STATICS

Forces in Beams and Cables

Introduction

- Preceding chapters dealt with:

  a) determining external forces acting on a structure and

  b) determining forces which hold together the various members of a structure.

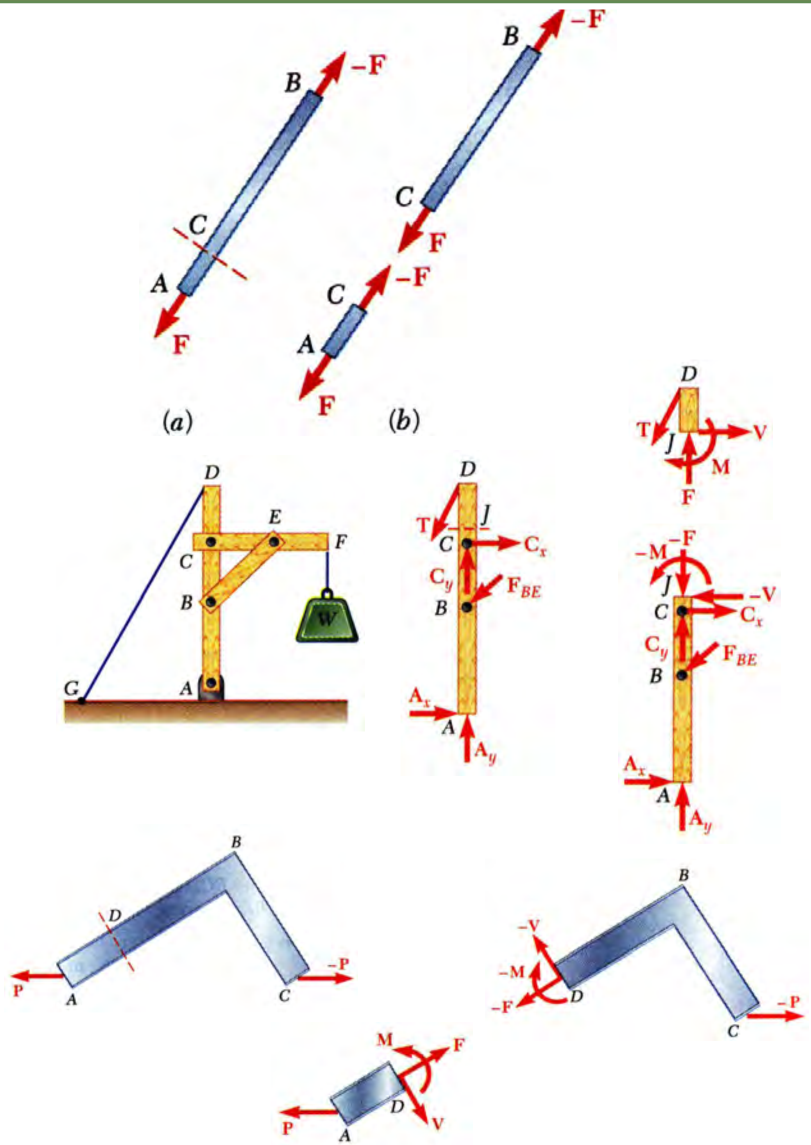
- The current chapter is concerned with determining the *internal forces* (i.e., tension/compression, shear, and bending) which hold together the various parts of a given member.

- Focus is on two important types of engineering structures:

  a) Beams - usually long, straight, prismatic members designed to support loads applied at various points along the member.

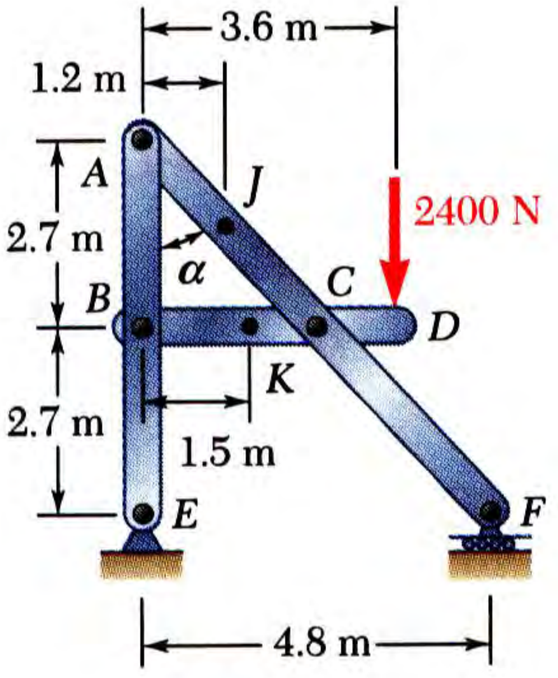
  b) Cables - flexible members capable of withstanding only tension, designed to support concentrated or distributed loads.

