T - \rho h + \nabla.\boldsymbol{q} + e^g_{og} H_g + e^o_{og} H_o = 0$$

$$\begin{cases} \rho C = (1-n)\rho_s C_s + n_w \rho_w C_w + n_o \rho_o C_o + n_g \rho_g C_g \\ \rho h = (1-n)\rho_s h_s + n_w \rho_w h_w + n_o \rho_o h_o + n_g \rho_g h_g \\ \boldsymbol{q} = (1-n)\boldsymbol{q}_s + n_w \boldsymbol{q}_w + n_o \boldsymbol{q}_o + n_g \boldsymbol{q}_g \end{cases}$$

$$\boldsymbol{q} = -\boldsymbol{\chi}_{eff} \left(\nabla T\right)^T$$

$$\boldsymbol{\chi}_{eff} = \boldsymbol{\chi}_S + \boldsymbol{\chi}_O + \boldsymbol{\chi}_W + \boldsymbol{\chi}_G$$

- **Constitutive law**
  - Stress-Strain

$$t_\alpha = p_\alpha \boldsymbol{I}, \qquad \alpha \neq s$$
$$\sigma = \sigma' + \frac{1}{n}\left(n_w p_w + n_o p_o + n_g p_g\right)\boldsymbol{I}$$

Total stress and Effective stress

$$d\boldsymbol{\sigma}' = \boldsymbol{D}_T\left[d\boldsymbol{\varepsilon} - d\boldsymbol{\varepsilon}_0\right]$$

Incremental relationship

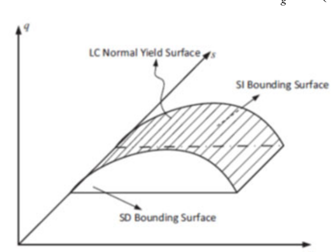
$$d\boldsymbol{\varepsilon}_0 = -(\frac{\beta_s}{3}\boldsymbol{I})dT$$

Strain due to temperature change

- **Constitutive law**
  - Prosity $\quad dn = -(1-n)d\left[\mathrm{tr}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0)\right] + (1-n)\left[\beta_s - \beta_m\right]dT$
  - Fluid density $\quad \rho_\alpha = f_\alpha(p_\alpha, T) \rightarrow d\rho_\alpha = \frac{d\rho_\alpha}{dp_\alpha}dp_\alpha + \frac{d\rho_\alpha}{dT}dT$
  - Capillary- volume fraction change

$$n_w = F(p_{cow}, T)$$
$$n_l = F(p_{cgo}, T)$$

$$dn_w = n'_w(dp_o - dp_w) + n'_{wT}dT$$
$$dn_o = n'_l(dp_g - dp_o) - n'_w(dp_o - dp_w) + (n'_{lT} - n'_{wT})dT$$
$$dn_g = -(1-n)d\left[\mathrm{tr}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0)\right] - n'_l(dp_g - dp_o) + \left\{(1-n)\left[\beta_S - \beta_m\right] - n'_{lT}\right\}dT$$



$$n_l = n_w + n_o, \qquad n_g = n - n_l$$
$$n'_l = \frac{\partial n_l}{\partial p_{cgo}}, \qquad n'_w = \frac{\partial n_w}{\partial p_{cow}}$$
$$n'_{lT} = \frac{\partial n_l}{\partial T}, \qquad n'_{wT} = \frac{\partial n_w}{\partial T}$$

Yield surfaces in q –p' -s space (Ghasemzadeh et al. 2017

Initial and boundary condition

- Initial condition

$$\boldsymbol{u} = \boldsymbol{u_0},\ \mathrm{p_w} = p_{w_0},\ \mathrm{p_o} = p_{o_0},\ \mathrm{p_g} = p_{g_0},\ T = T_0 \quad \text{in } \Omega \text{ and } \Gamma$$

- Essential condition

$$\boldsymbol{u} = \overline{\boldsymbol{u}} \text{ on } \Gamma_u^D,\ p_w = \overline{p}_w \text{ on } \Gamma_w^D,\ p_o = \overline{p}_o \text{ on } \Gamma_o^D,\ p_g = \overline{p}_g \text{ on } \Gamma_g^D,\ T = \overline{T} \text{ on } \Gamma_T^D$$

- Natural condition

$$\overline{\boldsymbol{t}} = \boldsymbol{l}^T \boldsymbol{\sigma} \quad \text{on } \Gamma_\mathrm{u}^\mathrm{N}$$

$$\overline{\boldsymbol{q}}_w = \left\{\rho_w \boldsymbol{K}_w \left(\rho_w \boldsymbol{g} - \nabla p_w\right)\right\}^T .\boldsymbol{n} \quad \text{on } \Gamma_\mathrm{w}^\mathrm{N}$$

$$\overline{\boldsymbol{q}}_o = \left\{\rho_o \boldsymbol{K}_o \left(\rho_o \boldsymbol{g} - \nabla p_o\right)\right\}^T .\boldsymbol{n} \quad \text{on } \Gamma_\mathrm{o}^\mathrm{N}$$

$$\overline{\boldsymbol{q}}_g = \left\{\rho_g \boldsymbol{K}_g \left(\rho_g \boldsymbol{g} - \nabla p_g\right)\right\}^T .\boldsymbol{n} \quad \text{on } \Gamma_\mathrm{g}^\mathrm{N}$$

$$\overline{\boldsymbol{q}}_T = \left(-\chi_{eff} \nabla T\right)^T .\boldsymbol{n} \quad \text{on } \Gamma_\mathrm{T}^\mathrm{N}$$

$$\boldsymbol{n} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T$$

$$\boldsymbol{l} = \begin{bmatrix} n_x & 0 & 0 & n_y & 0 & n_z \\ 0 & n_y & 0 & n_x & n_z & 0 \\ 0 & 0 & n_z & 0 & n_y & n_x \end{bmatrix}^T$$

Numerical solution

Equations solution method

- Equilibrium equations: Standard Galerkin Method
- Hydraulics and Thermal equations :CVFEM Method

FDM produces excessive numerical dispersion
and grid orientation problems
and difficulties in the treatment of complicated geometry
and boundary condition.

