

ژئومکانیک مخازن هیدرو کربوری

Petroleum Geomechanics

نفوذ پذیری

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نفوذ پذیری:

نفوذ پذیری خاصیتی از محیط متخلخل است که شاخص ظرفیت و توانایی سازند در عبور دادن سیالات را بیان می کند. این خاصیت توسط *Henry Darcy* در سال ۱۸۵۶ معرفی شد.

- Definition (ABW, Ref: API 27)
 - ... permeability is a property of the porous medium and is a measure of the capacity of the medium to transmit fluids
 - ... a measure of the fluid conductivity of the particular material
- Permeability is an INTENSIVE property of a porous medium (e.g. reservoir rock)

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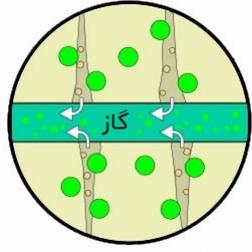
نفوذپذیری مطلق: توانایی محیط متخلخل برای عبور دادن یک گاز خنثی از میان خود را نفوذپذیری مطلق می‌گویند.

نفوذپذیری موثر: توانایی محیط متخلخل برای عبور دادن یک سیال از میان خود (در صورت حضور سیالات دیگر) را نفوذپذیری موثر می‌نامند. نفوذپذیری موثر تابعی از نفوذپذیری مطلق، نوع سیال، درجه اشباع و نحوه توزیع سیال در محیط متخلخل می‌باشد.

$$k_g + k_o + k_w \leq k \quad \rightarrow \quad \text{نفوذپذیری مطلق}$$

↓ نفوذپذیری موثر گاز
↓ نفوذپذیری موثر نفت
↓ نفوذپذیری موثر آب

$0 \leq k_o, k_g, k_w \leq k$



$$S_g + S_o + S_w = 1$$

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نفوذپذیری نسبی:

هرگاه در یک محیط متخلخل دو یا چند فاز به طور همزمان در حال جریان باشند، نفوذپذیری نسبی هر فاز در یک اشباع مشخص به صورت نسبت نفوذپذیری موثر آن فاز به نفوذپذیری مطلق تعریف می‌شود.

$$k_{ro} = \frac{k_o}{k} \quad \begin{array}{l} \rightarrow \text{نفوذپذیری موثر} \\ \rightarrow \text{نفوذپذیری مطلق} \end{array}$$

← نفوذپذیری نسبی

$$0 \leq k_{ro}, k_{rg}, k_{rw} \leq 1 \quad k_{ro} + k_{rg} + k_{rw} \leq 1$$

مثال: نفوذپذیری موثر نفت در محیط متخلخلی با درجه اشباع ۷۰ درصد 40 mD و نفوذپذیری مطلق 160 mD است. نفوذپذیری نسبی نفت در درجه اشباع ۷۰ درصد 25 درصد است.

Sources for Permeability Determination

- Core analysis
- Well test analysis (flow testing)
 - RFT (repeat formation tester) provides small well tests
- Production data
 - production logging measures fluid flow into well
- Log data
 - MRI (magnetic resonance imaging) logs calibrated via core analysis

The quality of the reservoir, as it relates to permeability can be classified as follows

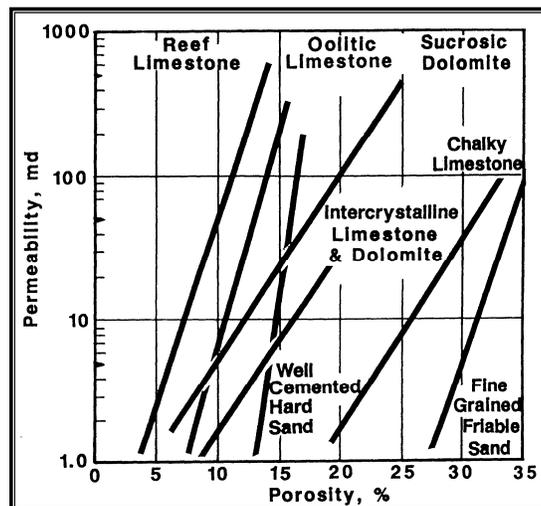
$k < 1$ md	poor
$1 < k < 10$ md	fair
$10 < k < 50$ md	moderate
$50 < k < 250$ md	good
$250 \text{ md} < k$	very good

This scale changes with time, for example 30 years ago $k < 50$ was considered poor.

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Examples, Typical Permeability-Porosity Relationship



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From Tiab and Donaldson, 1996

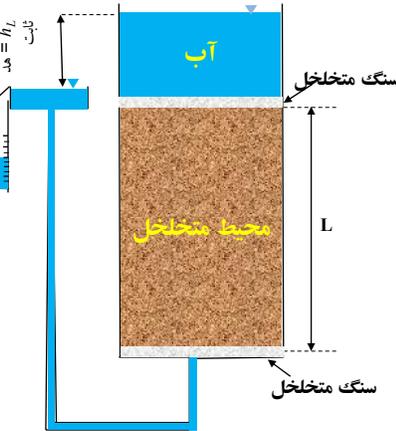
تعیین نفوذپذیری (هد ثابت)

Darcy's Apparatus for Determining Permeability

- Darcy's "K" was determined to be a combination of
 - k, permeability of the sand pack (porous medium, e.g. reservoir rock)
 - μ , viscosity of the liquid

$$K = \frac{k}{\mu}$$

$$v_s = \frac{q}{A} = -\frac{k}{\mu} \left[\frac{d\Phi}{ds} \right]$$



$$q = \frac{k A}{\mu L} (\Phi_1 - \Phi_2)$$

Φ : fluid potential with dimension of pressure

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Darcy Units

- Permeability is a derived dimension based on Darcy's Equation

$$k = (q \mu L) / (A \Delta p)$$

$$k = \frac{q \mu L}{A \Delta p}; \left[\frac{L^3}{T} \cdot \frac{P \cdot T}{1} \cdot \frac{L}{1} \cdot \frac{1}{L^2} \cdot \frac{1}{P} \right] = [L^2] \quad \text{m}^2 \text{ in SI units}$$

- The unit of permeability is the Darcy [d]
 - The oilfield unit is millidarcy [md]
- The Darcy is defined from Darcy's Equation, where:
 - q [cm³/s]
 - μ [cp]
 - L [cm]
 - A [cm²]
 - Δp [atm]

$$1 \text{ دارسی} = 9.86 \cdot 10^{-9} \text{ cm}^2$$

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Flow Potential

- Flow potential includes both pressure and gravity terms, simplifying Darcy's Law , $\Phi = p - \rho g z$

$$v_s = \frac{q}{A} = -\frac{k}{\mu} \left[\frac{d\Phi}{ds} \right]$$

- The generalized form of Darcy's Law includes pressure and gravity terms to account for horizontal or non-horizontal flow

$$v_s = \frac{q_s}{A} = -\frac{k}{\mu} \left[\frac{dp}{ds} - \rho g \frac{dz}{ds} \right]$$

- $z \uparrow \downarrow$; z is elevation measured from a datum

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تعیین نفوذپذیری (هد متغیر)

$$v_s = \frac{q}{A} = -\frac{k}{\mu} \left[\frac{d\Phi}{ds} \right]$$

$$q \int_0^L ds = -\frac{kA}{\mu} \int_{\Phi_1}^{\Phi_2} d\Phi$$

K : هدایت هیدرولیکی بر حسب متر بر ثانیه (m/s)

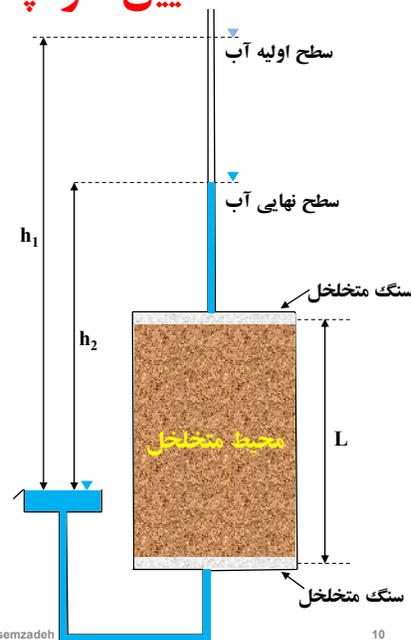
$$K = \frac{k\rho g}{\mu}$$

(kg/m³) چگالی سیال بر حسب کیلوگرم بر مترمکعب

(m/s²) شتاب جاذبه بر حسب متر بر مجذور ثانیه

(kg/m.s) ویسکوزیته دینامیک سیال بر حسب کیلوگرم بر

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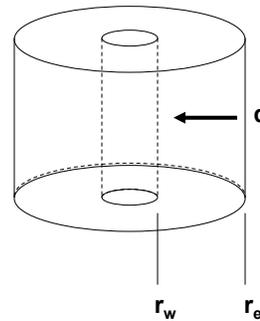


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Radial Flow, Incompressible Liquid

• 1-D Radial Flow System

- steady state flow
- incompressible fluid, $q(r_w \leq s \leq r_e) = \text{constant}$
- horizontal flow ($dZ/ds = 0 \therefore \Phi = p$)
- $A(r_w \leq s \leq r_e) = 2\pi rh$ where, $h = \text{constant}$
- Darcy flow (Darcy's Law is valid)
- $k = \text{constant}$ (non-reactive fluid)
- single phase ($S=1$)
- isothermal (constant μ)
- $ds = -dr$

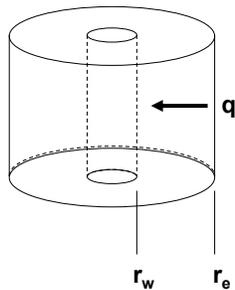


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Radial Flow, Incompressible Liquid

• Darcy's Law:



- $q_{e \rightarrow w} > 0$, if $p_e > p_w$

$$v_s = \frac{q}{A} = -\frac{k}{\mu} \left[\frac{d\Phi}{ds} \right]$$

$$\frac{q}{2\pi rh} dr = \frac{k}{\mu} dp$$

$$q \int_{r_e}^{r_w} \frac{1}{r} dr = \frac{2\pi kh}{\mu} \int_{p_e}^{p_w} dp$$

$$q = \frac{2\pi kh}{\mu \ln(r_e/r_w)} (p_e - p_w)$$

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تعیین نفوذپذیری محیط لایه‌ای

الف

$$\bar{k} = \frac{L}{\sum \frac{L_i}{k_i}}$$

ب

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تعیین نفوذپذیری محیط لایه‌ای

الف

$$\bar{k} = \frac{\sum k_i \cdot L_i}{L}$$

ب

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Radial, Parallel Flow

- Permeability varies across several (3) horizontal layers (k_1, k_2, k_3)
- Discrete changes in permeability
- Same pressure drop for each layer

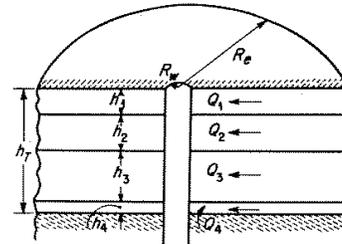
$$h = h_1 + h_2 + h_3 = \sum h_i$$

- Total flow rate is summation of flow rate for all layers

$$q = q_1 + q_2 + q_3 = \sum q_i$$

- Average permeability results in correct total flow rate

$$q = \frac{2\pi \bar{k} h}{\mu \ln(r_e/r_w)} \Delta p$$



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Radial, Parallel Flow

- Substituting,

$$q = \frac{2\pi \bar{k} h}{\mu \ln(r_e/r_w)} \Delta p$$

$$= \frac{2\pi k_1 h_1}{\mu \ln(r_e/r_w)} \Delta p + \frac{2\pi k_2 h_2}{\mu \ln(r_e/r_w)} \Delta p + \frac{2\pi k_3 h_3}{\mu \ln(r_e/r_w)} \Delta p$$

- Rearranging,

$$\bar{k} = \frac{\sum k_i \cdot h_i}{h}$$

- Average permeability reflects flow capacity of all layers

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Radial, Serial Flow

- Permeability varies across two vertical concentric cylindrical layers

$$[k(r_w \leq r \leq r_2) = k_1, \quad k(r_2 \leq r \leq r_e) = k_2]$$

- Discrete changes in permeability

$$r_e - r_w = \Delta r_1 + \Delta r_2 = \sum \Delta r_i$$

- Same flow rate passes through each layer

$$q = q_1 = q_2$$

- Total pressure drop is summation of pressure drops

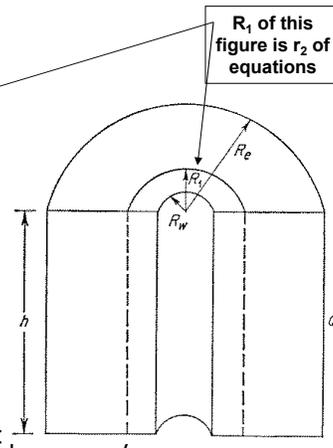
$$p_e - p_w = \Delta p_1 + \Delta p_2 = \sum \Delta p_i$$

- Average permeability results in correct total pressure drop

$$p_e - p_w = \frac{q \mu \ln(r_e/r_w)}{2\pi \bar{k} h}$$

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Radial, Serial Flow

- Substituting ($r_w=r_1, r_2, r_e=r_3$),

$$p_e - p_w = \frac{q \mu \ln(r_e/r_w)}{2\pi \bar{k} h} = \frac{q \mu \ln(r_2/r_w)}{2\pi k_1 h} + \frac{q \mu \ln(r_e/r_2)}{2\pi k_2 h}$$

- Rearranging,

$$\bar{k} = \frac{\ln(r_e/r_w)}{\sum_{\text{All Layers}} \frac{\ln(r_{i+1}/r_i)}{k_i}}$$

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Gas Flow vs. Liquid Flow

- Gas density is a function of pressure (for isothermal reservoir conditions)

- Real Gas Law

$$R = \left(\frac{pV}{znT} \right)_{\text{reservoir}} = \left(\frac{pV}{znT} \right)_{\text{standard conditions}}$$

- We cannot assume gas flow in the reservoir is incompressible

- Gas density determined from Real Gas Law

$$\rho_g = \frac{p \gamma_g M_{\text{air}}}{z R T}$$

- Darcy's Law describes volumetric flow rate of gas flow at reservoir conditions (*in situ*)

$$v_s = \frac{q_g}{A} = - \frac{k}{\mu_g} \left[\frac{dp}{ds} \right]$$

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Gas Formation Volume Factor

- Given a volumetric gas flow rate at reservoir conditions, q_g , we need to determine the mass flow rate, $q_{g,sc}$

- B_g has oilfield units of [rcf/scf]

- scf is a specified mass of gas (i.e. number of moles)

- reservoir cubic feet per standard cubic foot

- $(\text{ft}^3)_{\text{reservoir conditions}} / (\text{ft}^3)_{\text{standard conditions}}$

$$B_g = \frac{V_{\text{res}}}{V_{\text{sc}}} = \frac{(znRT/p)_{\text{res}}}{(znRT/p)_{\text{sc}}} = \frac{p_{\text{sc}} T z}{p T_{\text{sc}}}; \quad z_{\text{sc}} = 1$$

$$q_g = q_{g,sc} B_g$$

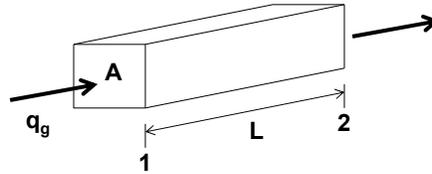
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Linear Gas Flow

• 1-D Linear Flow System

- Steady state flow (mass flow rate, $q_{g,sc}$, is constant)
- Gas density is described by real gas law,
 $\rho_g = (p\gamma_g M_{air}) / (zRT)$
- Horizontal flow path ($dZ/ds = 0 \therefore \Phi = p$)
- $A(0 \leq s \leq L) = \text{constant}$
- Darcy flow (Darcy's Law is valid)
- $k = \text{constant}$ (non-reactive fluid)
- single phase ($S_g = 1$)
- Isothermal ($T = \text{constant}$)



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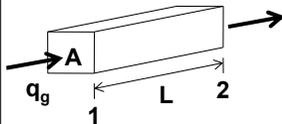
Linear Gas Flow

- Darcy's Law:

$$v_s = \frac{q_g}{A} = -\frac{k}{\mu_g} \left[\frac{dp}{ds} \right]$$

$$q_g ds = -\frac{kA}{\mu_g} dp$$

$$q_{g,sc} ds = -\frac{kA}{B_g \mu_g} dp$$



- $q_{1 \rightarrow 2} > 0$, if $p_1 > p_2$

$$q_{g,sc} ds = -kA \left(\frac{T_{sc}}{T p_{sc}} \right) \left[\frac{p}{z \mu_g} \right] dp$$

$$q_{g,sc} \int_0^L ds = -kA \left(\frac{T_{sc}}{T p_{sc}} \right) \int_{p_1}^{p_2} \frac{p}{z \mu_g} dp$$