Internal Forces in Members



- Straight two-force member *AB* is in equilibrium under application of *F* and -*F*.
- *Internal forces* equivalent to *F* and -*F* are required for equilibrium of free-bodies *AC* and *CB*.
- Multiforce member *ABCD* is in equil-ibrium under application of cable and member contact forces.
- Internal forces equivalent to a force-couple system are necessary for equil-ibrium of free-bodies *JD* and *ABCJ*.
- An internal force-couple system is required for equilibrium of two-force members which are not straight.

Sample Problem 7.1

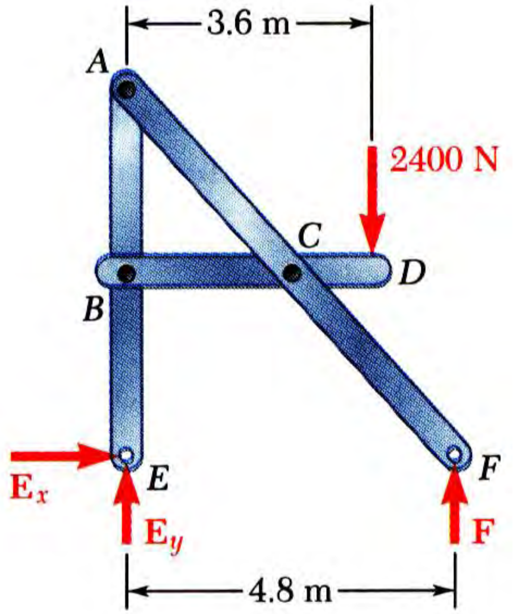


Determine the internal forces *(a)* in member *ACF* at point *J* and *(b)* in member *BCD* at *K*.

SOLUTION:

- Compute reactions and forces at connections for each member.
- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system which is determined by considering equilibrium of either part.
- Cut member *BCD* at *K*. Determine force-couple system equivalent to internal forces at *K* by applying equilibrium conditions to either part.

Sample Problem 7.1



SOLUTION:

- Compute reactions and connection forces.

Consider entire frame as a free-body:

$$\sum M_E = 0:$$

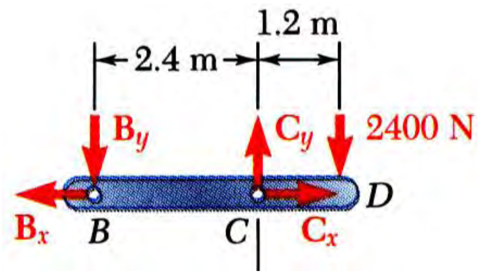
$$-(2400\,\text{N})(3.6\,\text{m}) + F(4.8\,\text{m}) = 0 \qquad F = 1800\,\text{N}$$

$$\sum F_y = 0:$$

$$-2400\,\text{N} + 1800\,\text{N} + E_y = 0 \qquad E_y = 600\,N$$

$$\sum F_x = 0: \qquad E_x = 0$$

Sample Problem 7.1



Consider member *BCD* as free-body:

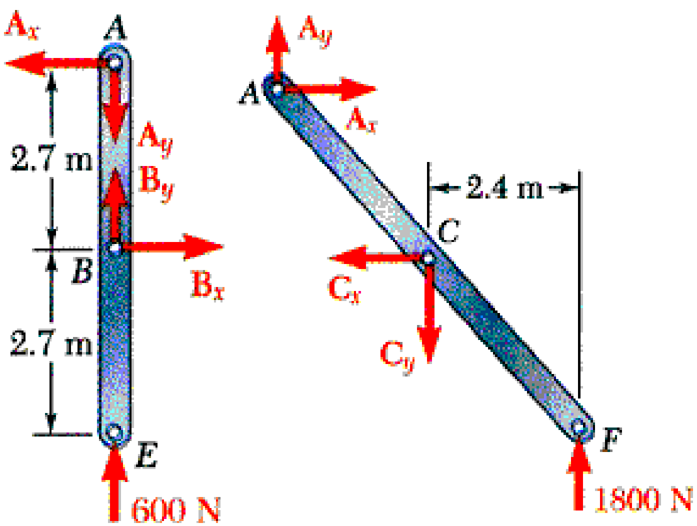
$\sum M_B = 0:$

$-(2400\text{ N})(3.6\text{ m}) + C_y(2.4\text{ m}) = 0 \qquad C_y = 3600\text{ N}$

$\sum M_C = 0:$

$-(2400\text{ N})(1.2\text{ m}) + B_y(2.4\text{ m}) = 0 \qquad B_y = 1200\text{ N}$

$\sum F_x = 0: \quad -B_x + C_x = 0$



Consider member *ABE* as free-body:

$\sum M_A = 0: \quad B_x(2.4\text{ m}) = 0 \qquad B_x = 0$

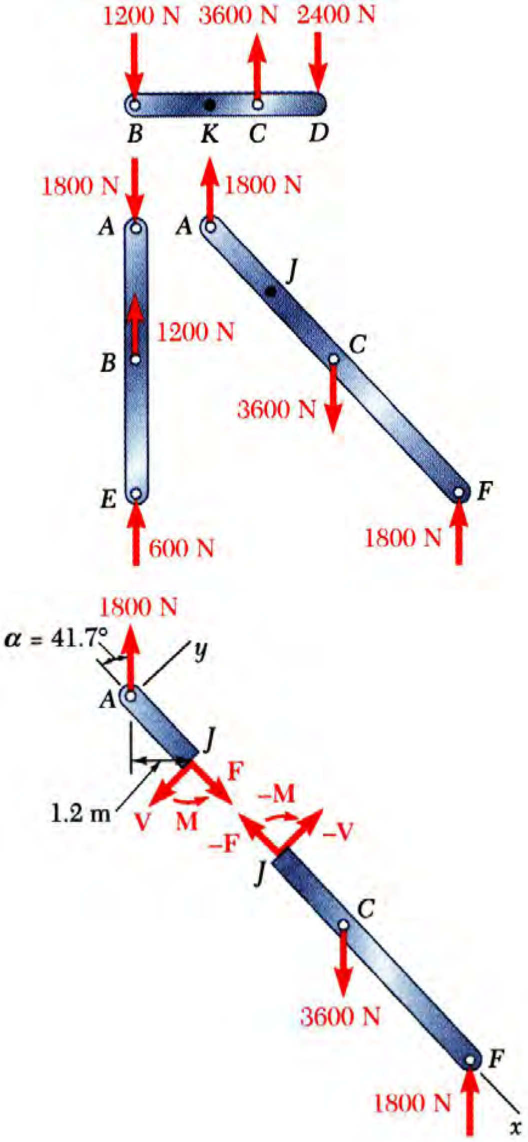
$\sum F_x = 0: \quad B_x - A_x = 0 \qquad A_x = 0$

$\sum F_y = 0: \quad -A_y + B_y + 600\text{ N} = 0 \qquad A_y = 1800\text{ N}$

From member *BCD*,

$\sum F_x = 0: \quad -B_x + C_x = 0 \qquad C_x = 0$

Sample Problem 7.1



- Cut member *ACF* at *J*. The internal forces at *J* are represented by equivalent force-couple system.