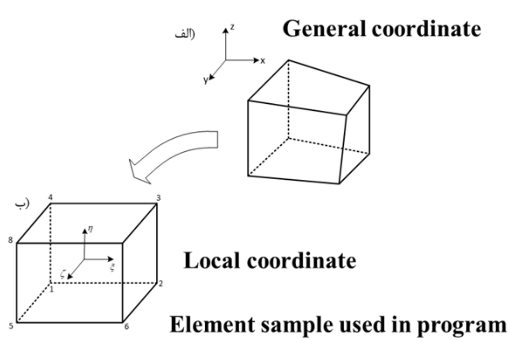
FEM (standard Galerkin) is not in general, locally mass conservative
And tends to generate numerical solutions with severe nonphysical oscillations.

CVFEM combines the mesh flexibility of FEM with the local conservative characteristic of the finite difference method at the control volume level.

Numerical Approximation

- Shape function $N_i = \frac{1}{8}(1+\xi\xi_i)(1+\eta\eta_i)(1+\zeta\zeta_i)$
- Global and local transformation $x_i(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\hat{\boldsymbol{x}}_i \quad ; \quad i = 1,2,3$
- Unknown function interpolation

$$\boldsymbol{u}(\xi,\eta,\zeta) = \boldsymbol{N}_u(\xi,\eta,\zeta)\hat{\boldsymbol{u}}$$
$$p_w(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\hat{\boldsymbol{p}}_w$$
$$p_o(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\hat{\boldsymbol{p}}_o$$
$$p_g(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\hat{\boldsymbol{p}}_g$$
$$T(\xi,\eta,\zeta) = \boldsymbol{N}(\xi,\eta,\zeta)\hat{\boldsymbol{T}}$$



Numerical solution: Equilibrium equations

$$\boldsymbol{L}^T\boldsymbol{\sigma} + \rho\boldsymbol{g}; \quad \boldsymbol{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}^T$$

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \frac{1}{n}\boldsymbol{m}\left(n_w p_w + n_o p_o + n_g p_g\right); \quad d\boldsymbol{\sigma}' = \boldsymbol{D}_T\left[d\boldsymbol{\varepsilon} - \frac{1}{3}\boldsymbol{m}\, d\varepsilon_0^s\right]$$

$$d\varepsilon_0^s = -\beta_s dT; \quad \boldsymbol{\varepsilon} = \boldsymbol{L}\boldsymbol{u}$$

$$\boldsymbol{C}_{uw}\dot{\hat{\boldsymbol{p}}}_w + \boldsymbol{C}_{uo}\dot{\hat{\boldsymbol{p}}}_o + \boldsymbol{C}_{ug}\dot{\hat{\boldsymbol{p}}}_g + \boldsymbol{K}_{uu}\dot{\hat{\boldsymbol{u}}} + \boldsymbol{C}_{uT}\dot{\hat{\boldsymbol{T}}} = \dot{\boldsymbol{f}}_u$$

$$\boldsymbol{C}_{uw} = \int_\Omega \boldsymbol{B}^T \frac{1}{n}\boldsymbol{m}\left(n_w + n'_w p_w + n'_w p_o\right)\boldsymbol{N}d\Omega$$

$$\boldsymbol{C}_{uo} = \int_\Omega \boldsymbol{B}^T \frac{1}{n}\boldsymbol{m}\left(n_o + n'_w p_w - (n'_w + n'_l)p_o + n'_l p_g\right)\boldsymbol{N}d\Omega$$

$$\boldsymbol{C}_{ug} = \int_\Omega \boldsymbol{B}^T \frac{1}{n}\boldsymbol{m}\left(n_g + n'_l p_o - n'_l p_g\right)\boldsymbol{N}d\Omega$$

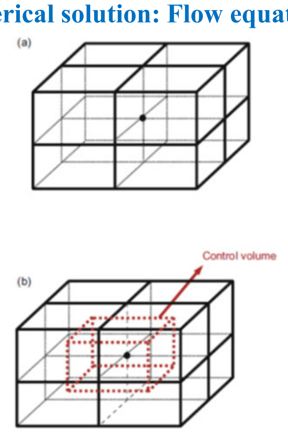
$$\boldsymbol{K}_{uu} = \int_\Omega \boldsymbol{B}^T\left(-\boldsymbol{D}_T\boldsymbol{B} - \frac{(1-n)}{n^2}\boldsymbol{m}\left[n_w p_w + n_o p_o - (n-n_g)p_g\right]\nabla^T\boldsymbol{N}_u\right)d\Omega$$

$$\boldsymbol{C}_{uT} = \int_\Omega \boldsymbol{B}^T\Bigg[\left\{\frac{(1-n)}{n^2}\boldsymbol{m}\left(n_w p_w + n_o p_o - (n-n_g)p_g\right) + \frac{1}{3}\boldsymbol{D}_T\boldsymbol{m}\right\}\beta_s + \frac{1}{n}\boldsymbol{m}\left\{n'_{wT}p_w + (n'_{lT} - n'_{wT})p_o + \left((1-n)[\beta_s - \beta_m] - n'_{lT}\right)p_g\right\} - \frac{(1-n)}{n^2}\boldsymbol{m}[\beta_s - \beta_m]\left(n_w p_w + n_o p_o + n_g p_g\right)\Bigg]\boldsymbol{N}d\Omega$$