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Integral of Pressure Dependent Terms

- Two commonly used approaches to the evaluating the integral of the pressure dependent terms:

- $(z\mu_g)=\text{Constant}$ approach, also called "p² Method"

- valid when pressure < 2,500 psia
- for Ideal Gas a subset of this approach is valid
 - $z = 1$; valid only for low pressures
 - μ_g depends on temperature only

$$I = \int_{p_1}^{p_2} \frac{p}{z \mu_g} dp$$

- Pseudopressure** approach

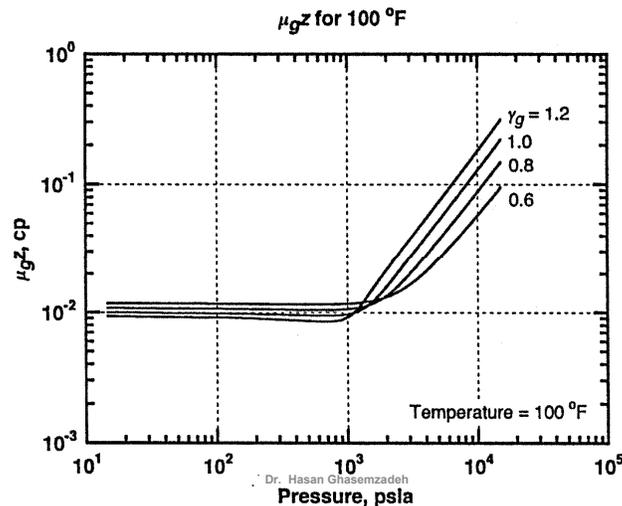
- the integral is evaluated *a priori* to provide the pseudopressure function, $m(p)$
 - specified gas gravity, γ_g
 - specified reservoir temperature, T
 - arbitrary base pressure, p_0
 - valid for any pressure range

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$(z\mu_g)=\text{Constant}$

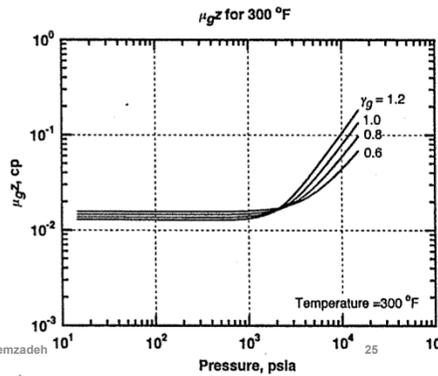
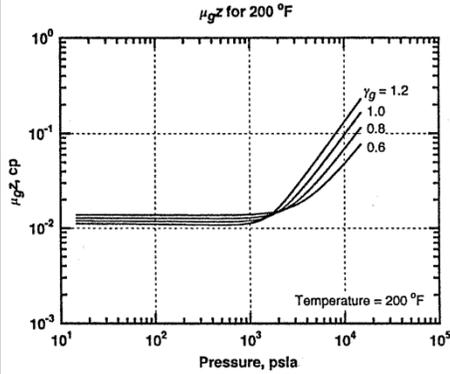
- Assumption that $(z\mu_g)$ is a constant function of pressure is valid for pressures < 2,500 psia, across the range of interest, for reservoir temperature and gas gravity



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$(z\mu_g) = \text{Constant}$

- At other temperatures in the range of interest



Reservoir Temperature Gradient
 $dT/dZ \approx 0.01 \text{ } ^\circ\text{F/ft}$

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$(z\mu_g) = \text{Constant, Linear Flow}$

- If $(z\mu_g) = \text{Constant}$

$$I = \int_{p_1}^{p_2} \frac{p}{z\mu_g} dp = \frac{1}{(z\mu_g)} \int_{p_1}^{p_2} p dp = \frac{1}{(z\mu_g)} \left[\frac{p^2}{2} \right]_{p_1}^{p_2}$$

- Gas Flow Rate (at standard conditions)

$$q_{g,sc} = \frac{k A}{L} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2 z \mu_g} \right) (p_1^2 - p_2^2)$$

valid when pressure < 2,500 psia=17.5 MPa=175 kg/cm²

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Real Gas Pseudopressure

- Recall piecewise integration:

$$\int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx$$

the ordering (position along x-axis) of the integral limits a,b and c is arbitrary

- pseudopressure, $m(p)$, is defined as:

$$m(p) = \int_{p_0}^p \frac{2p'}{z\mu_g} dp'$$

- Piecewise Integration of the pressure dependent terms:

$$I = \int_{p_1}^{p_2} \frac{p}{z\mu_g} dp = \frac{1}{2} \left[\int_{p_0}^{p_2} \frac{2p'}{z\mu_g} dp' - \int_{p_0}^{p_1} \frac{2p'}{z\mu_g} dp' \right] = \frac{1}{2} [m(p_2) - m(p_1)]$$

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Real Gas Pseudopressure, Linear Flow

- Recalling our previous equation for linear gas flow

$$q_{g,sc} \int_0^L ds = -k A \left(\frac{T_{sc}}{T p_{sc}} \right) \int_{p_1}^{p_2} \frac{p}{z\mu_g} dp$$

- And substituting for the pressure integral

$$q_{g,sc} = \frac{k A}{L} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2} \right) [m(p_1) - m(p_2)]$$

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Radial Gas Flow

- The radial equations for gas flow follow from the previous derivation for liquid flow and are left as self study

- $(z\mu_g) = \text{Constant}$

$$q_{g,sc} = \frac{2 \pi k h}{\ln(r_e/r_w)} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2 z \mu_g} \right) (p_e^2 - p_w^2)$$

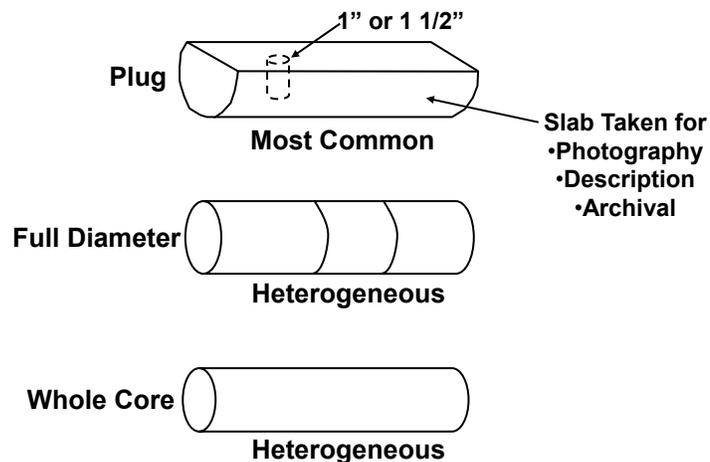
- Pseudopressure

$$q_{g,sc} = \frac{2 \pi k h}{\ln(r_e/r_w)} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2} \right) [m(p_e) - m(p_w)]$$

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Sample Size



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WHOLE-CORE METHOD

- Uses selected pieces from the full or whole core
 - Core sizes 2 1/2 to 5 1/2 inches in diameter
 - Several inches to several feet long
- Most applicable approach for very heterogeneous formations.
- Additional expense limits the practical number of tests.

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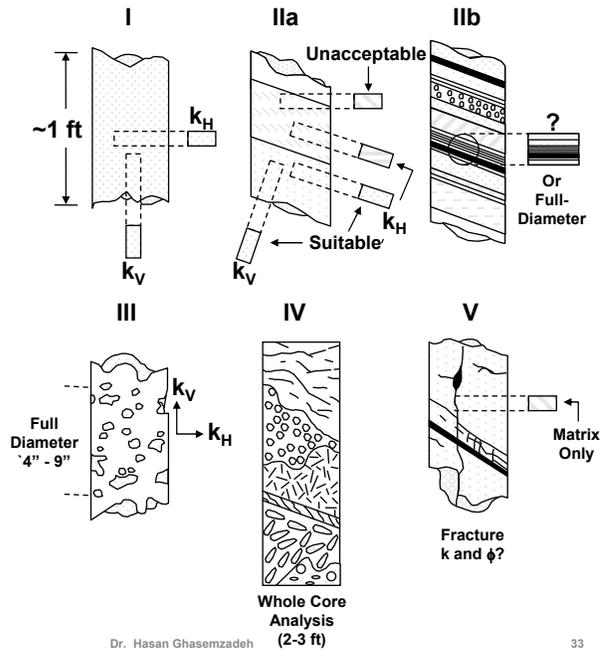
CORE PLUG METHOD

- Most commonly applied method.
- Uses small cylindrical core samples
 - 3/4 inch to 1 1/2 inch diameter
 - 1 to a few inches long
- May not apply to heterogeneous formations.

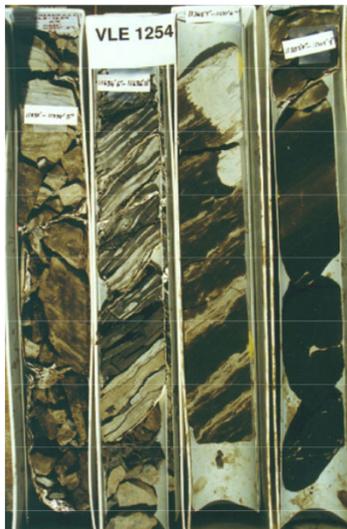
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Different Lithologies Require Careful Selection of Suitable Core Plugs or Require Whole-Core Analyses



WHOLE CORE



Whole Core Photograph, Misoa "C" Sandstone, Venezuela

Photo by W. Ayers

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LAB PROCEDURE FOR MEASURING PERMEABILITY

- Cut core plugs from whole core or use sample from whole core
- Clean core and extract reservoir fluids, then dry the core
- Flow a fluid through core at several flow rates
- Record inlet and outlet pressures for each rate

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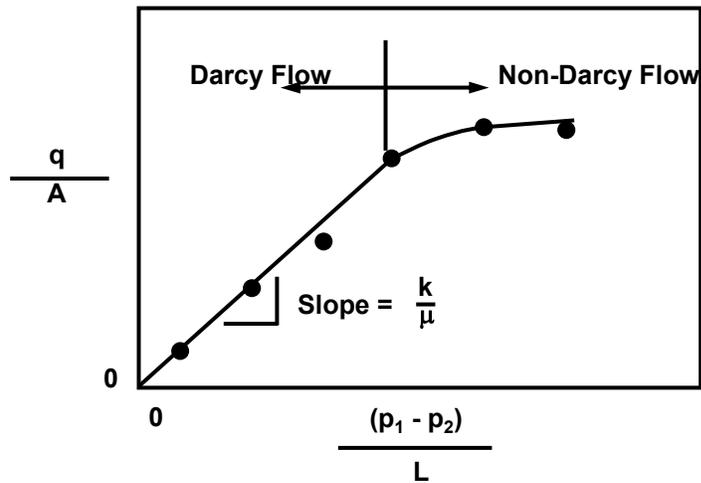
PERM PLUG METHOD LIQUID FLOW

- Measure inlet and outlet pressures (p_1 and p_2) at several different flow rates
- Graph ratio of flow rate to area (q/A) versus the pressure function $(p_1 - p_2)/L$
- For laminar flow, data follow a straight line with slope of k/μ
- At very high flow rates, turbulent flow is indicated by a deviation from straight line through origin

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Laboratory Determination of Absolute Permeability, Liquid Flow



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ISSUES AFFECTING LABORATORY MEASUREMENTS OF PERMEABILITY

- Core Handling, Cleaning, and Sampling
- Fluid-Rock Interactions
- Pressure Changes
- Rock Heterogeneities (Fractures)
- Gas Velocity Effects (Klinkenberg)

در دبی‌های کم نفوذپذیری‌ها گاز بیشتر از مایع می‌شود که بدلیل چسبیدن مایع به جداره محیط متخلخل است که اثر کلینکنبرگ نام دارد و در سنگهای با نفوذپذیری کم بیشتر می‌شود

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CORE HANDLING PROCESSES AFFECT PERMEABILITY MEASUREMENTS

- Core Handling
- Cleaning
- Drying (Clay Damage)
- Storage (Freezing)
- Sampling

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FLUID-ROCK INTERACTIONS AFFECT MEASUREMENTS OF PERMEABILITY

- Fresh water may cause clay swelling, reducing permeability
- Tests may cause fines migration, plugging pore throats and reducing permeability
- Reservoir or synthetic reservoir fluids are generally preferred

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PRESSURES AFFECT LABORATORY MEASUREMENTS OF PERMEABILITY

- Core alterations resulting from loss of Confining Pressure during core recovery
- Core testing may be conducted by applying a range of net overburden pressures

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CORE HETEROGENEITIES AFFECT MEASUREMENTS OF PERMEABILITY

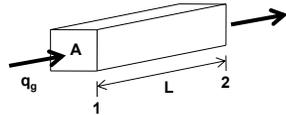
- Naturally-fractured reservoirs
 - Core plugs represent matrix permeability
 - Total system permeability (matrix + fractures) is higher
- Core Mineralogy problems (Salts, Gypsum)

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Laboratory Analysis of Gas Flow Mean Pressure Method

- Beginning from $(z\mu_g)=\text{Constant}$ Equation for Linear Flow



$$q_{g,sc} = \frac{k A}{L} \left(\frac{T_{sc}}{T p_{sc}} \right) \left(\frac{1}{2 z \mu_g} \right) (p_1^2 - p_2^2)$$

- From Real Gas Law, we can evaluate q at any pressure

$$q_{g,sc} = \frac{\bar{q} \bar{p} T_{sc}}{\bar{z} p_{sc} T} = \frac{k A}{L} \left(\frac{T_{sc}}{p_{sc} T} \right) \left(\frac{1}{2 z \mu_g} \right) (p_1^2 - p_2^2)$$

- where,

$$\bar{p} = \frac{(p_1 + p_2)}{2}$$

- and,

$$(z\mu_g) = \text{Constant} = \bar{z} \cdot \bar{\mu}_g$$

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Laboratory Analysis of Gas Flow Mean Pressure Method

- canceling terms and substituting mean pressure

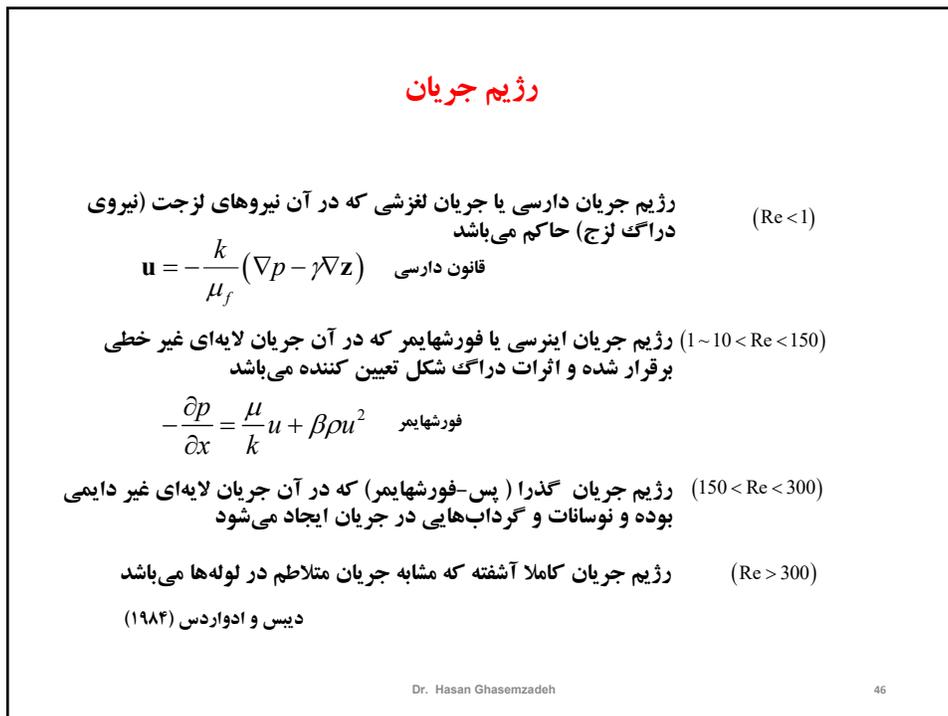
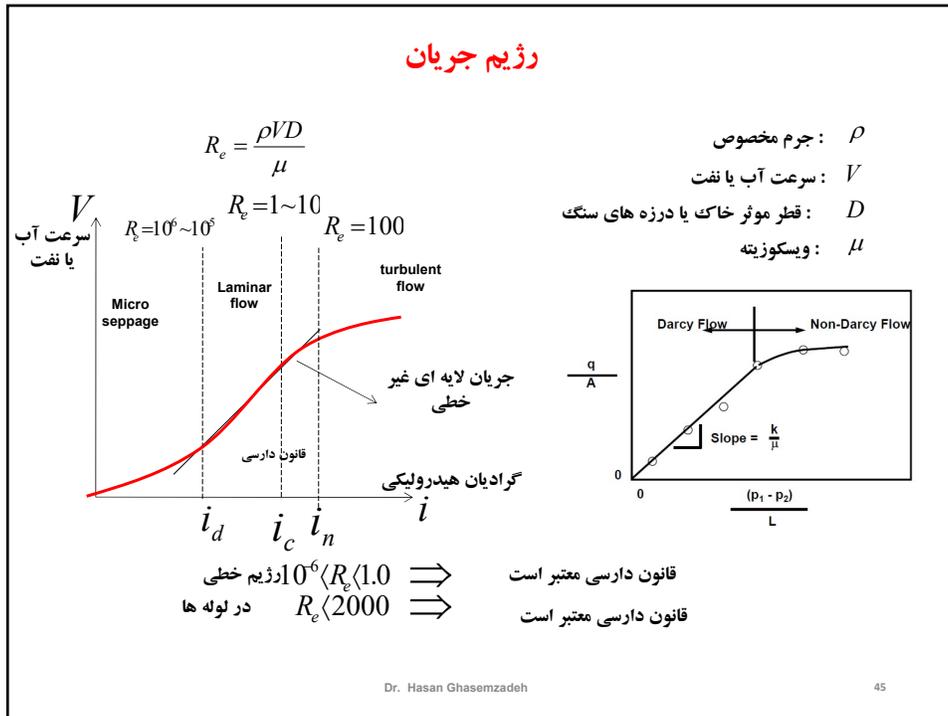
$$\bar{q} = \frac{k A}{\bar{\mu}_g L} \left(\frac{1}{\bar{p}} \right) \frac{(p_1^2 - p_2^2)}{2}$$

$$\bar{q} = \frac{k A}{\bar{\mu}_g L} (p_1 - p_2)$$

- The Mean Pressure Method is commonly used to analyze laboratory flow (low pressure)
 - flowing temperature is isothermal
 - Mean flow rate is volumetric rate at point in core where pressure is mean pressure value