Consider free-body *AJ*:

$$\sum M_J = 0:$$

$$-(1800\text{ N})(1.2\text{ m}) + M = 0 \qquad \boxed{M = 2160\text{ N}\cdot\text{m}}$$

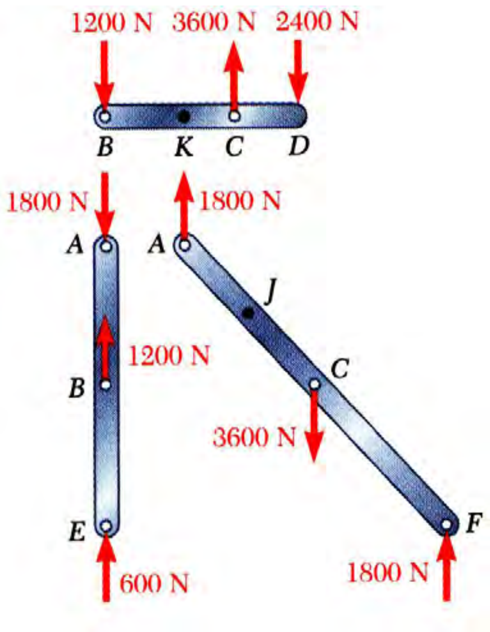
$$\sum F_x = 0:$$

$$F - (1800\text{ N})\cos 41.7° = 0 \qquad \boxed{F = 1344\text{ N}}$$

$$\sum F_y = 0:$$

$$-V + (1800\text{ N})\sin 41.7° = 0 \qquad \boxed{V = 1197\text{ N}}$$

Sample Problem 7.1



- Cut member *BCD* at *K*. Determine a force-couple system equivalent to internal forces at *K* .

Consider free-body *BK*:

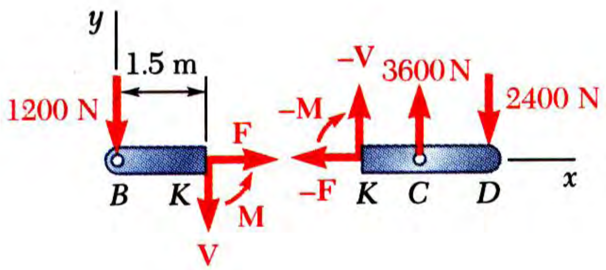
$$\sum M_K = 0:$$

$$(1200\,\mathrm{N})(1.5\,\mathrm{m}) + M = 0 \qquad \boxed{M = -1800\,\mathrm{N}\cdot\mathrm{m}}$$

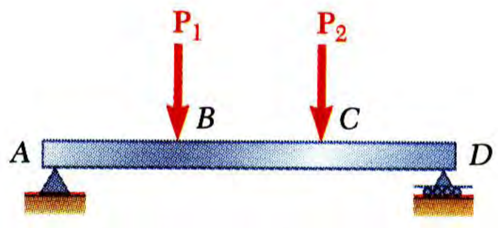
$$\sum F_x = 0: \qquad \boxed{F = 0}$$

$$\sum F_y = 0:$$

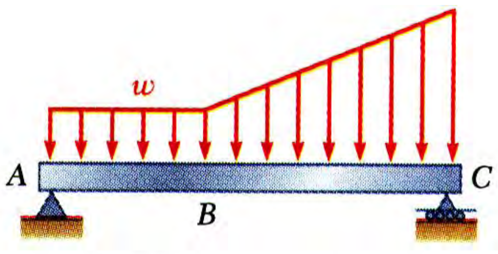
$$-1200\,\mathrm{N} - V = 0 \qquad \boxed{V = -1200\,\mathrm{N}}$$



Various Types of Beam Loading and Support



(a) Concentrated loads



(b) Distributed load

- *Beam* - structural member designed to support loads applied at various points along its length.