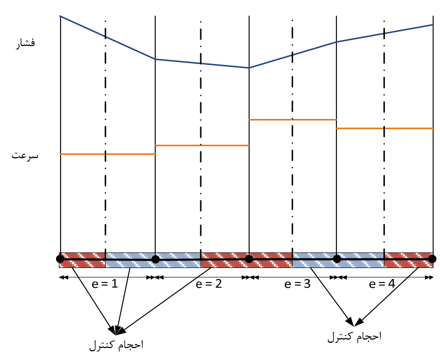
$$\boldsymbol{f}_u = -\int_\Omega \boldsymbol{N}_u^T \rho\boldsymbol{g}d\Omega + \int_{\Gamma_u^N}\boldsymbol{N}_u^T\bar{\boldsymbol{t}}d\Gamma$$

$$\boldsymbol{B} = \boldsymbol{L}\boldsymbol{N}_u$$

Numerical solution: Flow equation

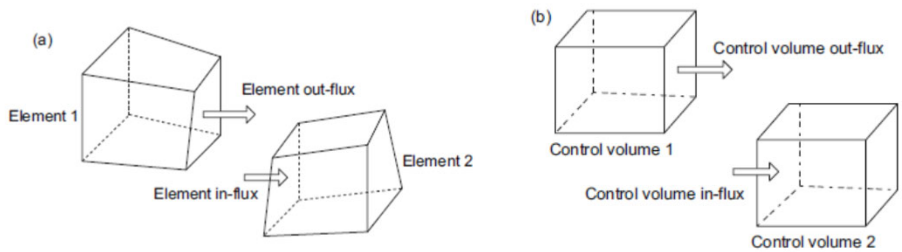


a) System of finite element mesh in the transformed space; b) representation of a control volume around a node



Discontinuity of velocity field across element faces and continuity of velocity field across control volume faces for a one dimensional finite element mesh

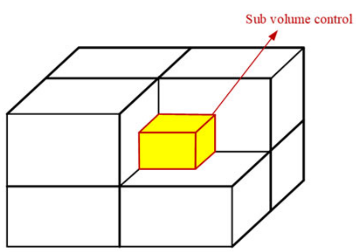
➢ **Numerical solution: Flow equation**



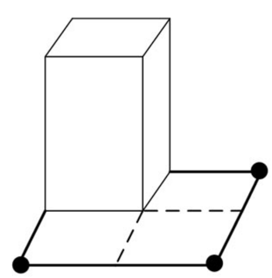
Mass in-flux and out-flux through the elements are not necessarily equal.

owing to continuity of the velocity field at the control volume faces, if the mass flux is integrated on each control volume surface, conservation of mass will be obtained on the control volumes

➢ **Numerical solution: Flow equation**



Control volume definition in FEM



Weighted function in CVFEM

$$W_i = \begin{cases} 1 & \text{in the sub-control volume belongs to node } i \\ 0 & \text{elsewhere} \end{cases}$$

Numerical solution: Flow equation

$$\bar{\boldsymbol{P}}_{ww}\hat{\boldsymbol{p}}_w + \boldsymbol{P}_{ww}\dot{\hat{\boldsymbol{p}}}_w + \boldsymbol{C}_{wo}\dot{\hat{\boldsymbol{p}}}_o + \boldsymbol{C}_{wu}\dot{\hat{\boldsymbol{u}}} + \boldsymbol{C}_{wT}\dot{\hat{\boldsymbol{T}}} = \boldsymbol{f}_w$$

$$\bar{\boldsymbol{P}}_{oo}\hat{\boldsymbol{p}}_o + \boldsymbol{C}_{ow}\dot{\hat{\boldsymbol{p}}}_w + \boldsymbol{P}_{oo}\dot{\hat{\boldsymbol{p}}}_o + \boldsymbol{C}_{og}\dot{\hat{\boldsymbol{p}}}_g + \boldsymbol{C}_{ou}\dot{\hat{\boldsymbol{u}}} + \boldsymbol{C}_{oT}\dot{\hat{\boldsymbol{T}}} = \boldsymbol{f}_o$$

$$\bar{\boldsymbol{P}}_{gg}\hat{\boldsymbol{p}}_g + \boldsymbol{C}_{go}\dot{\hat{\boldsymbol{p}}}_o + \boldsymbol{P}_{gg}\dot{\hat{\boldsymbol{p}}}_g + \boldsymbol{C}_{gu}\dot{\hat{\boldsymbol{u}}} + \boldsymbol{C}_{gT}\dot{\hat{\boldsymbol{T}}} = \boldsymbol{f}_g$$

$$\bar{\boldsymbol{P}}_{ww} = -\int_{\Gamma_{\text{S.C.V.}}-\Gamma_w^N} \boldsymbol{W}^T \left\{\left(\rho_w \boldsymbol{K}_w \nabla \boldsymbol{N}\right)^T . \boldsymbol{n}\right\}^T d\Gamma$$

$$\boldsymbol{P}_{ww} = \int_\Omega \boldsymbol{W}^T \left(n_w \frac{\partial \rho_w}{\partial p_w} - n'_w \rho_w \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{C}_{wo} = \int_\Omega \boldsymbol{W}^T \left(n'_w \rho_w\right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{C}_{wu} = \int_\Omega \boldsymbol{W}^T \left(n_w \rho_w\right) \boldsymbol{m}^T \boldsymbol{B} d\Omega$$

$$\boldsymbol{C}_{wT} = \int_\Omega \boldsymbol{W}^T \left(n_w \frac{\partial \rho_w}{\partial T} + n'_{wT} \rho_w \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{f}_w = \int_\Omega \dot{M}_w \boldsymbol{W}^T d\Omega - \int_{\Gamma_w^N} \bar{q}_w \boldsymbol{W}^T d\Gamma - \int_{\Gamma_{\text{S.C.V.}}-\Gamma_w^N} \left[\rho_w^2 \left(\boldsymbol{K}_w \boldsymbol{g}\right)^T . \boldsymbol{n}\right] \boldsymbol{W}^T d\Gamma$$

$$\bar{\boldsymbol{P}}_{oo} = -\int_{\Gamma_{\text{S.C.V.}}-\Gamma_o^N} \boldsymbol{W}^T \left\{\left(\rho_o \boldsymbol{K}_o \nabla \boldsymbol{N}\right)^T . \boldsymbol{n}\right\}^T d\Gamma$$