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Non-Darcy Flow - Forchheimer Equation

- Forchheimer proposed a flow equation to account for the non-linear effect of turbulence by adding a second order term

$$\frac{-dp}{ds} = \frac{\mu_g}{k} \left(\frac{q_g}{A} \right) + \beta \rho_g \left(\frac{q_g}{A} \right)^2$$

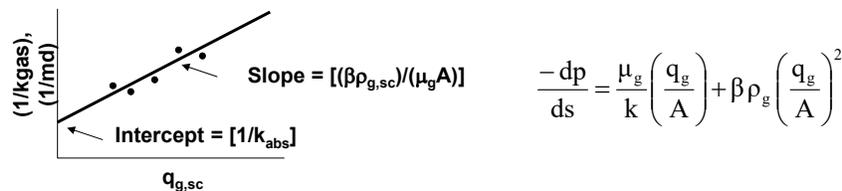
- Note that unit corrections factors would be required for non-coherent unit systems.
- As flow rate decreases, we approach Darcy's Law (2nd order term approaches zero)

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Non-Darcy Flow - Forchheimer Plot

- Based on Forchheimer's Equation a plotting method was developed to determine absolute permeability even with Non-Darcy effects
 - $(1/k_{gas})$ vs. $q_{g,sc}$
 - k_{gas} determined from Darcy's Law (incorrectly assuming Darcy flow) and is a function of $q_{g,sc}$
 - intercept = $(1/k_{abs})$; absolute permeability

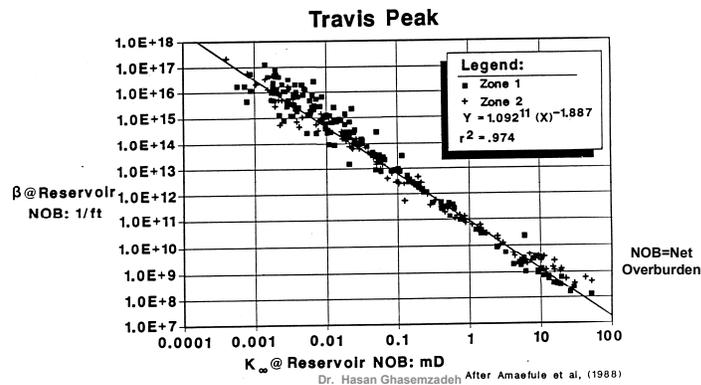


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Non-Darcy Flow - Forchheimer Equation

- Non-Darcy Coefficient, β , is an empirically determined function of absolute permeability
 - For Travis Peak (Texas)



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Air Permeability Measurement

- Measurement of permeability in the laboratory is most commonly done with air
 - Convenient and inexpensive
 - Problem: low values of mean flowing pressure
 - downstream pressure, p_{atm}
 - upstream pressure, just a few psi higher than p_{atm}
- At low mean flowing pressure, gas slippage occurs
 - Diameter of flow path through porous media approaches the "mean free path" of gas molecules
 - mean free path is a function of molecule size
 - mean free path is a function of gas density
 - Increasing mean flowing pressure results in less slippage
 - as $p_{\text{mean}} \rightarrow \infty$, we obtain absolute (equivalent liquid) permeability

mean free path (λ) is the average distance travelled by a moving particle between successive impacts (collisions), which modify its direction or energy or other particle properties

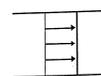
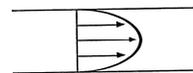
mean free path for air in ambient pressure is 68nm ($\lambda_{\text{air}} = 68 \text{ nm}$).

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Non-Darcy Flow - Gas Slippage

- Liquid flow and gas flow at high mean flowing pressure is laminar
 - Darcy's Law is valid
 - flow velocity at walls is zero
- At low mean flowing pressure gas slippage occurs
 - Non-Darcy flow is observed
 - flow at walls is not zero



Klinkenberg developed a method to correct gas permeability measured at low mean flowing pressure to equivalent liquid permeability

$$P_m = (P_1 + P_2) / 2$$

$$K_g = K_L + C * (1 / P_m)$$



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Reciprocal Mean Pressure 1/p

Knudsen number

$$Kn = \frac{\lambda}{L}$$

The ratio of molecular mean free path (λ) to the characteristic length of porous medium (L)

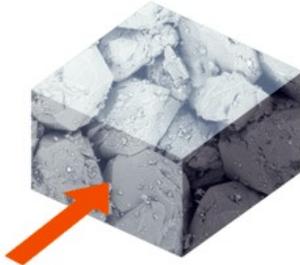
- Based on the Knudsen number magnitude, flow regimes can be classified as follows :
 - Continuum Regime : $Kn < 0.001$
 - Slip Flow Regime : $0.001 < Kn < 0.1$
 - Transition Regime : $0.1 < Kn < 10$
 - Free Molecular Regime : $Kn > 10$
- In continuum regime no-slip conditions are valid.
- In slip flow regime first order slip boundary conditions are applicable.
- In transition regime (according to the literature present) higher order slip boundary conditions may be valid.
- Transition regime with high Knudsen number and free molecular regime need molecular dynamics.

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PERMEABILITY FROM MICRO-CT SCANS

Fluid forced through digital sample



LBM simulates the Navier-Stokes equation in complex pore space

$$\rho \bar{u} \nabla \bar{u} = -\nabla p + \mu \Delta \bar{u} + \bar{F}$$

Density
Pressure
Velocity of flow
Viscosity
Body force

Lattice-Boltzmann method (LBM)

The pore volume and pore size determined from the CT Scan are manipulated mathematically by simulating the Navier-Stokes equation using the Lattice-Boltzmann Method

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آزمایشها بر روی مغزه

آنالیز معمولی مغزه (R CAL) Routine Core Analysis

با انجام آزمایشهای معمولی روی مغزه نظیر تعیین چگالی، تخلخل و نفوذپذیری می توان آنالیز معمولی مغزه را انجام داد

آنالیز ویژه مغزه (S CAL) Special Core Analysis

با انجام آزمایشهای پیشرفته روی مغزه نفوذپذیری نسبی، زاویه تماس و ... می توان آنالیز ویژه مغزه را انجام داد. خواص پتروفیزیکی تا خواص جریان سیالات (fluid flow properties)، آسیب سازند (formation damage) در این آزمایشها انجام می شود

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نفوذپذیری در برابر چند سیال

تر شوندگی:

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نفوذپذیری در برابر چند سیال

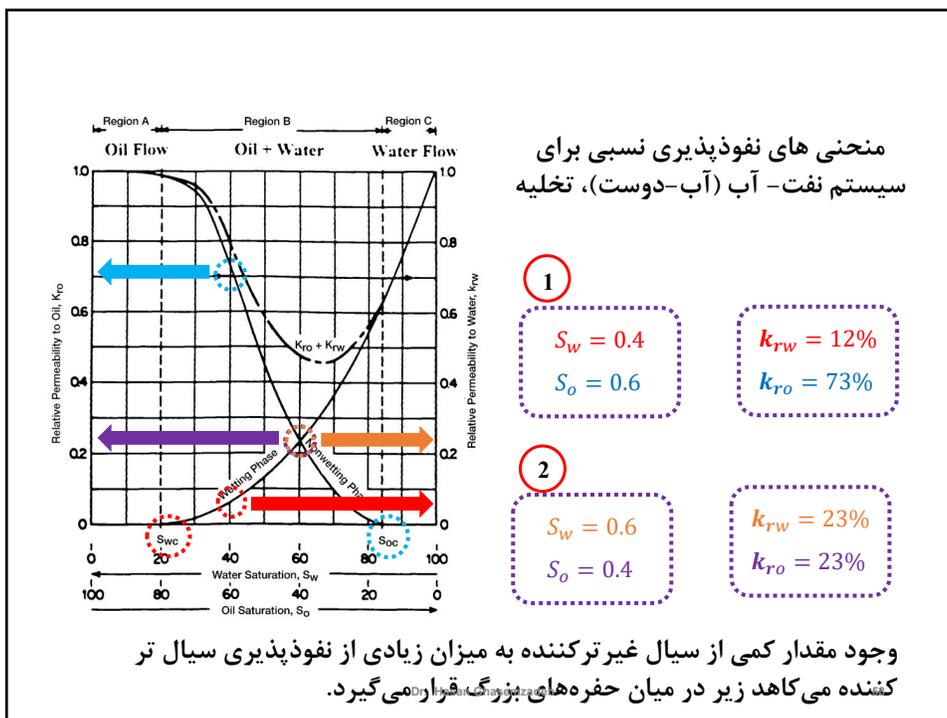
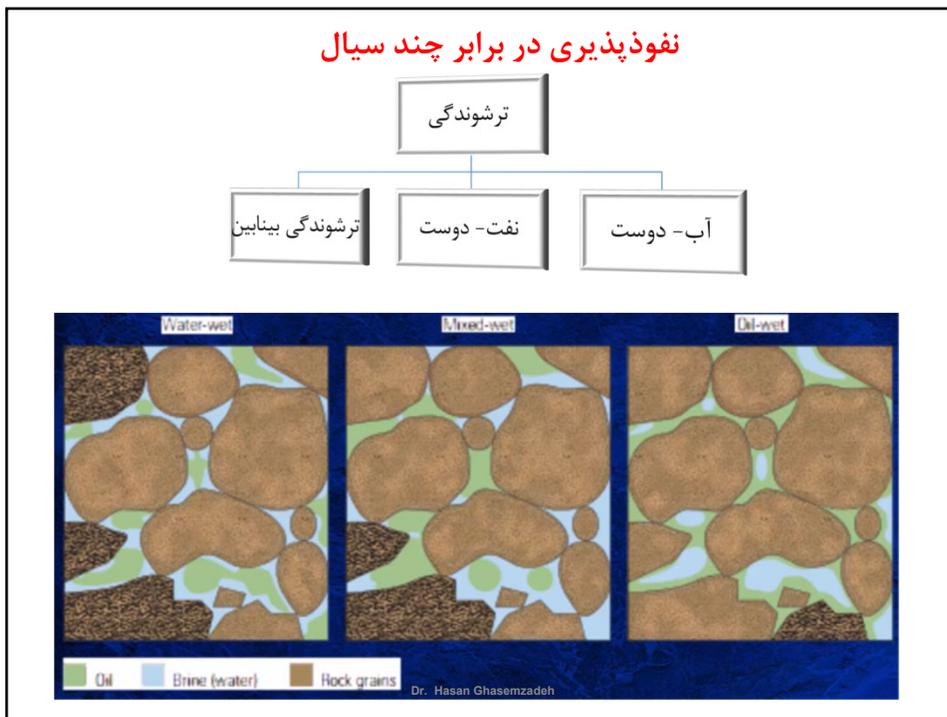
WATER-WET OIL-WET

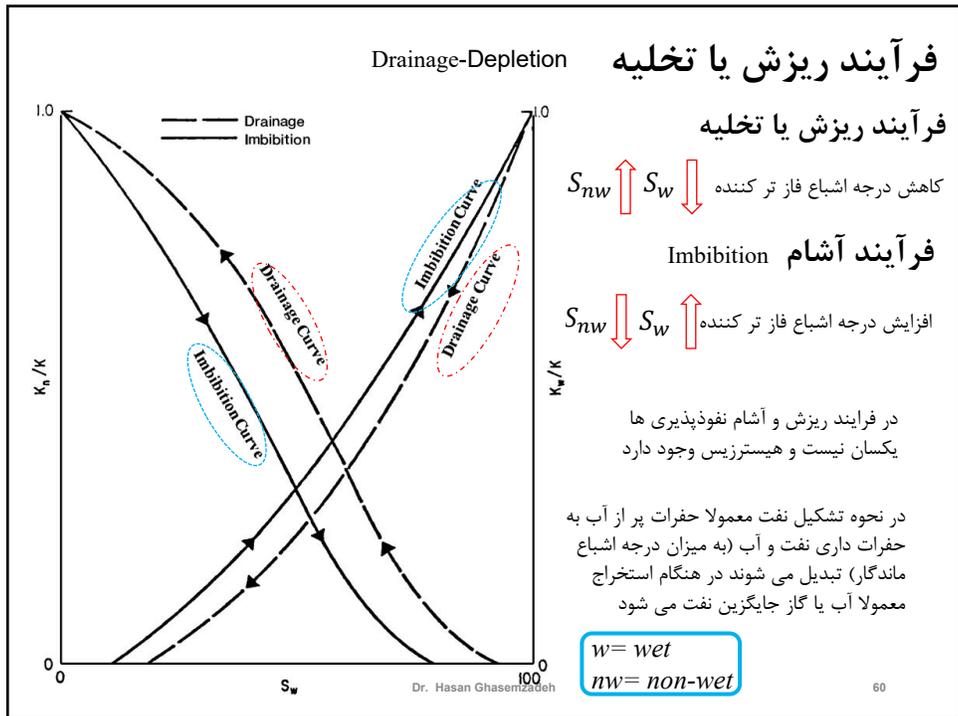
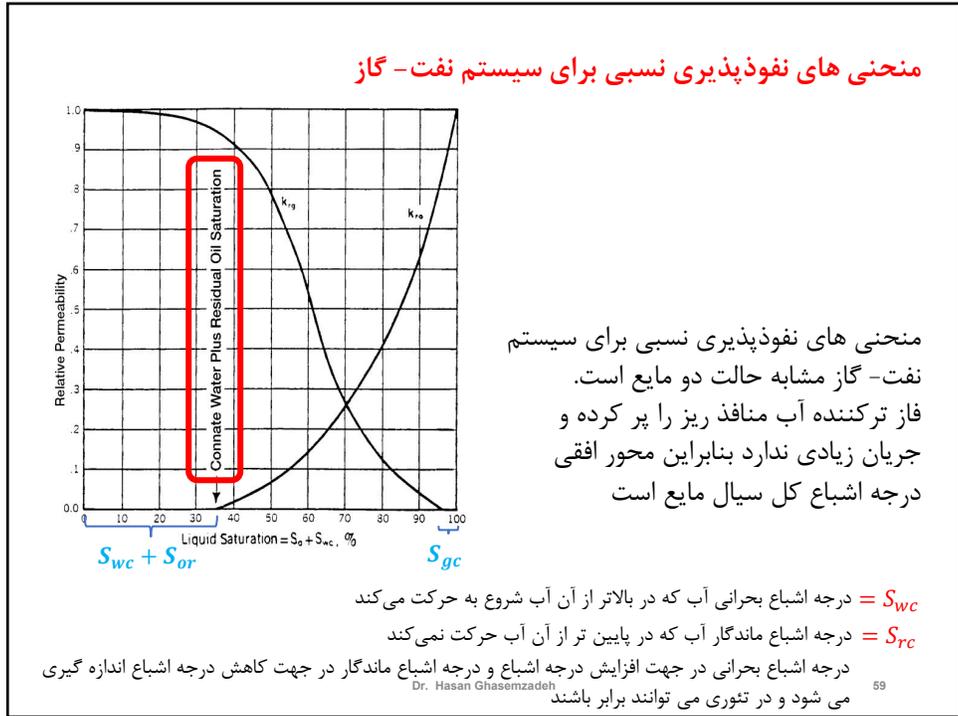
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ماسه
سنگ