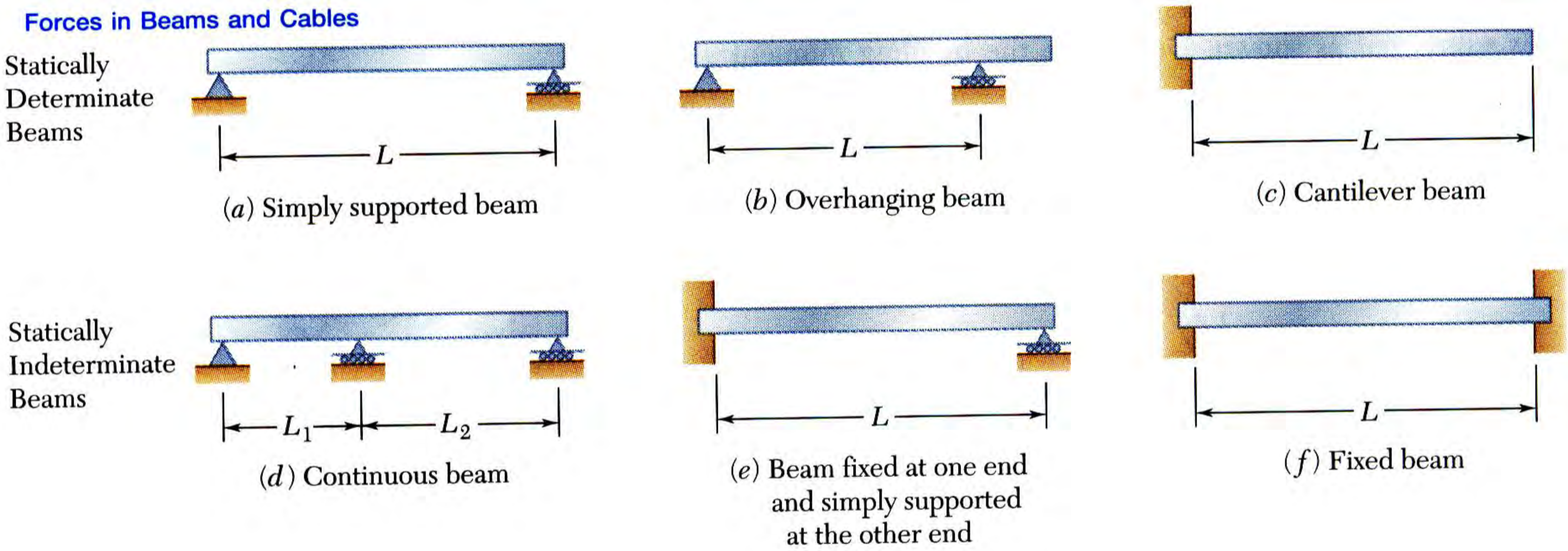
- Beam can be subjected to *concentrated* loads or *distributed* loads or combination of both.

- *Beam design* is two-step process:

    1) determine shearing forces and bending moments produced by applied loads

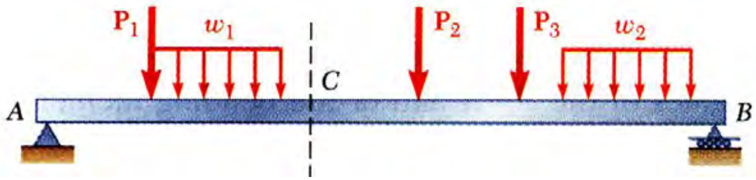
    2) select cross-section best suited to resist shearing forces and bending moments

Various Types of Beam Loading and Support

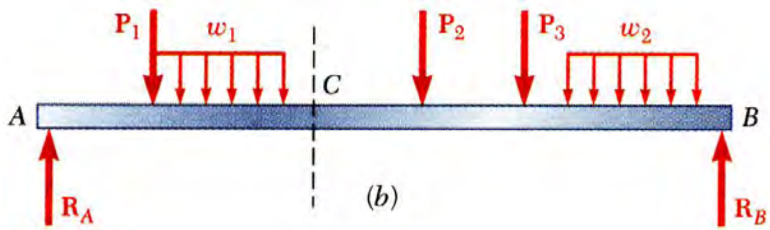


- Beams are classified according to way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.

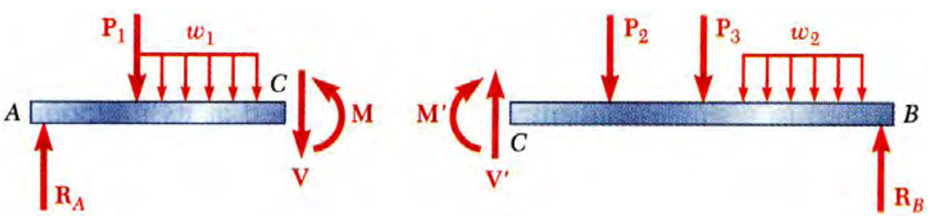
Shear and Bending Moment in a Beam



- Wish to determine bending moment and shearing force at any point in a beam subjected to concentrated and distributed loads.