$$\boldsymbol{C}_{ow} = \int_\Omega \boldsymbol{W}^T \left(n'_w \rho_o\right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{P}_{oo} = \int_\Omega \boldsymbol{W}^T \left(n_o \frac{\partial \rho_o}{\partial p_o} - (n'_l + n'_w)\rho_o - \frac{\partial m_{og}}{\partial p_o} \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{C}_{og} = \int_\Omega \boldsymbol{W}^T \left(n'_l \rho_o\right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{C}_{ou} = \int_\Omega \boldsymbol{W}^T \left(n_o \rho_o\right) \boldsymbol{m}^T \boldsymbol{B} d\Omega$$

$$\boldsymbol{C}_{oT} = \int_\Omega \boldsymbol{W}^T \left(n_o \frac{\partial \rho_o}{\partial T} + n'_{lT} \rho_o - n'_{wT} \rho_o - \frac{\partial m_{og}}{\partial T} \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{f}_o = \int_\Omega \dot{M}_o \boldsymbol{W}^T d\Omega - \int_{\Gamma_o^N} \bar{q}_o \boldsymbol{W}^T d\Gamma - \int_{\Gamma_{\text{S.C.V.}}-\Gamma_o^N} \left[\rho_o^2 \left(\boldsymbol{K}_o \boldsymbol{g}\right)^T . \boldsymbol{n}\right] \boldsymbol{W}^T d\Gamma$$

$$\bar{\boldsymbol{P}}_{gg} = -\int_{\Gamma_{\text{S.C.V.}}-\Gamma_g^N} \boldsymbol{W}^T \left\{\left(\rho_g \boldsymbol{K}_g \nabla \boldsymbol{N}\right)^T . \boldsymbol{n}\right\}^T d\Gamma; \quad \boldsymbol{C}_{go} = \int_\Omega \boldsymbol{W}^T \left(n'_l \rho_g + \frac{\partial m_{og}}{\partial p_o} \right) \boldsymbol{N} d\Omega; \quad \boldsymbol{P}_{gg} = \int_\Omega \boldsymbol{W}^T \left(n_g \frac{\partial \rho_g}{\partial p_g} - n'_l \rho_g \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{C}_{gu} = \int_\Omega \boldsymbol{W}^T \left((1-n)\rho_g + n_g \rho_g\right) \boldsymbol{m}^T \boldsymbol{B} d\Omega; \quad \boldsymbol{C}_{gT} = \int_\Omega \boldsymbol{W}^T \left(n_g \frac{\partial \rho_g}{\partial T} - \left((1-n)\beta_m + n'_{lT} - n'_{wT}\right)\rho_g + \frac{\partial m_{og}}{\partial T} \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{f}_g = \int_\Omega \dot{M}_g \boldsymbol{W}^T d\Omega - \int_{\Gamma_g^N} \bar{q}_g \boldsymbol{W}^T d\Gamma - \int_{\Gamma_{\text{S.C.V.}}-\Gamma_g^N} \left[\rho_g^2 \left(\boldsymbol{K}_g \boldsymbol{g}\right)^T . \boldsymbol{n}\right] \boldsymbol{W}^T d\Gamma$$

Numerical solution: Heat transfer equation

$$\bar{\boldsymbol{T}}_{TT}\hat{\boldsymbol{T}} + \boldsymbol{C}_{To}\dot{\hat{\boldsymbol{p}}}_o + \boldsymbol{T}_{TT}\dot{\hat{\boldsymbol{T}}} = \boldsymbol{f}_T$$

$$\bar{\boldsymbol{T}}_{TT} = \int_{\Omega_{\text{C.V.}}} \boldsymbol{W}^T \left[\rho_w C_w \left\{ \boldsymbol{K}_w \left(\rho_w \boldsymbol{g} - \nabla p_w\right)\right\}^T + \rho_o C_o \left\{ \boldsymbol{K}_o \left(\rho_o \boldsymbol{g} - \nabla p_o\right)\right\}^T + \rho_g C_g \left\{ \boldsymbol{K}_g \left(\rho_g \boldsymbol{g} - \nabla p_g\right)\right\}^T \right] \nabla \boldsymbol{N} d\Omega - \left[\int_{\Gamma_{\text{C.V.}}-\Gamma_T^N} \boldsymbol{W}^T \left\{\left(\boldsymbol{\chi}_{eff} \nabla \boldsymbol{N}\right)^T . \boldsymbol{n}\right\}^T d\Gamma \right]^T$$

$$\boldsymbol{C}_{To} = \int_{\Omega_{\text{C.V.}}} \boldsymbol{W}^T \left(\frac{\partial m_{og}}{\partial p_o} \Delta H \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{T}_{TT} = \int_{\Omega_{\text{C.V.}}} \boldsymbol{W}^T \left(\rho C + \frac{\partial m_{og}}{\partial T} \Delta H \right) \boldsymbol{N} d\Omega$$

$$\boldsymbol{f}_T = \int_{\Omega_{\text{C.V.}}} \rho h \boldsymbol{W}^T d\Omega - \int_{\Gamma_T^N} \bar{q}_T \boldsymbol{W}^T d\Gamma$$