دولومیت
لایم استون





روابط نفوذپذیری

$$S_o^* = \frac{S_o}{1 - S_{wc}} \quad S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc}} \quad S_g^* = \frac{S_g}{1 - S_{wc}}$$

تعریف درجه اشباع موثر

نوع سازند	k_{ro}	k_{rw}
ماسه نامستحکم، دانه بندی خوب	$(1 - S_w^*)$	$(S_w^*)^3$
ماسه نامستحکم، دانه بندی ضعیف	$(1 - S_w^*)^2 (1 - S_w^{*1.5})$	$(S_o^*)^{3.5}$
ماسه سنگ مستحکم، آهک اولیتیکی	$(1 - S_o^*)^2 (1 - S_w^{*2})$	$(S_o^*)^4$

روابط ویلی و گاردنر

برای آب - نفت

برای نفت- گاز کافیسیت نفوذپذیری نسبی فازهای ترکننده و غیرترکننده را به ترتیب برای نفت و گاز قرار دهیم

$$k_{rw} = (S_w^*)^2 - k_{ro} \left[\frac{S_w^*}{1 - S_w^*} \right]$$

روابط بین نفوذپذیری های نسبی

$$k_{ro} = (S_o^*)^2 - k_{rg} \left[\frac{S_o^*}{1 - S_o^*} \right]$$

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روابط نفوذپذیری

$$k_{ro} = k_{rg} \left[\frac{(S_o^*)^4}{(1 - S_o^*)^2 (1 - (S_o^*)^2)} \right]$$

رابطه تورکاسو - ویلی برای نفت- گاز

$$k_{rw} = \sqrt{S_w^*} S_w^3$$

رابطه پیروسون

برای فاز آب در دو حالت تخلیه و آشام

$$(k_r)_{nonwetting} = \left[1 - \left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{nw}} \right) \right]^2$$

آشام

$$(k_r)_{nonwetting} = (1 - S_w^*) \left[1 - (S_w^*)^{0.25} \sqrt{S_w} \right]^{0.5}$$

تخلیه

$$k_{ro} = (1 - S_g^*)^4$$

رابطه کوری

$$k_{rg} = (S_g^*) (2 - S_g^*)$$

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روابط نفوذپذیری بر اساس فشار موینگی

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^2 \frac{\int_{S_{wc}}^{S_w} dS_w / p_c^2}{\int_{S_{wc}}^1 dS_w / p_c^2}$$

$$k_{ro} = \left(\frac{1 - S_w}{1 - S_{wc}} \right)^2 \frac{\int_{S_{wc}}^1 dS_w / p_c^2}{\int_{S_{wc}}^{S_w} dS_w / p_c^2}$$

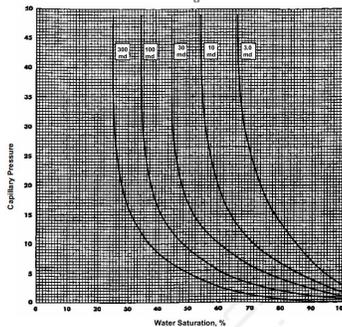
روابط ویلی و گاردنر

برای آب - نفت

$$k_{rg} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^2 \frac{\int_0^{S_o} dS_o / p_c^2}{\int_0^1 dS_o / p_c^2}$$

$$k_{ro} = \left(1 - \frac{S_o - S_{or}}{S_g - S_{gc}} \right)^2 \frac{\int_{S_{gc}}^1 dS_o / p_c^2}{\int_0^{S_o} dS_o / p_c^2}$$

برای نفت- گاز



S_{gc} درجه اشباع بحرانی گاز؛

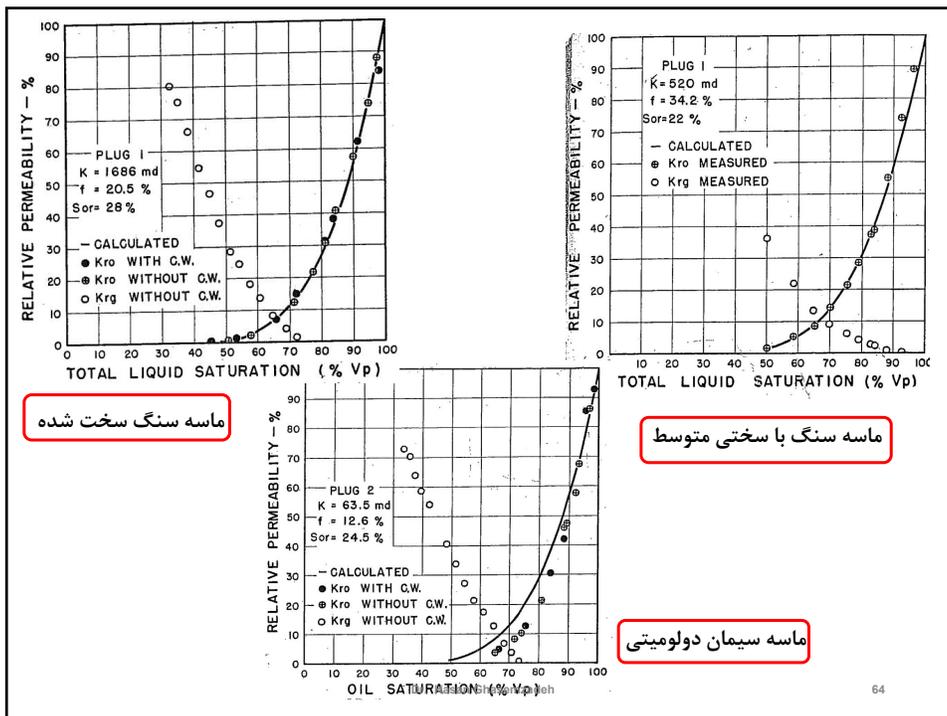
S_{wc} درجه اشباع آب محتوایی؛

S_{or} درجه اشباع نفت باقی مانده.

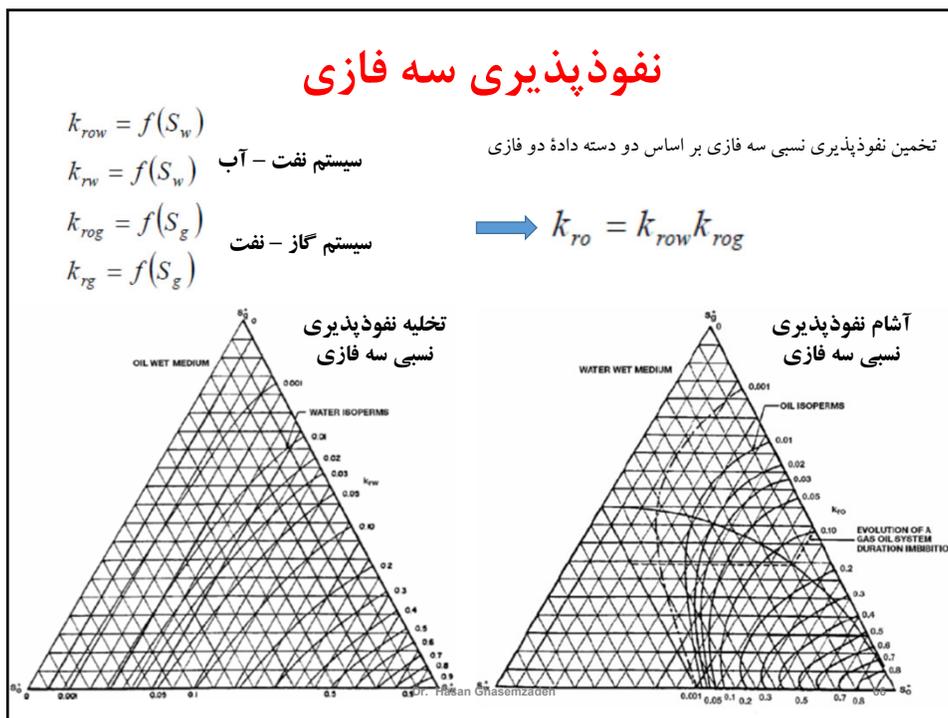
$$P_c = P_{nw} - P_w$$

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روابط وابلی - نفوذ پذیری سه فازی سامانه آب دوست

(Oolitic (egg stone)) در ماسه سنگ مستحکم، سنگ حفره دار یا سنگ آهک اولیتیک

$$k_{rg} = \frac{S_g^2 [(1 - S_{wc})^2 - (S_w + S_o - S_{wc})^2]}{(1 - S_{wc})^4}$$

$$k_{ro} = \frac{S_o^3 (2S_w + S_o - 2S_{wc})}{(1 - S_{wc})^4}$$

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wc}} \right)^4$$



در ماسه سنگ مستحکم با جورشدگی مناسب دانه ها

$$k_{rg} = \frac{S_o^3 (2S_w + S_o - 2S_{wc})^4}{(1 - S_{wi})^4}$$

$$k_{ro} = \frac{S_o^3}{(1 - S_{wc})^3}$$

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wi}} \right)^3$$

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مدل احتمالی برای تخمین نفوذ پذیری نسبی نفت در محیط سه فاز نفت - آب - گاز

مدل استون (I)

$$S_o^* = \frac{S_o - S_{om}}{(1 - S_{wc} - S_{om})} \quad (\text{for } S_o \geq S_{om})$$

درجات اشباع نرمالیزه استون

$$S_w^* = \frac{S_w - S_{wc}}{(1 - S_{wc} - S_{om})} \quad (\text{for } S_w \geq S_{wc})$$

$$S_g^* = \frac{S_g}{(1 - S_{wc} - S_{om})}$$

$$S_g^* + S_w^* + S_o^* = 1$$

S_{om} درجه اشباع نفت ماندگار زمانی که نفت همزمان با گاز و آب وجود دارد

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org}$$

$$\alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}}$$

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مدل احتمالی برای تخمین نفوذپذیری نسبی نفت در محیط سه فاز نفت-آب-گاز

$$K_{ro} = S_o^* \beta_w \beta_g$$

$$\beta_w = \frac{k_{row}}{1 - S_w^*}$$

$$S_o^* = 1 \Rightarrow K_{ro} = 1$$

$$\beta_g = \frac{k_{rog}}{1 - S_g^*}$$

با کاهش S_o^* (افزایش درجه اشباع آب یا گاز) K_{ro} با نسبت بیشتری نسبت به S_o^* کاهش می یابد.

برای رفع مشکل نرمالیزه در این مدل

$$k_{ro} = \frac{S_o^*}{(1 - S_w^*)(1 - S_g^*)} \left(\frac{k_{row} k_{rog}}{(k_{ro})_{Swc}} \right)$$

شکل نرمالیزه عزیز و ستاری

مقدار نفوذپذیری نفت در درجه اشباع آب محتوای سیستم نفت-آب $(k_{ro})_{Swc}$

$$k_{ro} = (k_{ro})_{Swc} \left[\left(\frac{k_{row}}{(k_{ro})_{Swc}} + k_{rw} \right) \left(\frac{k_{rog}}{(k_{ro})_{Swc}} + k_{rg} \right) - (k_{rw} + k_{rg}) \right] \quad \text{مدل استون (II)}$$

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رابطه هوستاد- هولت

$$k_{ro} = \left[\frac{k_{row} k_{rog}}{(k_{ro})_{Swc}} \right] \beta^{n_1}$$

به رابطه عزیز و ستاری تبدیل می شود $n = 1$

$$\beta = \frac{S_o^*}{(1 - S_w^*)(1 - S_g^*)} \quad 0 \leq \beta \leq 1$$

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om} - S_{gc}}$$

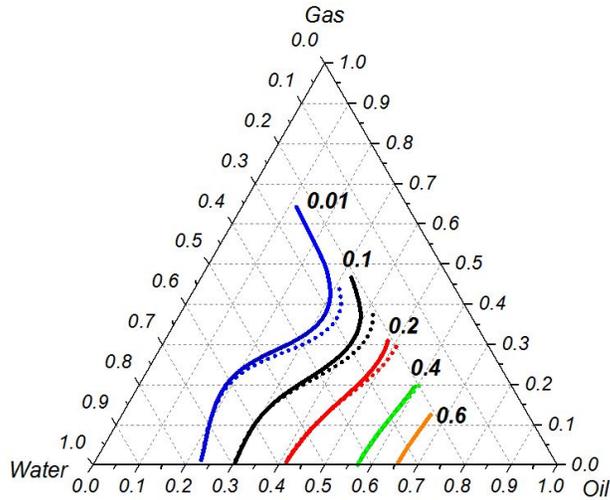
$$S_g^* = \frac{S_g - S_{gc}}{1 - S_{wc} - S_{om} - S_{gc}}$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om} - S_{gc}}$$

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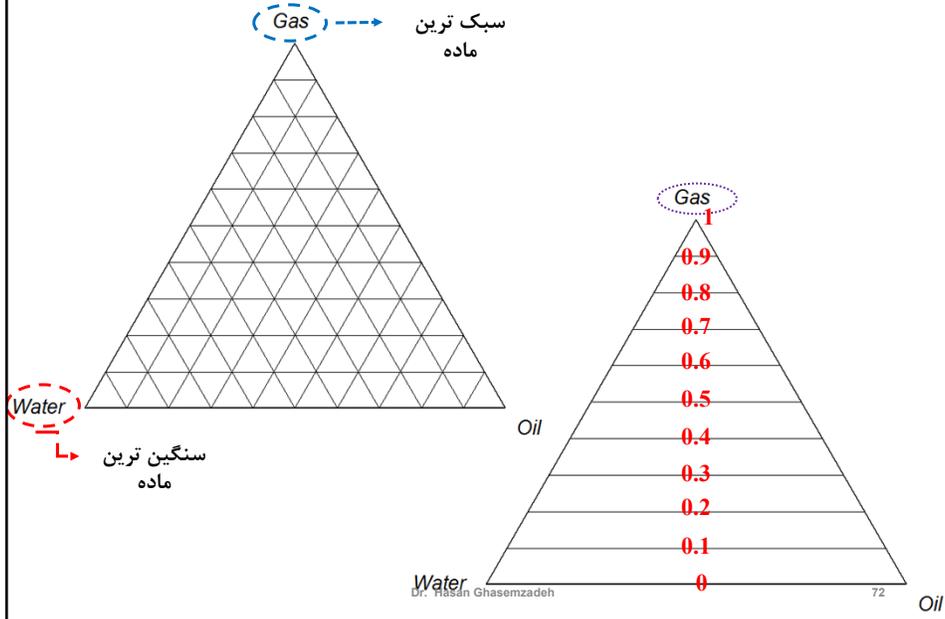
نمودارهایی نفوذپذیری سه فازی