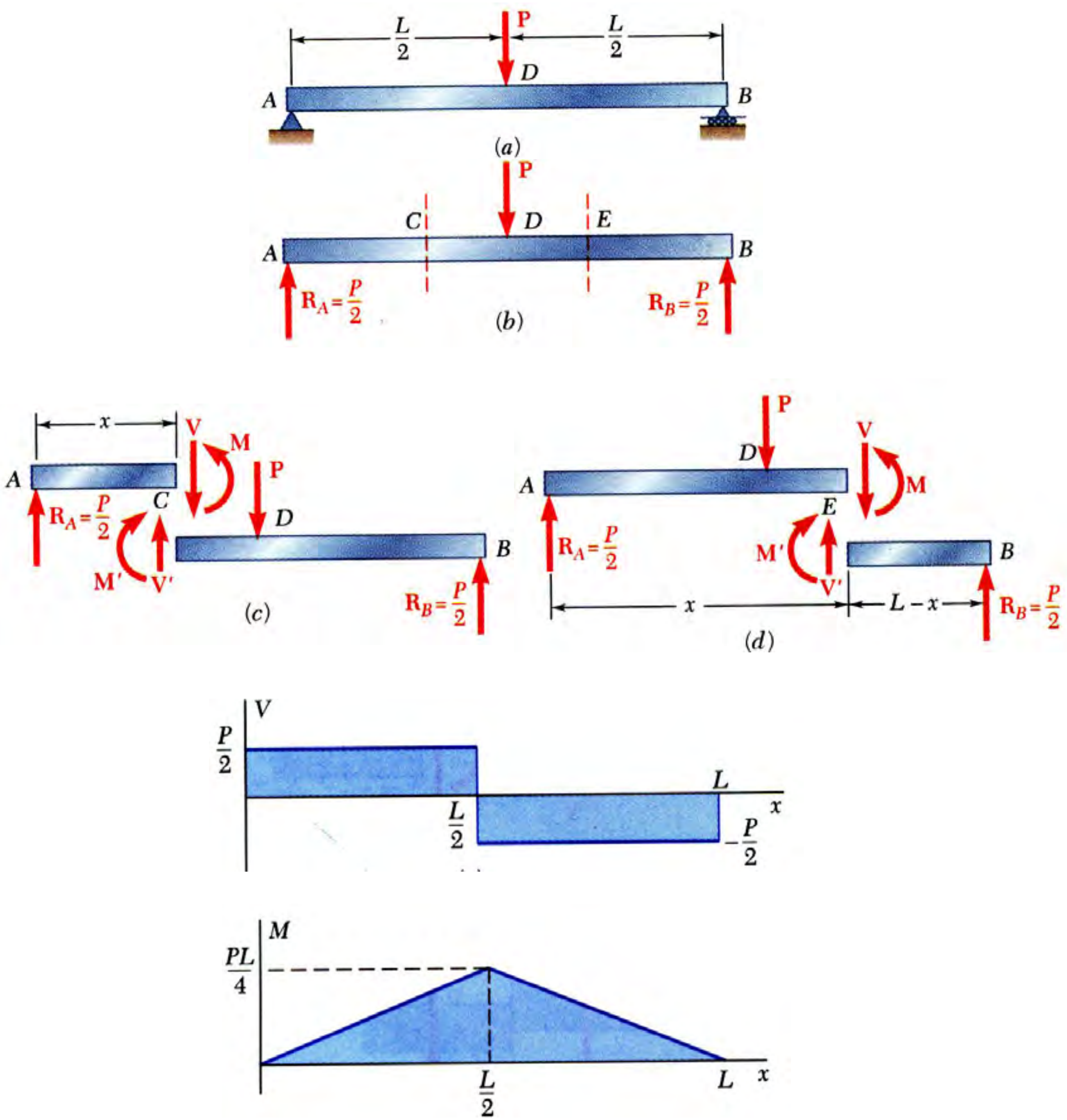
- Determine reactions at supports by treating whole beam as free-body.