System of equations

$$\begin{bmatrix} \bar{\boldsymbol{P}}_{ww} & 0 & 0 & 0 & 0 \\ 0 & \bar{\boldsymbol{P}}_{oo} & 0 & 0 & 0 \\ 0 & 0 & \bar{\boldsymbol{P}}_{gg} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{T}}_{TT} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{p}}_o \\ \hat{\boldsymbol{p}}_g \\ \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{T}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{P}_{ww} & \boldsymbol{C}_{wo} & 0 & \boldsymbol{C}_{wu} & \boldsymbol{C}_{wT} \\ C_{ow} & \boldsymbol{P}_{oo} & \boldsymbol{C}_{og} & \boldsymbol{C}_{ou} & \boldsymbol{C}_{oT} \\ 0 & \boldsymbol{C}_{go} & \boldsymbol{P}_{gg} & \boldsymbol{C}_{gu} & \boldsymbol{C}_{gT} \\ \boldsymbol{C}_{uw} & \boldsymbol{C}_{uo} & \boldsymbol{C}_{ug} & \boldsymbol{K}_{uu} & \boldsymbol{C}_{uT} \\ 0 & \boldsymbol{C}_{To} & 0 & 0 & \boldsymbol{T}_{TT} \end{bmatrix} \begin{bmatrix} \dot{\hat{\boldsymbol{p}}}_w \\ \dot{\hat{\boldsymbol{p}}}_o \\ \dot{\hat{\boldsymbol{p}}}_g \\ \dot{\hat{\boldsymbol{u}}} \\ \dot{\hat{\boldsymbol{T}}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{f}_w \\ \boldsymbol{f}_o \\ \boldsymbol{f}_g \\ \dot{\boldsymbol{f}}_u \\ \boldsymbol{f}_T \end{bmatrix}$$

- **Temporal discretization (fully implicit)**

$$\begin{bmatrix} \boldsymbol{P}_{ww}+\Delta t\bar{\boldsymbol{P}}_{ww} & \boldsymbol{C}_{wo} & 0 & \boldsymbol{C}_{wu} & \boldsymbol{C}_{wT} \\ C_{ow} & \boldsymbol{P}_{oo}+\Delta t\bar{\boldsymbol{P}}_{oo} & \boldsymbol{C}_{og} & \boldsymbol{C}_{ou} & \boldsymbol{C}_{oT} \\ 0 & \boldsymbol{C}_{go} & \boldsymbol{P}_{gg}+\Delta t\bar{\boldsymbol{P}}_{gg} & \boldsymbol{C}_{gu} & \boldsymbol{C}_{gT} \\ \boldsymbol{C}_{uw} & \boldsymbol{C}_{uo} & \boldsymbol{C}_{ug} & \boldsymbol{K}_{uu} & \boldsymbol{C}_{uT} \\ 0 & \boldsymbol{C}_{To} & 0 & 0 & \boldsymbol{T}_{TT}+\Delta t\bar{\boldsymbol{T}}_{TT} \end{bmatrix}_{n+1} \begin{bmatrix} \Delta\hat{\boldsymbol{p}}_w \\ \Delta\hat{\boldsymbol{p}}_o \\ \Delta\hat{\boldsymbol{p}}_g \\ \Delta\hat{\boldsymbol{u}} \\ \Delta\hat{\boldsymbol{T}} \end{bmatrix}_{n+1} = \Delta t \begin{bmatrix} \boldsymbol{f}_w \\ \boldsymbol{f}_o \\ \boldsymbol{f}_g \\ \dot{\boldsymbol{f}}_u \\ \boldsymbol{f}_T \end{bmatrix}_{n+1} - \Delta t \begin{bmatrix} \bar{\boldsymbol{P}}_{ww} & 0 & 0 & 0 & 0 \\ 0 & \bar{\boldsymbol{P}}_{oo} & 0 & 0 & 0 \\ 0 & 0 & \bar{\boldsymbol{P}}_{gg} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{\boldsymbol{T}}_{TT} \end{bmatrix}_{n+1} \begin{bmatrix} \hat{\boldsymbol{p}}_w \\ \hat{\boldsymbol{p}}_o \\ \hat{\boldsymbol{p}}_g \\ \hat{\boldsymbol{u}} \\ \hat{\boldsymbol{T}} \end{bmatrix}_{n}$$

- **Newton method for linearization of equation**

$$\mathbb{F}(\mathbf{X}) = (\mathbf{A}+\Delta t\mathbf{Q})\Delta\mathbf{X}_{n+1} - (\Delta t\mathbf{F})_{n+1} + (\Delta t\mathbf{Q})\mathbf{X}_n = 0.$$

$$\frac{\partial\mathbb{F}}{\partial X} \approx (\mathbf{A}+\Delta t\mathbf{Q}) \rightarrow \text{leads to diverge}$$

$$\frac{\partial\mathbb{F}}{\partial X} = (\mathbf{A}+\Delta t\mathbf{Q}) + \left(\frac{\partial\mathbf{A}}{\partial\Delta\mathbf{X}} + \Delta t\frac{\partial\mathbf{Q}}{\partial\Delta\mathbf{X}}\right)\Delta\mathbf{X}^i_{n+1} + \left(\Delta t\frac{\partial\mathbf{Q}}{\partial\Delta\mathbf{X}}\right)\mathbf{X}_n \rightarrow \text{high computational cost}$$

$$\frac{\partial\mathbb{F}}{\partial X} \approx (\mathbf{A}+\Delta t\mathbf{Q}) + \left(\Delta t\frac{\partial\mathbf{Q}}{\partial\Delta\mathbf{X}}\right)\mathbf{X}^i_{n+1}$$

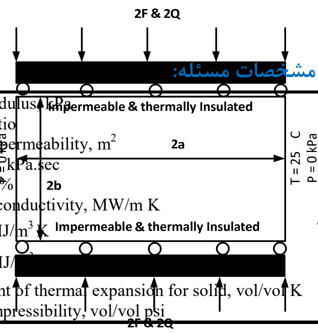
Results

- **Non isothermal Mandel problem**
- **Two phases consolidation**
- **Heterogeneous deformable porous media**
- **Reservoir application**

➢ Non isothermal Mandel problem(Pao et al. 2001)

✓ THM problem

مشخصات مسئله:

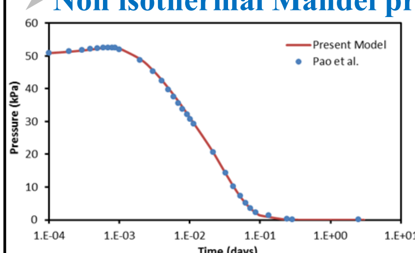


Shear modulus, kPa	2.38e4
Poison ratio	0.2
Absolute permeability, m^2	1.e-6
Viscosity, kPa.sec	1.e-3
Porosity, %	20
Thermal conductivity, MW/m K	0.5
$\rho_s C_s$, MJ/m^3 K	3.0
$\rho_f C_f$, MJ/	4.0
Coefficient of thermal expansion for solid, vol/vol K	0.9e-6
Fluid compressibility, vol/vol psi	3e-6

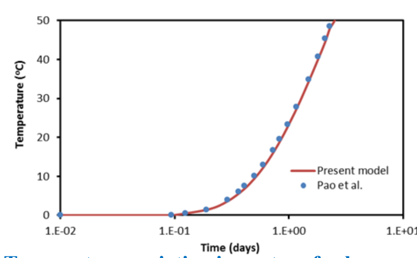
a=b= 1 m
F=1e6 kN
Q=1000 MJ/D

Geometry and boundary condition

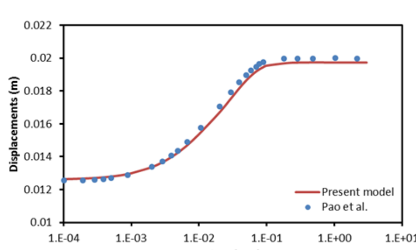
➢ Non isothermal Mandel problem



Pore Pressure variation in center of column



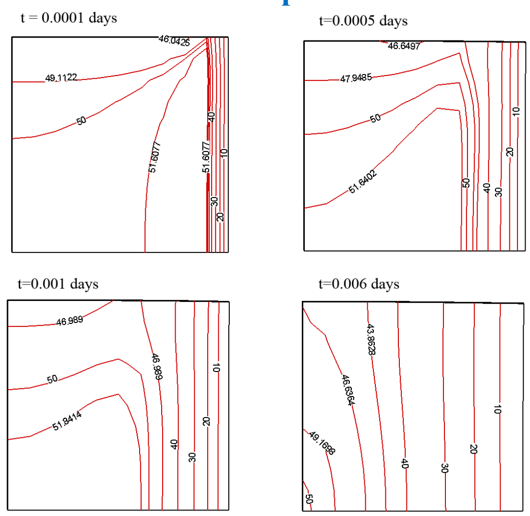
Temperature variation in center of column



Displacement variation in center of column

به دلیل تقارن فقط یک چهارم دامنه مساله شبیه سازی و حل شده است

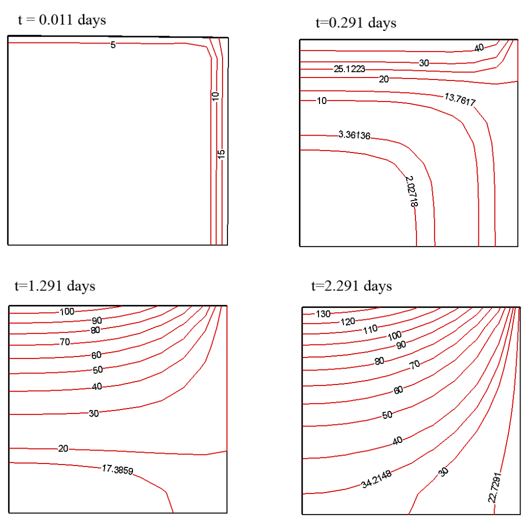
Non isothermal Mandel problem



Pore Pressure Distribution

به دلیل تقارن فقط یک چهارم دامنه مساله شبیه سازی و حل شده است

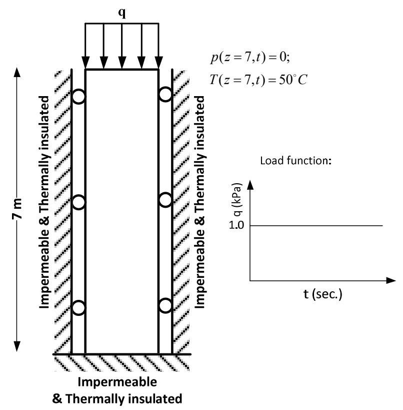
Non isothermal Mandel problem



Temperature Distribution

به دلیل تقارن فقط یک چهارم دامنه مساله شبیه سازی و حل شده است

Two phases consolidation(Pao et al. 2001)



Geometry and boundary condition

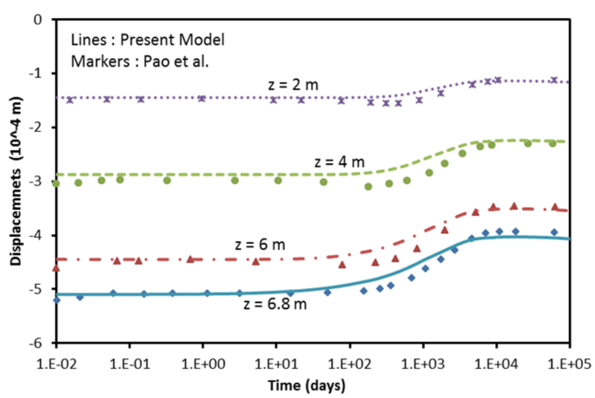
Problem specification

Elastic modulus, kPa	6.e3
Poison ratio	0.4
Density of soil at stc, ton/m^3	2.0
Thermal conductivity, kJ/m C sec	1.15
Specific heat of soil, kJ/kg C	125
Thermal expansion for soil, vol/vol C	1.e-6
Porosity, %	30.0
Absolute permeability, m^2	0.46e-11
Residual water saturation, %	20.0
Initial water saturation, %	92.0
Density of water at stc, ton/m^3	1.0
Thermal expansion for water, vol/vol C	2.1e-4
Water compressibility, 1/kPa	0.43e-11
Gas viscosity, kPa sec	1.e-6
Density of gas at stc, ton/m^3	1.22

$$p_c = 1.68 \times \left[\frac{(n_w - n_{w_{res}})}{(n_{w_{sat}} - n_{w_{res}})} \right]^{-1/3}$$