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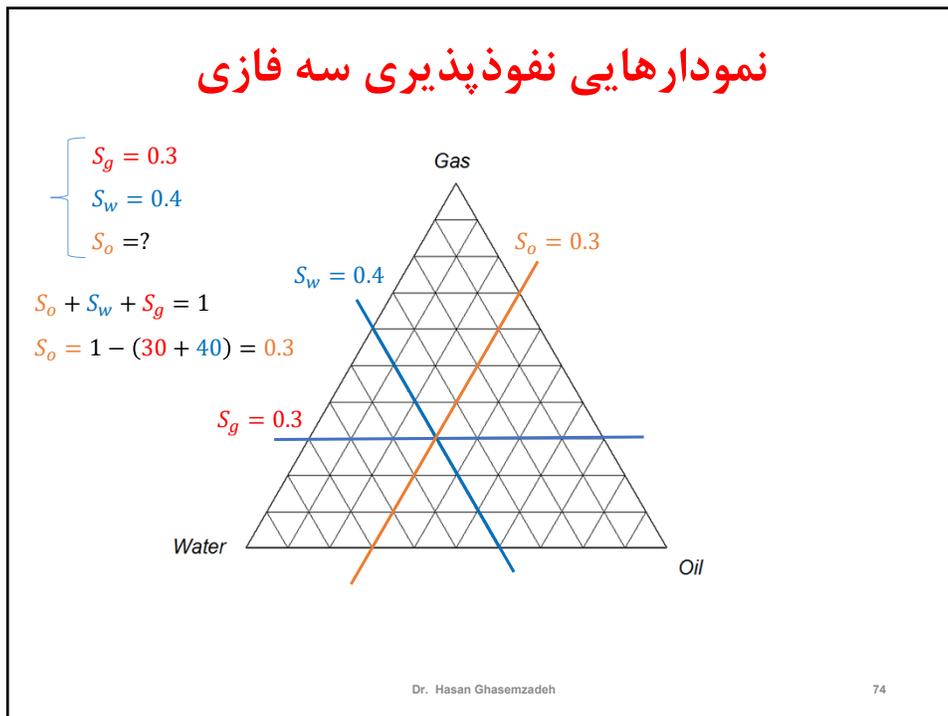
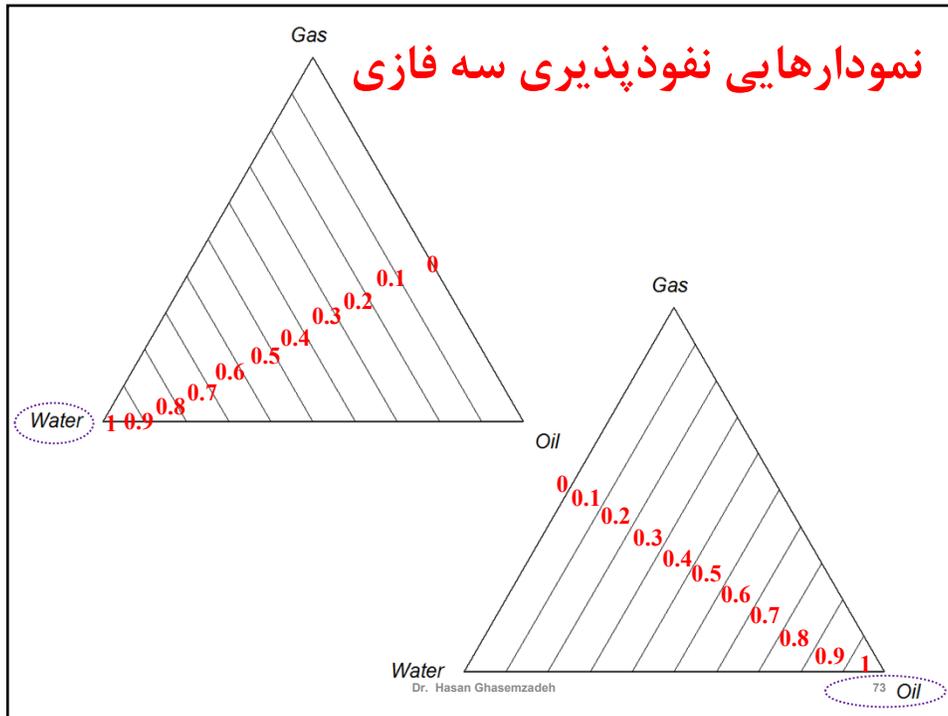
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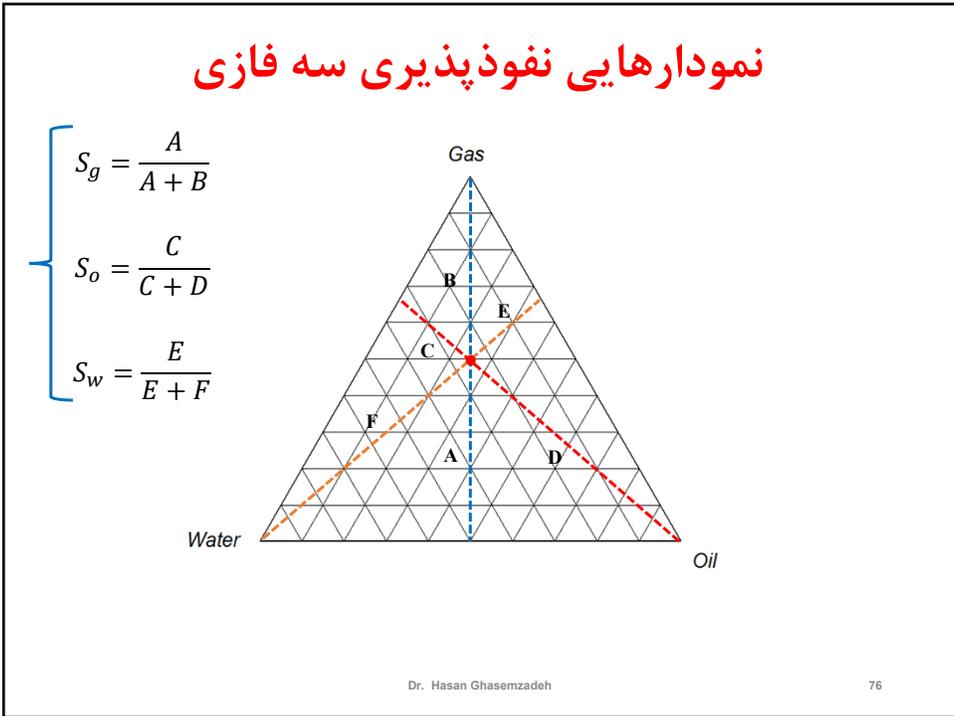
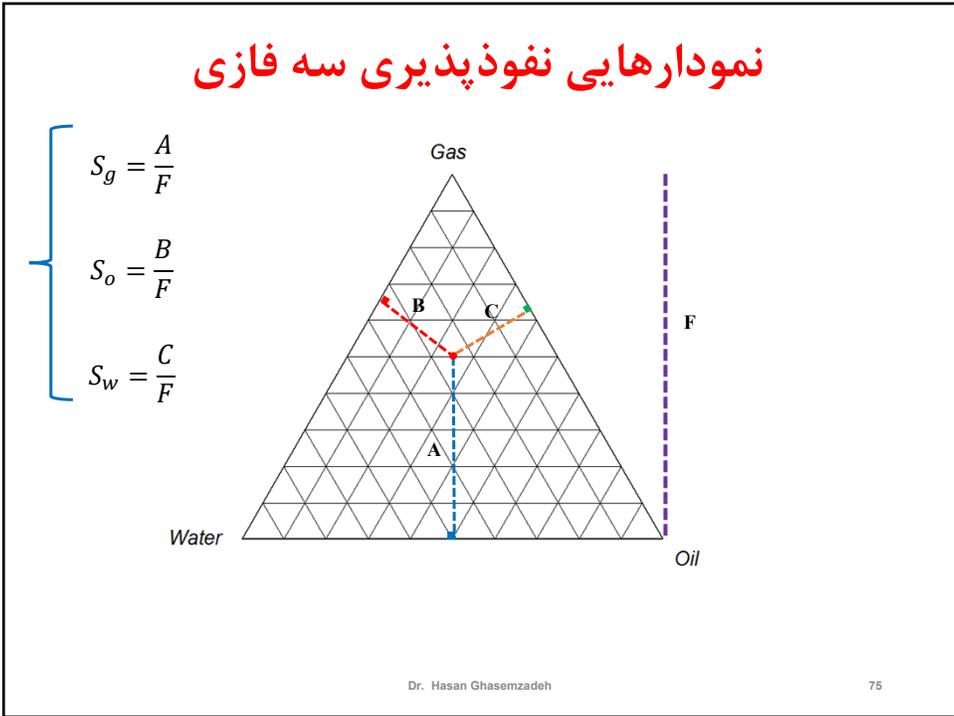
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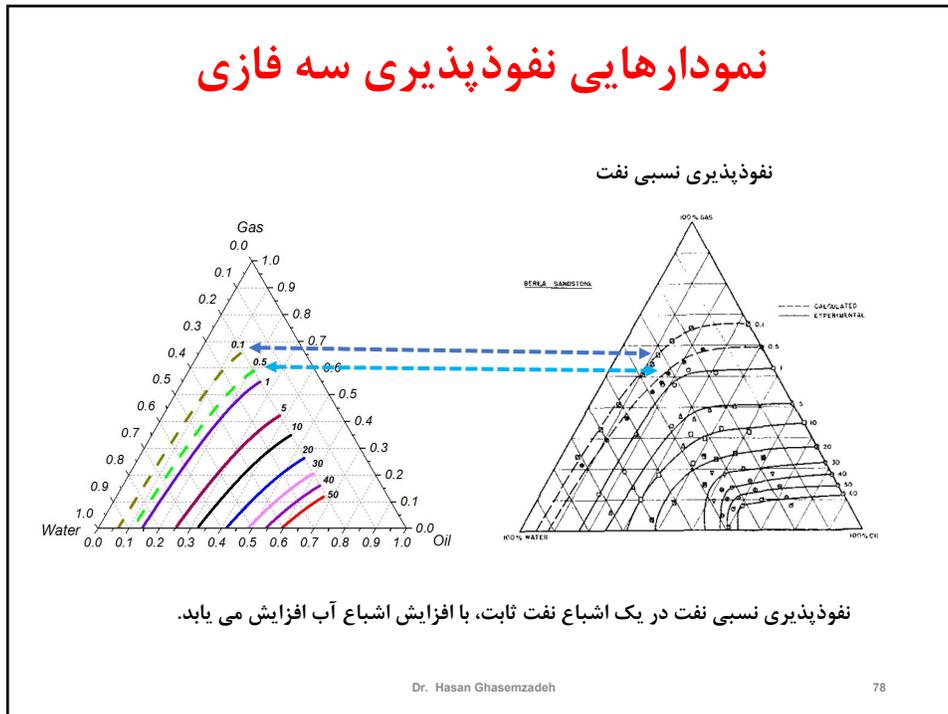
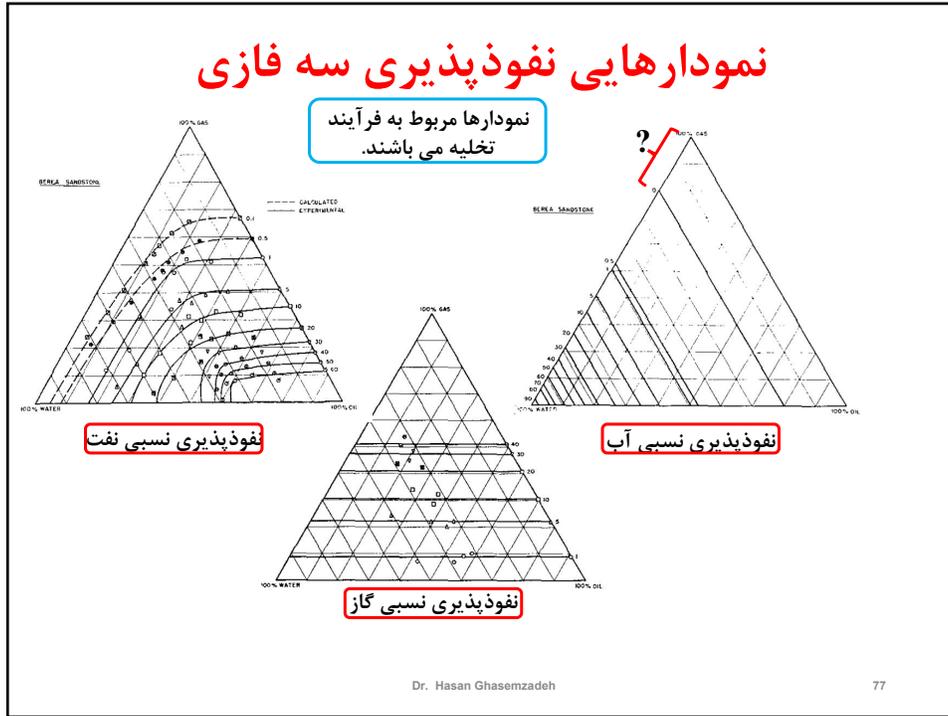


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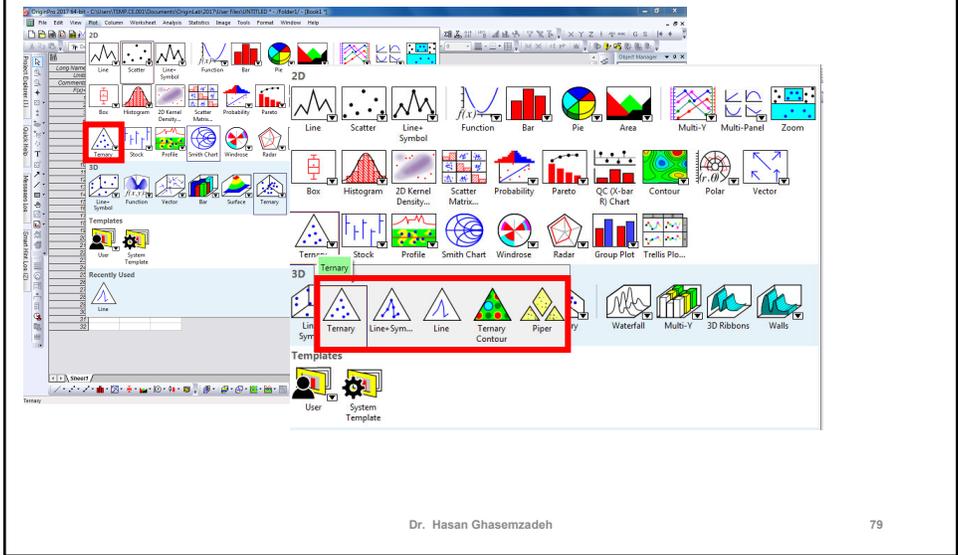






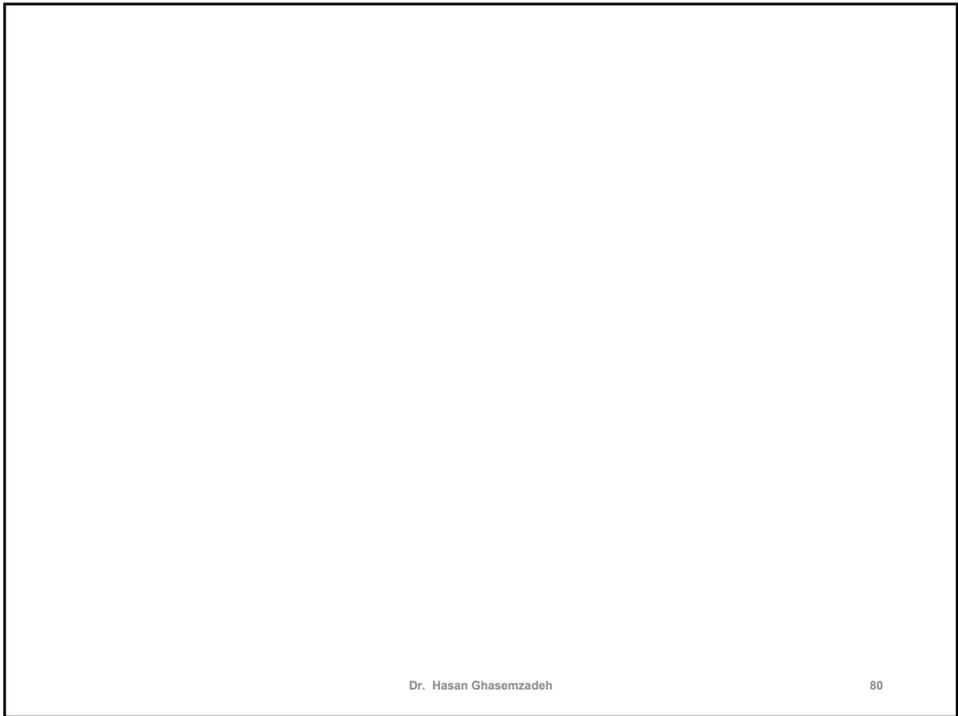
نمودارهایی نفوذپذیری سه فازی

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تعیین نفوذپذیری نسبی نفت در محیط دو فاز نفت - گاز

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^2 \frac{\int_0^{S_o} \frac{dS_o}{P_c^2}}{\int_0^1 \frac{dS_o}{P_c^2}}$$

1 **Burnide** بر اساس معادله Kozeny-Carman روابط زیر را برای نفوذپذیری ارائه داد.

$$K_{rg} = \left(\frac{S_o - S_{or}}{S_m - S_{or}} \right)^2 \frac{\int_{S_o}^1 \frac{dS_o}{P_c^2}}{\int_0^1 \frac{dS_o}{P_c^2}}$$

2 فشار موینگی P_c

$$\frac{1}{P_c^2} = \begin{cases} \bar{C}(S_o - S_{or}) & \text{for } S_o > S_{or} \\ 0 & \text{for } S_o < S_{or} \end{cases}$$

3

$$\frac{1}{P_c^2} = CS_{oe}$$

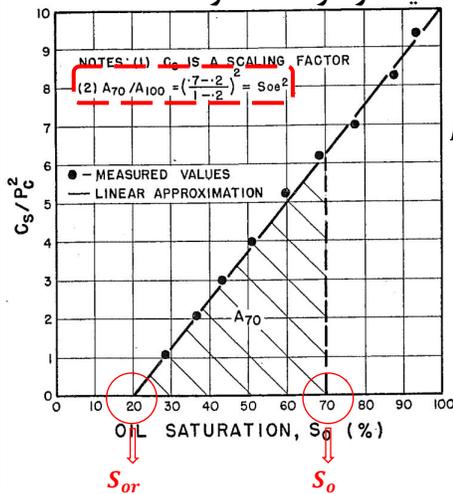
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$$C = \bar{C}(1 - S_{or}) \quad S_{oe} = \frac{S_o - S_{or}}{1 - S_{or}}$$

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Corey (1954)

تعیین نفوذپذیری نسبی نفت در محیط دو فاز نفت - گاز



$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^2 \frac{\int_0^{S_o} \frac{dS_o}{P_c^2}}{\int_0^1 \frac{dS_o}{P_c^2}}$$

$$\frac{\int_0^{S_o} \frac{dS_o}{P_c^2}}{\int_0^1 \frac{dS_o}{P_c^2}} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^2$$

اگر $S_m = 1$ فرض شود $\Rightarrow K_{ro} = S_{oe}^4$ 5
 $\Rightarrow k_{rg} = (1 - S_{oe})^2 (1 - S_{oe}^2)$ 6

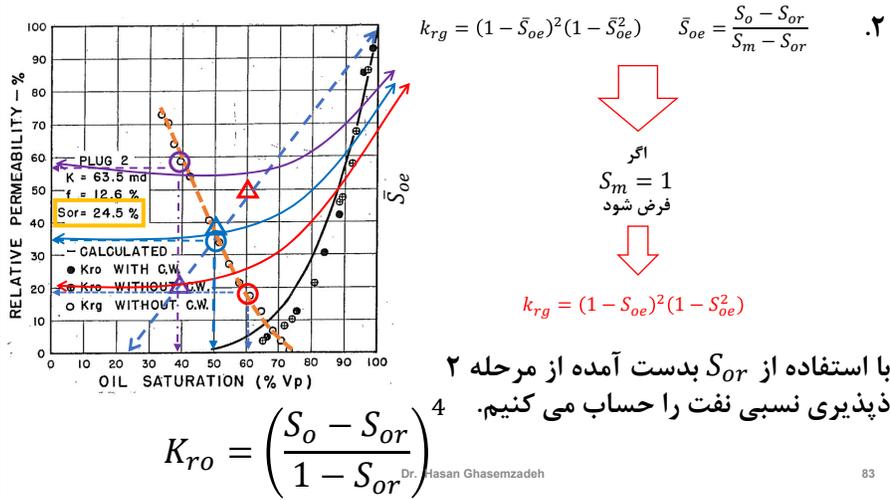
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نحوه استفاده از روابط:

۱. رسم نمودار k_{rg} بر حسب S_o

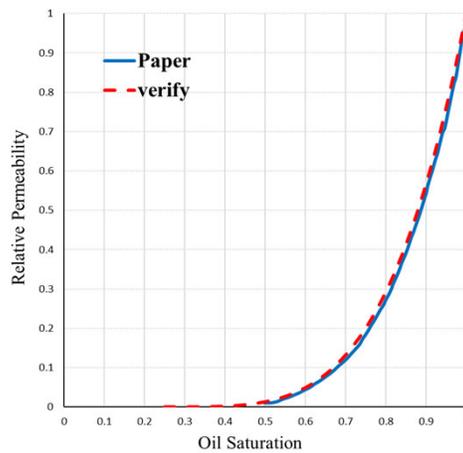
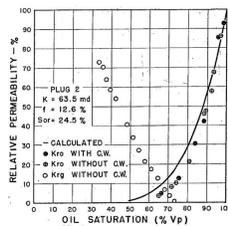
(استفاده از روش Capillary pressure technique، یک روش ساده برای اندازه گیری سریع نفوذپذیری نسبی گاز می باشد).



تعیین نفوذپذیری نسبی نفت در محیط دو فاز نفت-گاز

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^4$$

$$S_{or} = 0.245$$



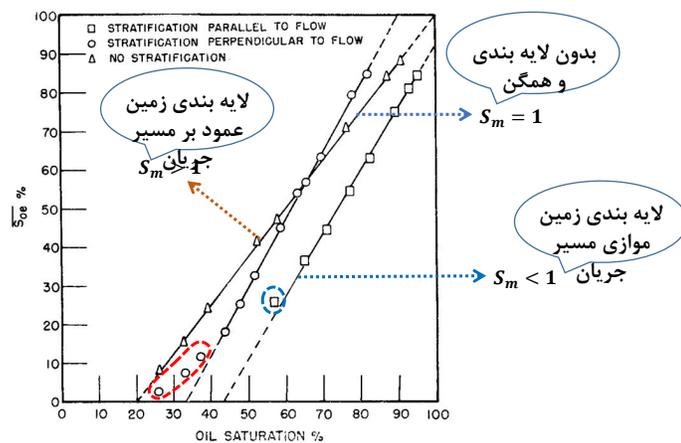
تعیین نفوذپذیری نسبی نفت در محیط دو فاز نفت- گاز

- ❖ از این روش نمی توان در سیستم های دوفازی آب- گاز یا آب- نفت استفاده کرد.
- ❖ از این روش برای حالت آشام نمی توان استفاده کرد.
- ❖ این روابط برای مخازن ماسه سخت شده مناسب هست.
- ❖ دلیل استفاده از منحنی های K_{rg} صرفا برای بدست آوردن پارامترهای S_{or} و S_m هست و اگر بتوان مقادیر این پارامترها را پیدا کرد، دیگر نیازی به منحنی های K_{rg} نیست.
- ❖ S_m در حقیقت می تواند معنای فیزیکی نداشته باشد و صرفا برای بدست آوردن S_{or} استفاده می شود.

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تأثیر لایه بندی بر نفوذپذیری نسبی



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Corey (1956)

تعیین نفوذپذیری نسبی نفت در محیط سه فاز نفت- آب- گاز

$$\frac{1}{P_c^2} = \begin{cases} \bar{C}(S_o - S_{or}) & \text{for } S_o > S_{or} \\ 0 & \text{for } S_o < S_{or} \end{cases}$$



$$\frac{1}{P_c^2} = \begin{cases} \bar{C}(S_L - S_{Lr}) & \text{for } S_L > S_{Lr} \\ 0 & \text{for } S_L \leq S_{Lr} \end{cases}$$

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تعیین نفوذپذیری نسبی نفت در محیط سه فاز نفت- آب- گاز

$$K_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}} \right)^2 \frac{\int_0^{S_o} \frac{dS_o}{P_c^2}}{\int_0^1 \frac{dS_o}{P_c^2}}$$



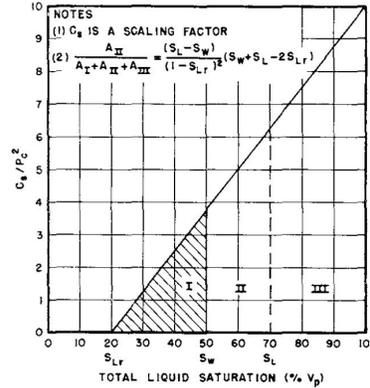
$$K_{ro} = \left(\frac{S_L - S_{Lr}}{1 - S_{Lr}} \right)^2 \frac{\int_{S_{Lr}}^{S_L} \frac{dS_L}{P_c^2}}{\int_0^1 \frac{dS_L}{P_c^2}}$$

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تعیین نفوذپذیری نسبی نفت در محیط سه فاز نفت- آب- گاز

$$\int_{S_W}^{S_L} \frac{dS_L}{P_c^2} = \frac{A_{II}}{A_I + A_{II} + A_{III}} = \frac{(S_L - S_W)}{(1 - S_{Lr})^2} (S_W + S_L - S_{Lr})$$



$$k_{ro} = \frac{(S_L - S_W)^3}{(1 - S_{Lr})^4} (S_W + S_L - 2S_{Lr})$$

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مدل احتمالی برای تخمین نفوذپذیری نسبی نفت در محیط سه فاز نفت- آب- گاز

- ✓ ارائه نفوذپذیری نسبی نفت در محیط سه فاز با استفاده از دو دسته از داده های دو فاز (نفت- آب و نفت- گاز)
- ✓ به عبارت دیگر تعیین نفوذپذیری نسبی فاز ترشوندگی بینابینی با استفاده از دو دسته از داده های نفوذپذیری نسبی در یک سیستم سه فازی با استفاده از درون یابی
- ✓ مدل بر اساس مفهوم احتمالات و تعاریف تجربی است
- ✓ اگرچه این مدل برای هر دو سیستم ترجیحا آب- دوست یا نفت- دوست قابل کاربرد است اما در این مقاله صرفا برای حالت آب- دوست ارائه شده است.

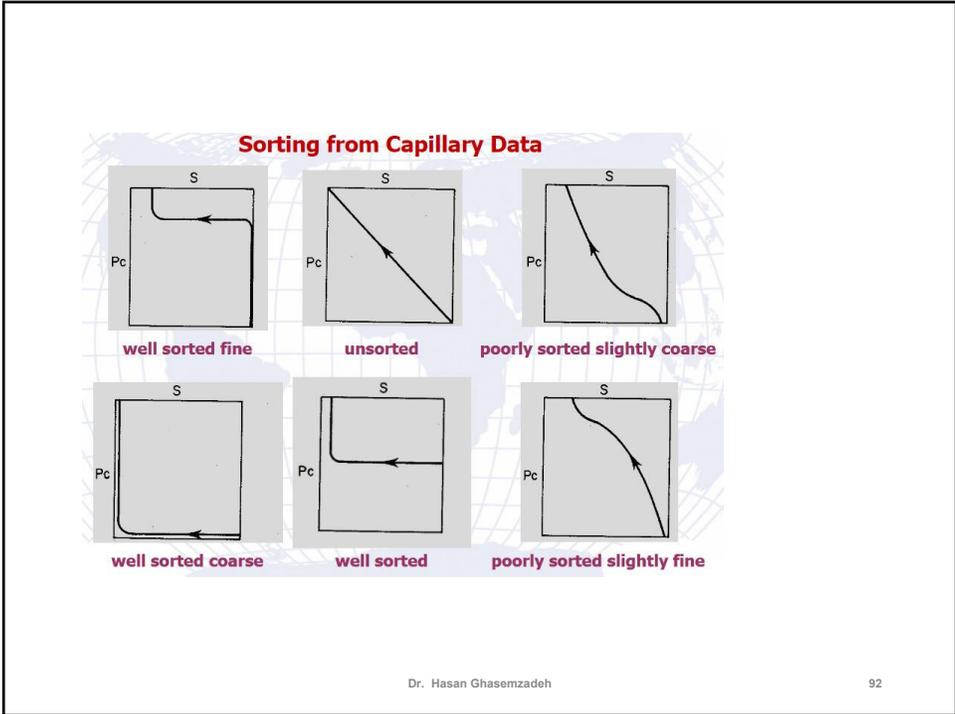
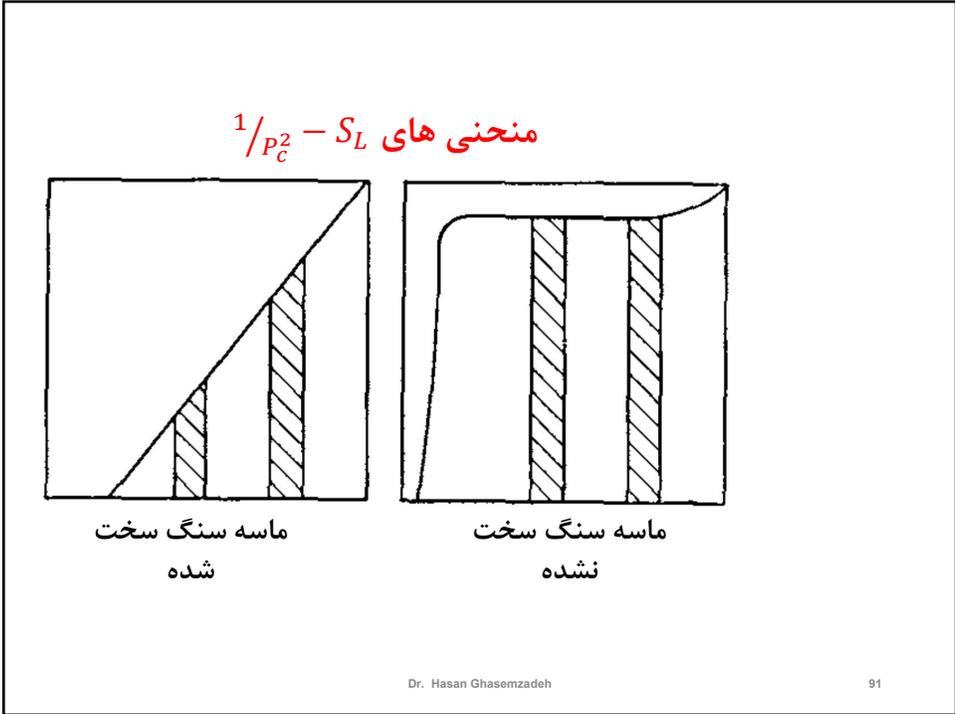
داده های مورد نیاز:

- بدست آوردن مقادیر K_{rOw} و K_{rW} از داده های نفت- آب که تابع اشباع آب هستند (K_{rOw} یعنی نفوذپذیری نسبی نفت در محیط دو فاز نفت- آب).
- بدست آوردن مقادیر K_{rOg} و K_{rG} از داده های نفت- گاز که تابع اشباع گاز هستند (K_{rOg} یعنی نفوذپذیری نسبی نفت در محیط دو فاز نفت- گاز).
- همچنین در این روش سعی شده است اثرات هیستریزس در نظر گرفته شود.
- ✓ مطالعات قبلی نشان داده است که در سیستم آب- دوست نفوذپذیری نسبی فاز تر (آب) فقط تابع اشباع آب هست و نفوذپذیری نسبی فاز نادر (گاز) فقط تابع اشباع گاز هست و نفوذپذیری فاز ترشوندگی میانی پیچیده تر است.

Stone (1970)

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صحت سنجی:

✓ سه دسته داده نفوذپذیری نسبی سه فاز برای صحت رابطه ارائه شده با مقالات Corey et al ، Dalton et al و Saraf ارائه شده است.

✓ با توجه به ارائه مقاله Corey در بخش قبل فقط این قسمت ارائه و صحت سنجی شده است.