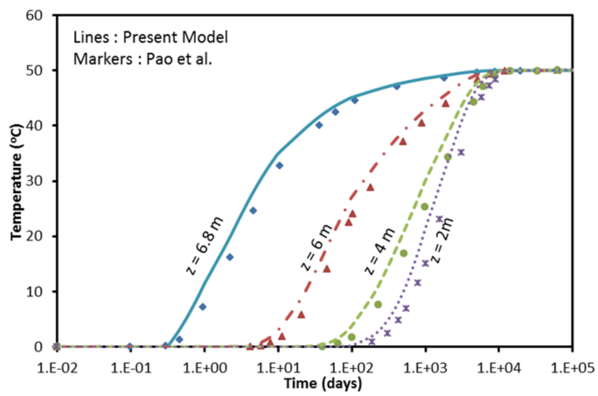
$$\mu_w = 661.2(T-229)^{-1.562}$$

Two phases consolidation



Displacement in different points of column

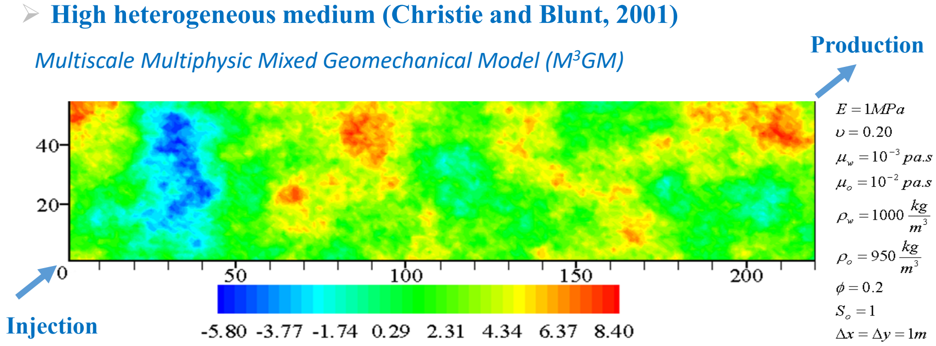
- **Two phases consolidation**(Pao et al. 2001)



Temperature in different points of column

- **High heterogeneous medium (Christie and Blunt, 2001)**

Multiscale Multiphysic Mixed Geomechanical Model (M³GM)



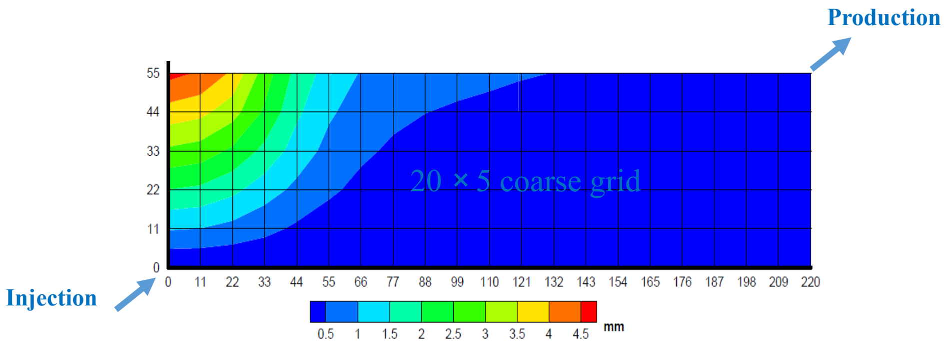
$E = 1MPa$
$\upsilon = 0.20$
$\mu_w = 10^{-3}\, pa.s$
$\mu_o = 10^{-2}\, pa.s$
$\rho_w = 1000\, \frac{kg}{m^3}$
$\rho_o = 950\, \frac{kg}{m^3}$
$\phi = 0.2$
$S_o = 1$
$\Delta x = \Delta y = 1m$

Natural logarithm of the permeability derived from the top layer of SPE10 test case

This layer belongs to a Tarbert formation with isotropic horizontal permeability that varies over 6 orders of magnitude.

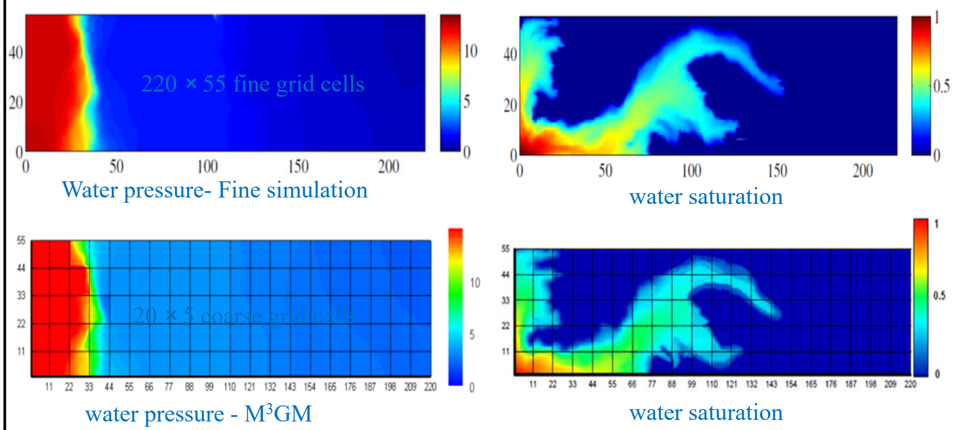
- **Heterogeneous deformable porous media**

Multiscale Multiphysic Mixed Geomechanical Model (M³GM)