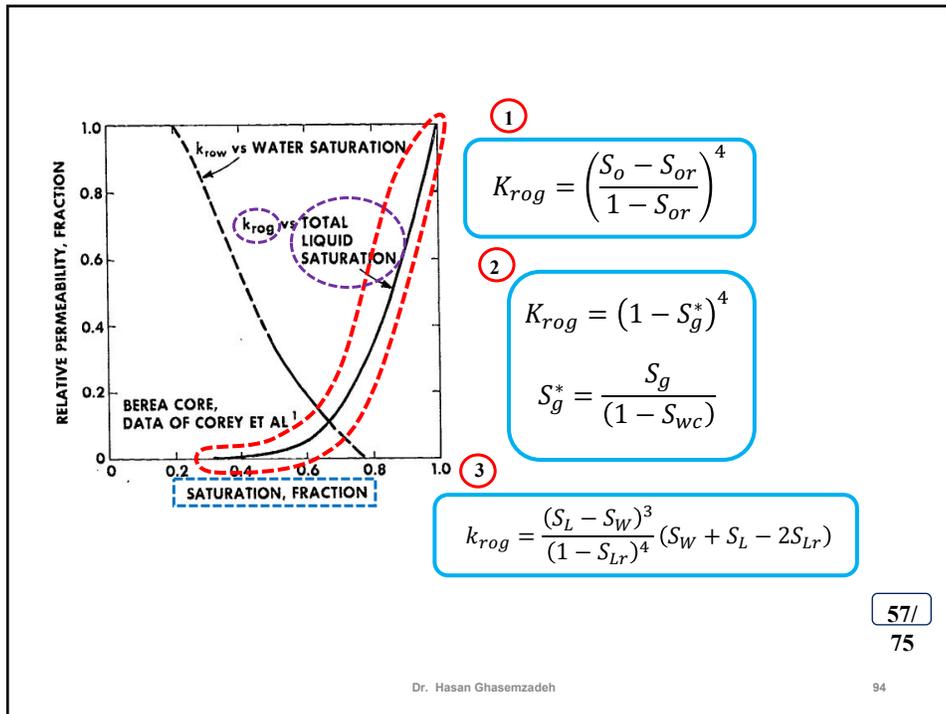
صحت سنجی با داده های مقاله Corey:

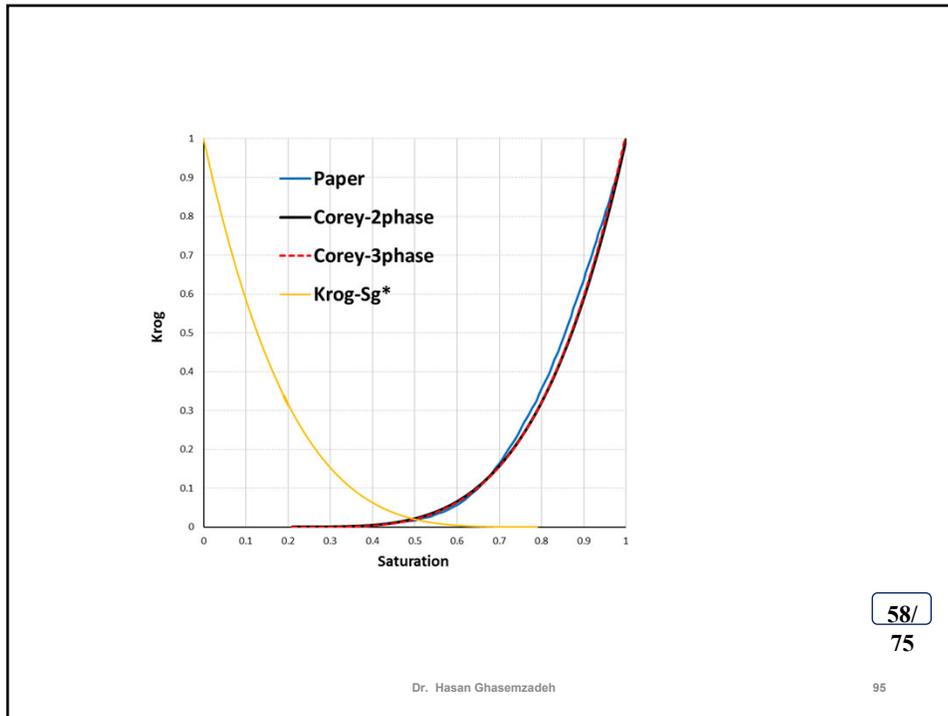
$$S_{om} = 0$$

$$S_{om} = \frac{1}{2} S_{wc} = 0.1$$

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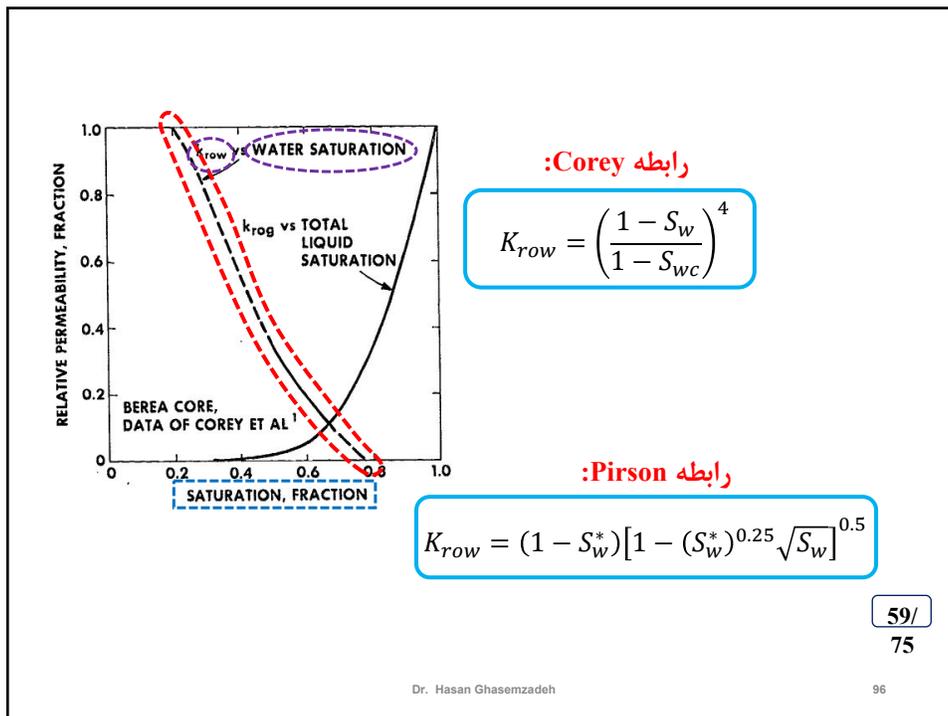
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75

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رابطه Corey:

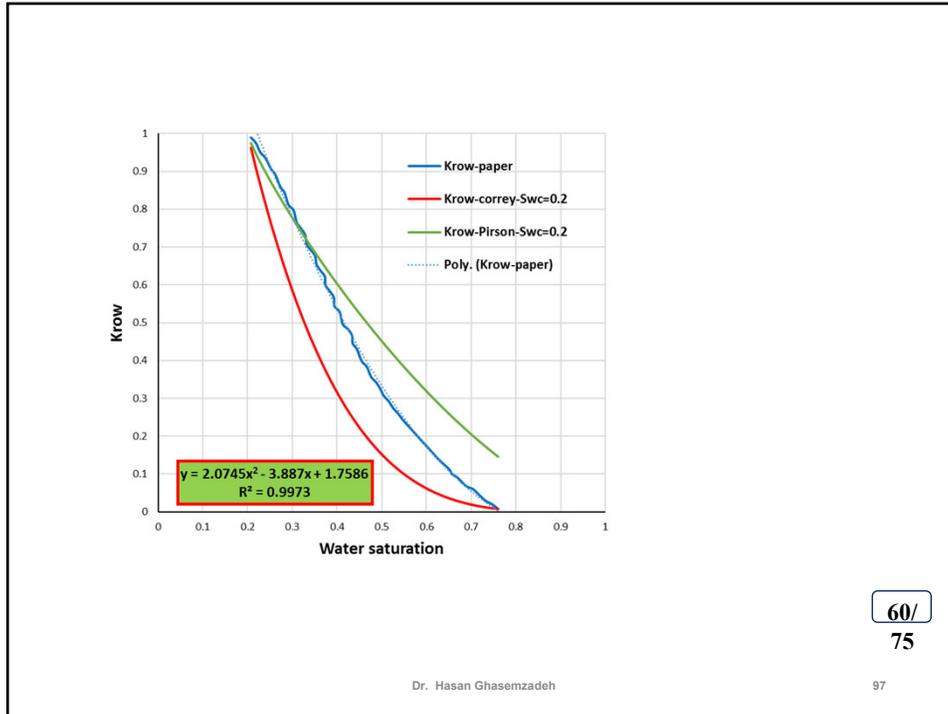
$$K_{row} = \left(\frac{1 - S_w}{1 - S_{wc}} \right)^4$$

رابطه Pirson:

$$K_{row} = (1 - S_w^*) [1 - (S_w^*)^{0.25} \sqrt{S_w}]^{0.5}$$

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75

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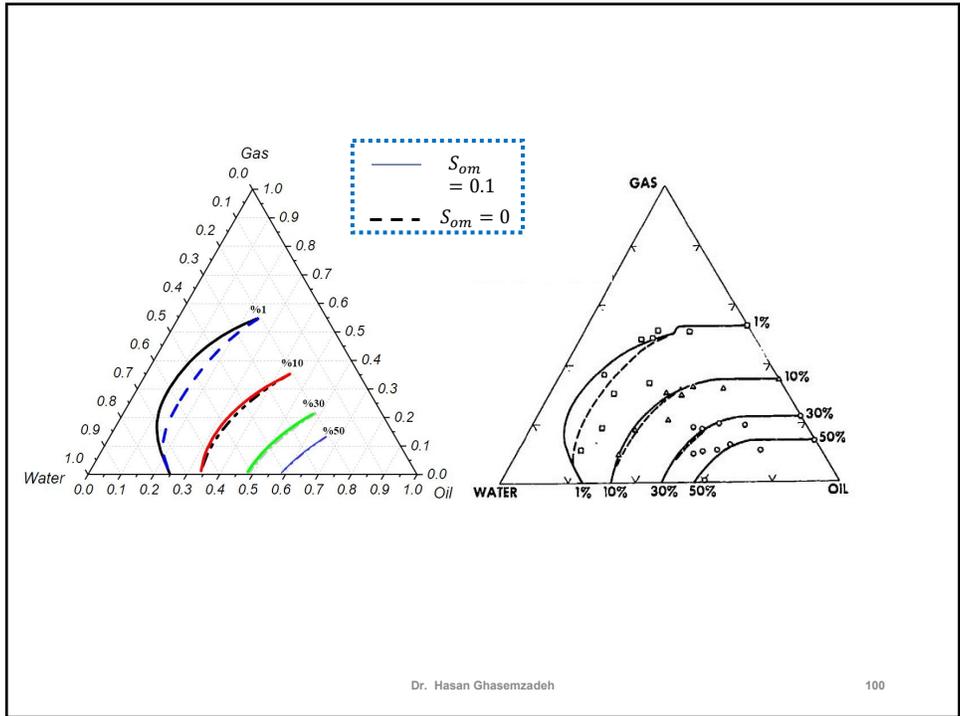
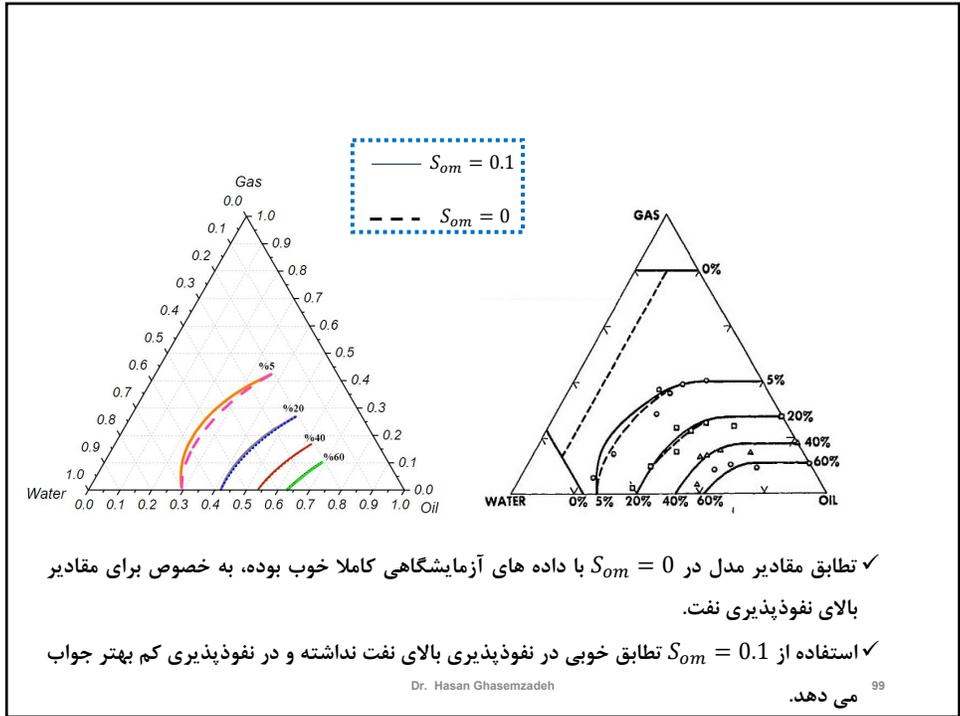
```

StoneIm
1 - |clc
2 - |clear all
3
4 - |syms So
5
6
7 - |Kro = 0.2;
8
9 - |Som = 0.1;
10 - |Swc = 0.2;
11 - |Sw = 0.21:0.01:1.0;
12 - |Sw = Sw';
13 - |Answer = zeros(numel(Sw), 1);
14
15 - |for i = 1:numel(Sw)
16
17 - |Sww = (Sw(i) - Swc)/(1 - Swc - Som);
18 - |Krow = ((2.0745*(Sww(i)^2) - (3.887*Sww(i)) + 1.7586);
19 - |Bw = Krow/(1 - Sww);
20 - |Soo = (So - Som)/(1 - Swc - Som);
21 - |Sg = 1 - (Sw(i) + So);
22 - |Sgg = Sg/(1 - Swc - Som);
23 - |Krog = (1 - (Sg/(1-Swc)))^4;
24 - |Bg = Krog/(1 - Sgg);
25
26 - |Eq = Soo*Bw*Bg - Kro;
27 - |SS = solve(Eq, So);
28 - |Answer(i) = double(SS(1));
29 - |% SS(1)
30 - |% if SS > 0.0 && SS <= 1 - Sw(i)
31 - |%
32 - |%
33 - |% end
34
35 - |end
    
```

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JCPT73-04-06

RESERVOIR SIMULATION

Estimation of Three-Phase Relative Permeability and Residual Oil Data

H. L. STONE, Research Supervisor, Reservoir Simulation, Esso Production Research Company, Houston, Texas.

ABSTRACT

The probability model presented in an earlier paper has been revised to yield improved estimates of three-phase relative permeabilities. Both the old and the new models use two sets of two-phase data — water-oil and gas-oil — as a basis for estimating the more difficult-to-measure three-phase data. The relative permeabilities predicted by the new model compare favorably with the experimental data available in the literature, including data on the dependence of water/oil residual oil saturations on trapped gas saturations.

INTRODUCTION

A PROBABILITY MODEL for the estimation of three-phase relative permeability data, which was described in an earlier paper⁽¹⁾ has been revised and improved. The revised model uses two sets of two-phase data — water displacing oil and gas displacing oil — as the basis for making such estimates. Hysteresis is taken into consideration by employing the appropriate two-phase data, as discussed in the section titled "Data Required."

Relative permeabilities predicted by this model are in better agreement with experimental data than those given from the earlier model, especially in the region of low oil saturations. In fact, the new approach is capable of providing estimates of residual oil data in the three-phase region which are needed, for example, to determine the effect on oil recovery of an initial free gas saturation during a waterflood. In the previous model, these residual oil data had to be provided as input to supplement the two sets of two-phase data.

Both the old and the new model have a desirable property in that they yield the correct two-phase data when only two phases are flowing and yet provide consistent and continuous functions of the phase saturations. As will be shown later, these interpolated values agree with the available three-phase data within experimental uncertainty.

The method can be used to estimate the relative permeability to oil in a preferentially water-wet system in which water and gas relative permeabilities depend on only the water and gas saturations, respectively. Its applicability to preferentially oil-wet and mixed-wetability systems is a matter of conjecture, since the data used to evaluate it were measured on cores or sand packs which the experimenters attempted to keep fully waterwet. However, the probabilistic basis of the model suggests that it should be useful for estimating the relative permeability of one phase if the relative permeability of the other two phases is a function only of their respective phase relative permeability. Thus, in a fully oil-wet system where oil and gas relative permeability is a function of gas saturation, one would expect the probability model to be effective in predicting the water relative permeability. For systems in which mixed wettability is caused by a fraction of the pore spaces being strongly oil-wettable, but not proven, hypothesis is that in one saturation range equations applicable to a water-wet system can be used, whereas elsewhere the oil-wet equations will apply. Appendix B is devoted to a discussion of this hypothesis.

In the first section of this paper, the data necessary to use the model will be discussed. This is followed by presentation of the equations which comprise the model, an outline of the physical basis of the model, and finally a comparison of predicted and experimental relative permeabilities for several systems reported in the literature. Included in this latter section are experimental data of Høegsberg and Kjørne on residual oil saturation by waterflooding in the presence of an initial free gas saturation. For simplicity, the discussion of data required, equations, etc., will be that which applies to predicting the oil-phase relative permeability in a preferentially water-wet system.

DATA REQUIRED

Data required for the estimation of the oil-phase relative permeability are two sets of two-phase data — water-oil and gas-oil. To allow for hysteresis, the water and gas saturations should be changing in the same direction in the two sets of two-phase data, as desired in the three-phase system. For example, if both the water and gas saturations are expected to increase in the three-phase system, then drainage gas-oil data would be used and imbibition water-oil data. However, if oil is displacing gas, followed by water, then imbibition gas-oil data, followed by drainage water-oil data, would be used for both the gas-oil and water-oil systems. In this latter case, the particular gas relative permeability curve used would probably depend on the maximum gas saturation present prior to waterflooding.