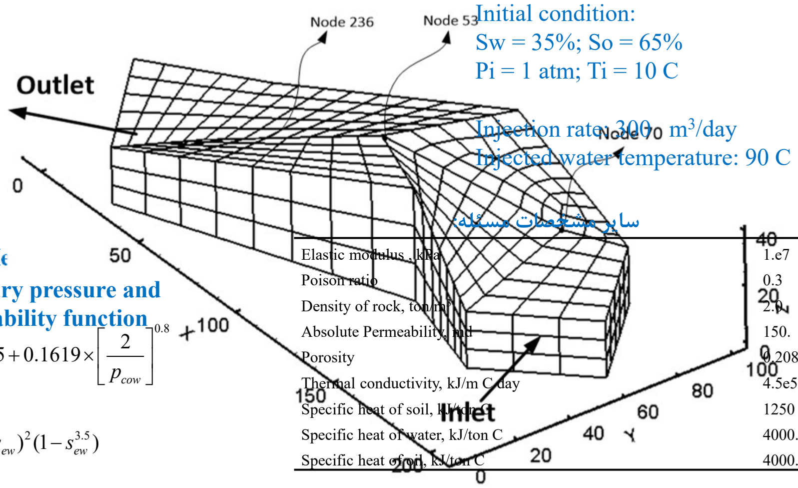
M³GM deformation fields for the SPE 10 top layer test case after 0.132 PVI

- **High heterogeneous medium**



Water pressure and water saturation fields for the test case after 0.132 PVI.

➢ Reservoir application



Initial condition:
Sw = 35%; So = 65%
Pi = 1 atm; Ti = 10 C

Injection rate: 300 m^3/day
Injected water temperature: 90 C

سایر مشخصات مسئله:

Elastic modulus , kPa	1.e7
Poison ratio	0.3
Density of rock, ton/m^3	2.0
Absolute Permeability, md	150.
Porosity	0.208
Thermal conductivity, kJ/m C day	4.5e5
Specific heat of soil, kJ/ton C	1250
Specific heat of water, kJ/ton C	4000.
Specific heat of oil, kJ/ton C	4000.

Me

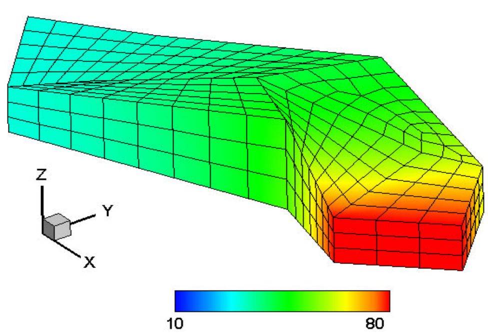
Capillary pressure and permeability function

$$n_w = 0.045 + 0.1619 \times \left[\frac{2}{p_{cow}} \right]^{0.8}$$

$$k_{rw} = s_{ew}^{5.5}$$

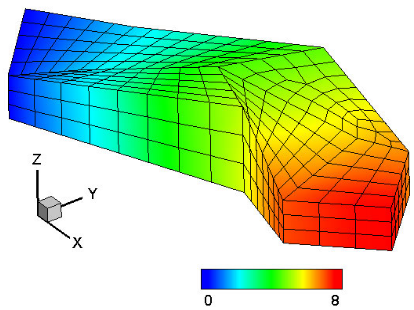
$$k_{ro} = (1 - s_{ew})^2 (1 - s_{ew}^{3.5})$$

➢ Reservoir application

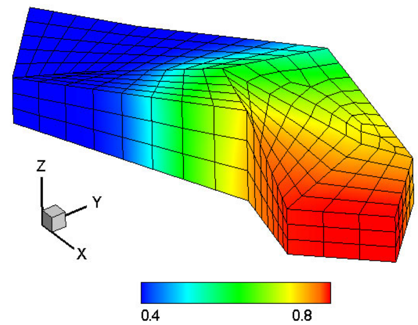


Temperature distribution after 50 days of injection

➢ **Reservoir application**

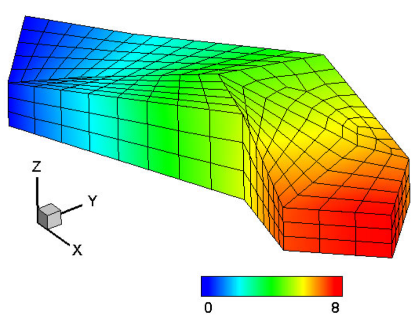


Pressure distribution
after 50 days of injection

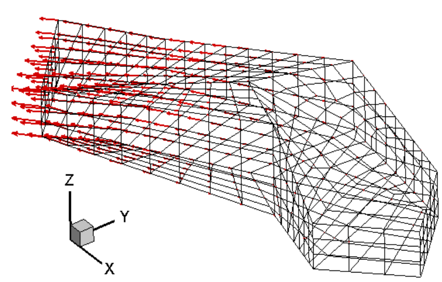


Degree of saturation distribution
after 50 days of injection

➢ **Reservoir application**

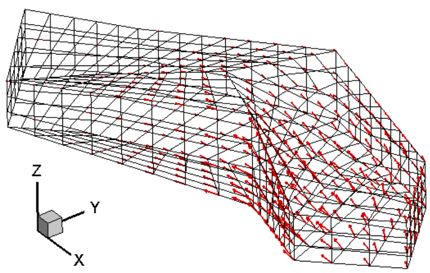


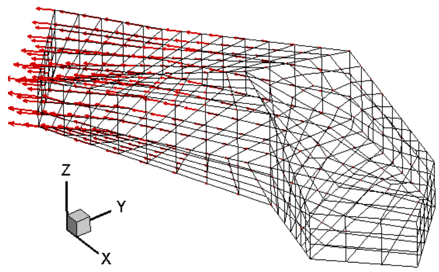
Vertical displacement
after 50 days of injection



Vertical displacement vectors
after 50 days of injection

- **Reservoir application**





Water velocity vectors after 50 days of injection

Oil velocity vectors after 50 days of injection

Thermal and mechanical data are needed for modeling real reservoir