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Technology, October-December, 1973, Montreal

اصلاح و بهبود رابطه قبلی خود

Stone (1973)

:Stone 2

- ❖ رابطه قبلی اصلاح و بهبود یافت.
- ❖ نتایج مدل جدید تطابق بهتری با داده های آزمایشگاهی دارد به خصوص در نواحی اشباع کم نفت.
- ❖ مدل جدید قادر به تخمین نفت ماندگار در ناحیه سه فاز مورد نیاز برای استفاده هست، در حالیکه در مدل قبلی این داده باید به مدل وارد می شد.
- ❖ هر دو مدل قبلی و جدید از دو دسته داده های نفوذپذیری نسبی برای تخمین استفاده می کنند.
- ❖ مدل می تواند برای نفوذپذیری نسبی نفت در یک سیستم ترجیحاً آب-دوست که نفوذپذیری نسبی آب و گاز به ترتیب فقط به اشباع آب و گاز بستگی دارد استفاده شود.
- ❖ در محیط نفت-دوست با ترشوندگی بینابینی هم می توان از این روش استفاده کرد.
- ❖ اساس مدل احتمال پیشنهاد می کند که این روش برای تخمین نفوذپذیری نسبی یک فاز در صورتیکه نفوذپذیری نسبی دو فاز دیگر فقط به اشباع خودشان وابسته باشند مفید است.

$$K_{ro} + K_{rg} + K_{rw} = \sigma_w \sigma_g$$

$\sigma_w = ?$
 $\left\{ \begin{array}{l} K_{rg} = 0 \\ \sigma_g = 1 \end{array} \right.$
 $\xrightarrow{K_{ro} + K_{rg} + K_{rw} = \sigma_w \sigma_g}$
 $K_{row} + K_{rw} = \sigma_w$

$\sigma_g = ?$
 $\left\{ \begin{array}{l} K_{rw} = 0 \\ \sigma_w = 1 \end{array} \right.$
 $\xrightarrow{K_{ro} + K_{rg} + K_{rw} = \sigma_w \sigma_g}$
 $K_{rog} + K_{rg} = \sigma_g$

$$K_{ro} = (K_{row} + K_{rw})(K_{rog} + K_{rg}) - (K_{rw} + K_{rg})$$

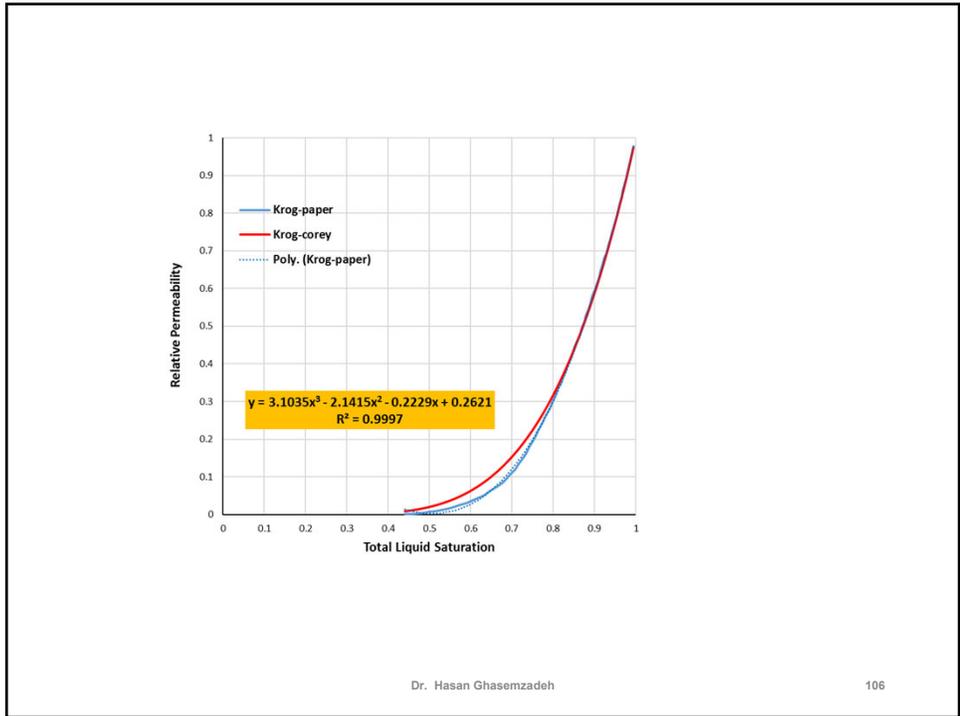
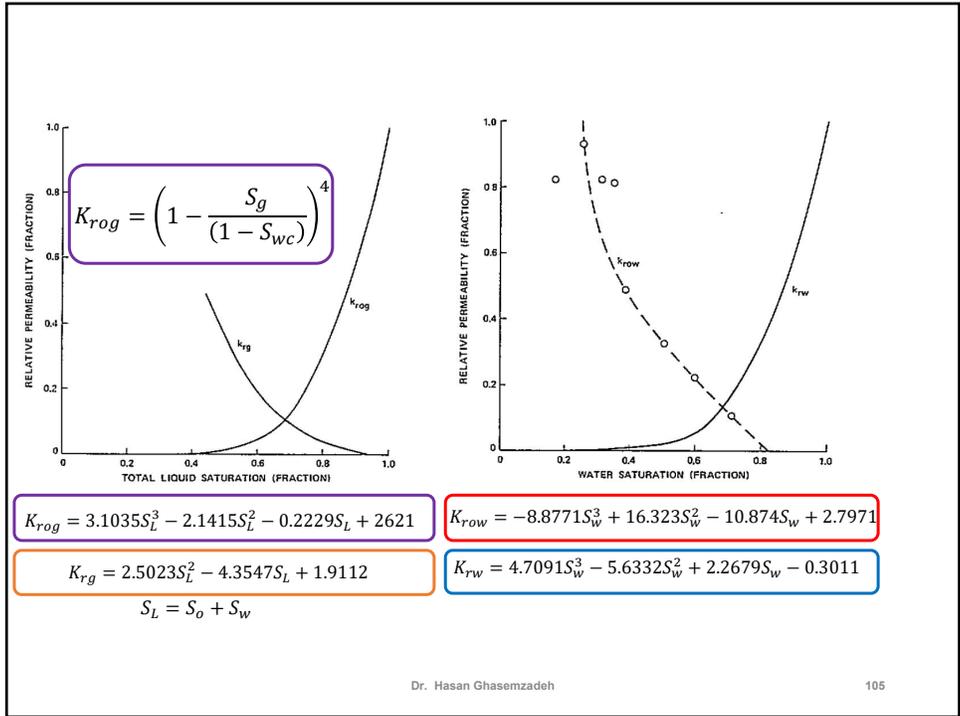
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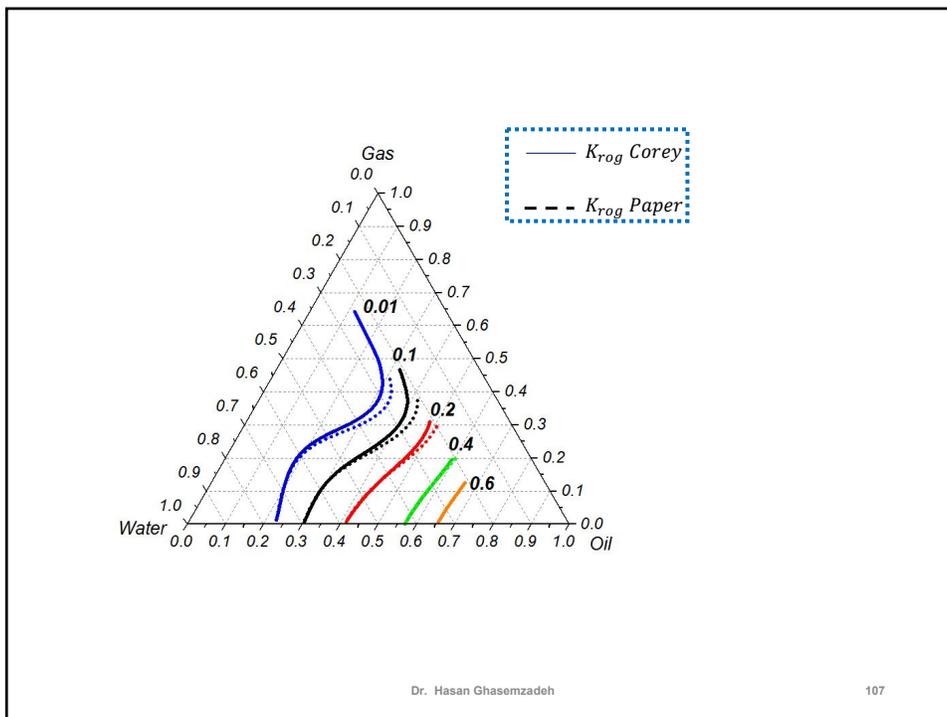
داده های مورد نیاز:

$\diamond K_{rog}$
 $\diamond K_{rg}$

$\diamond K_{row}$
 $\diamond K_{rw}$

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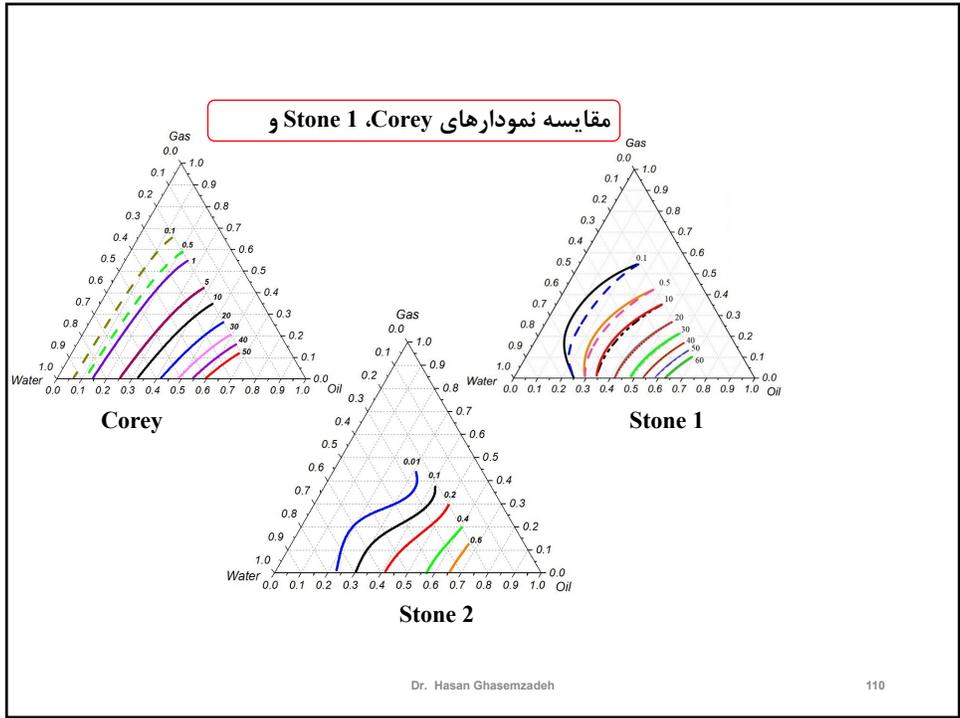
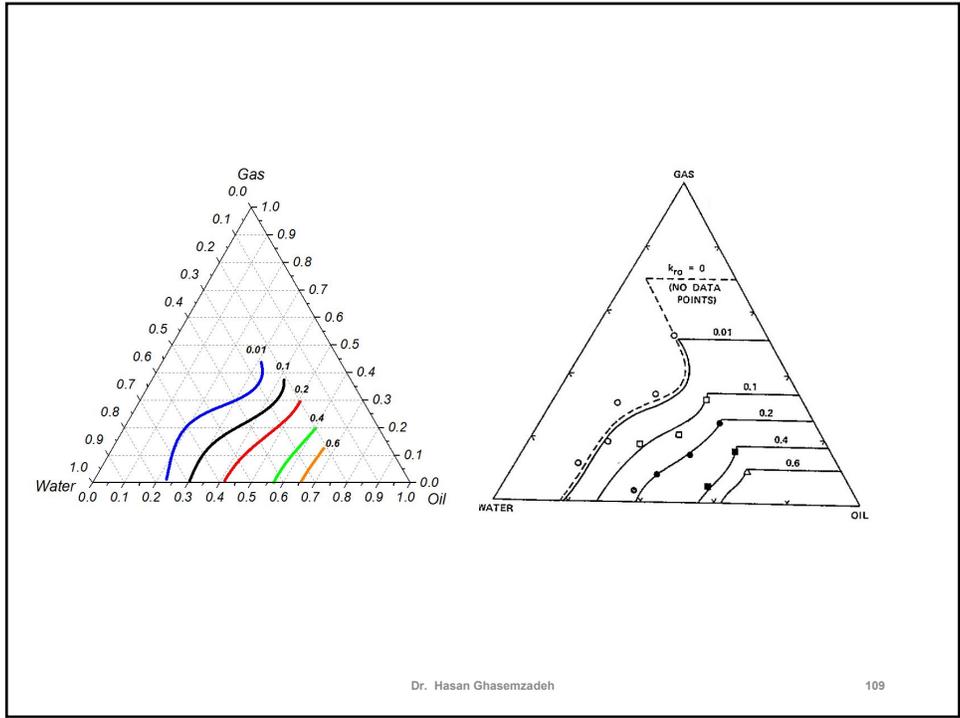


```

Stone2.m x
1 -   clc
2 -   clear all
3
4 -   syms So
5
6 -   Kro = 0.4;
7 -   Sw = 0.2:0.01:1.0;
8 -   Sw = Sw';
9 -   Answer = zeros(numel(Sw), 1);
10
11 -   for i = 1:numel(Sw)
12 -       SL = Sw(i)+So
13 -       Krw = (4.7091*(Sw(i)^3)-(5.6332*(Sw(i)^2))+2.2679*Sw(i))-0.3011
14 -       Krg = (2.5023*(SL^2)-(4.354*SL)+1.9112
15 -       Krow = (-8.8771*(Sw(i)^3)+(16.323*(Sw(i)^2)-(10.874*Sw(i))+2.7971;
16 -       Krog = (3.1035*(SL^3)-(2.1415*(SL^2)-(0.2229*SL)+0.2621;
17
18 -       Eq = ((Krow + Krw)*(Krog + Krg)-(Krw + Krg)) - Kro;
19 -       SS = solve(Eq, So);
20 -       Answer(i) = double(SS(1));
21 -       % SS(1)
22 -       % if SS > 0.0 && SS <= 1 - Sw(i)
23 -       %
24 -       %
25 -       % end
26
27 -   end
    
```

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Gas Flow vs. Liquid Flow

- Gas value is appraised at standard conditions

- Standard Temperature and Pressure
- $q_{g,sc}$ [scf/day] is mass flow rate (for specified γ_g)

$$\text{Income}[\$] = \int_{t_1}^{t_2} \text{Price}[\$/\text{Mscf}] \cdot q_{g,sc}[\text{Mscf/day}] \, dt[\text{days}]$$

- For steady state flow conditions in the reservoir, as flow proceeds along the flow path:

- Mass flow rate, $q_{g,sc}$, is constant
- Pressure decreases
- Density decreases
- Volumetric flow rate, q_g , increases

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اشباع:

- اشباع بحرانی نفت S_{OC}
- اشباع نفت باقی مانده S_{OR}
- اشباع بحرانی آب S_{WC}
- اشباع آب همزاد S_{WC}
- اشباع بحرانی گاز S_{GC}

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سیستم های نفت - آب

$$k_{ro} = (k_{ro})_{Swc} \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{no}$$

$$k_{rw} = (k_{rw})_{Sorw} \left[\frac{S_w - S_{wc}}{1 - S_{wc} - S_{orw}} \right]^{nw}$$

$$p_{cwo} = (p_c)_{Swc} \left[\frac{1 - S_w - S_{orw}}{1 - S_{wc} - S_{orw}} \right]^{np}$$

سیستم های گاز - نفت

$$k_{rg} = (k_{rg})_{Sgc} \left[\frac{1 - S_g - S_{lc}}{1 - S_{gc} - S_{lc}} \right]^{ngo}$$

$$k_{rg} = (k_{rg})_{Swc} \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{ng}$$

$$p_{cgo} = (p_c)_{Sgc} \left[\frac{S_g - S_{gc}}{1 - S_{gc} - S_{lc}} \right]^{npg}$$

$$S_{lc} = S_{wc} + S_{org} \quad 113$$

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