- Cut beam at *C* and draw free-body diagrams for *AC* and *CB*. By definition, positive sense for internal force-couple systems are as shown.

- From equilibrium considerations, determine *M* and *V* or *M'* and *V'*.

Shear and Bending Moment Diagrams



- Variation of shear and bending moment along beam may be plotted.
- Determine reactions at supports.
- Cut beam at *C* and consider member *AC*,

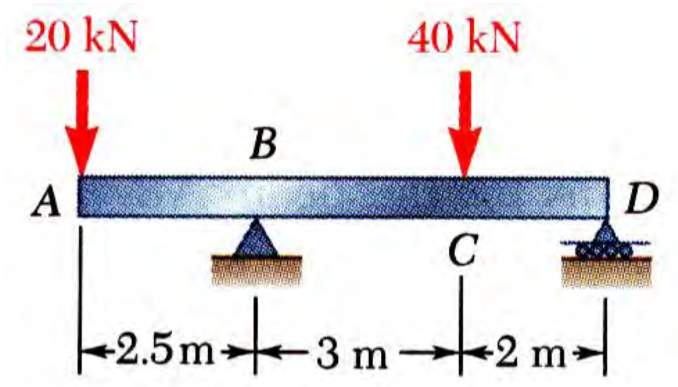
  $V = +P/2 \quad M = +Px/2$

- Cut beam at *E* and consider member *EB*,

  $V = -P/2 \quad M = +P(L-x)/2$

- For a beam subjected to <u>concentrated loads</u>, shear is constant between loading points and moment varies linearly.

Sample Problem 7.2

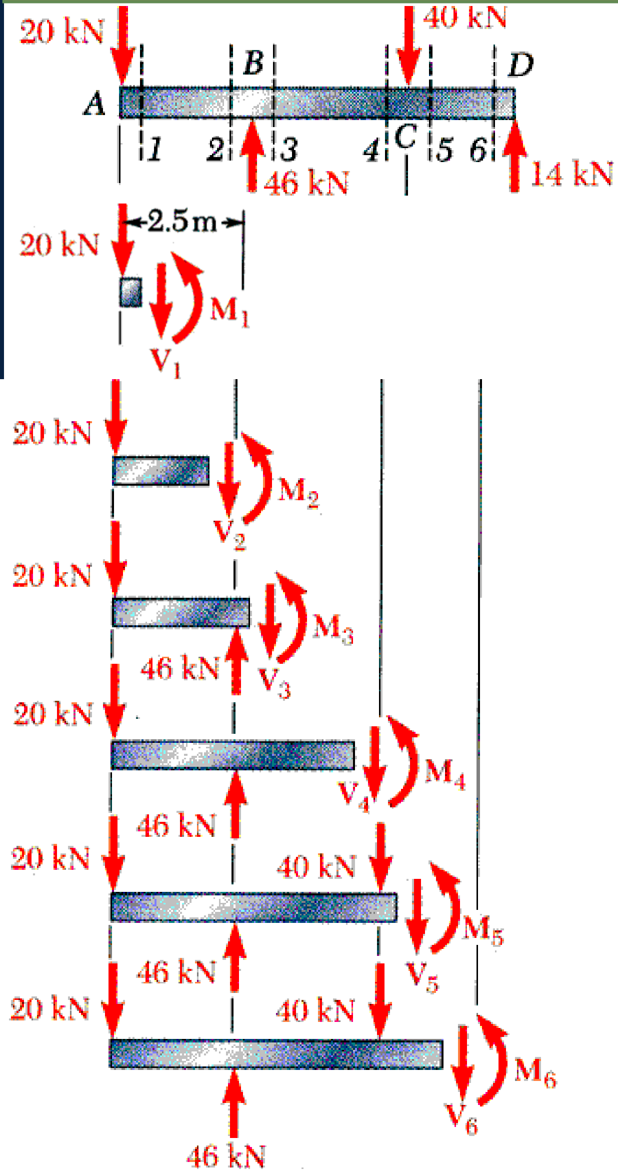


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results.

Sample Problem 7.2



SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems at sections on either side of load application points.

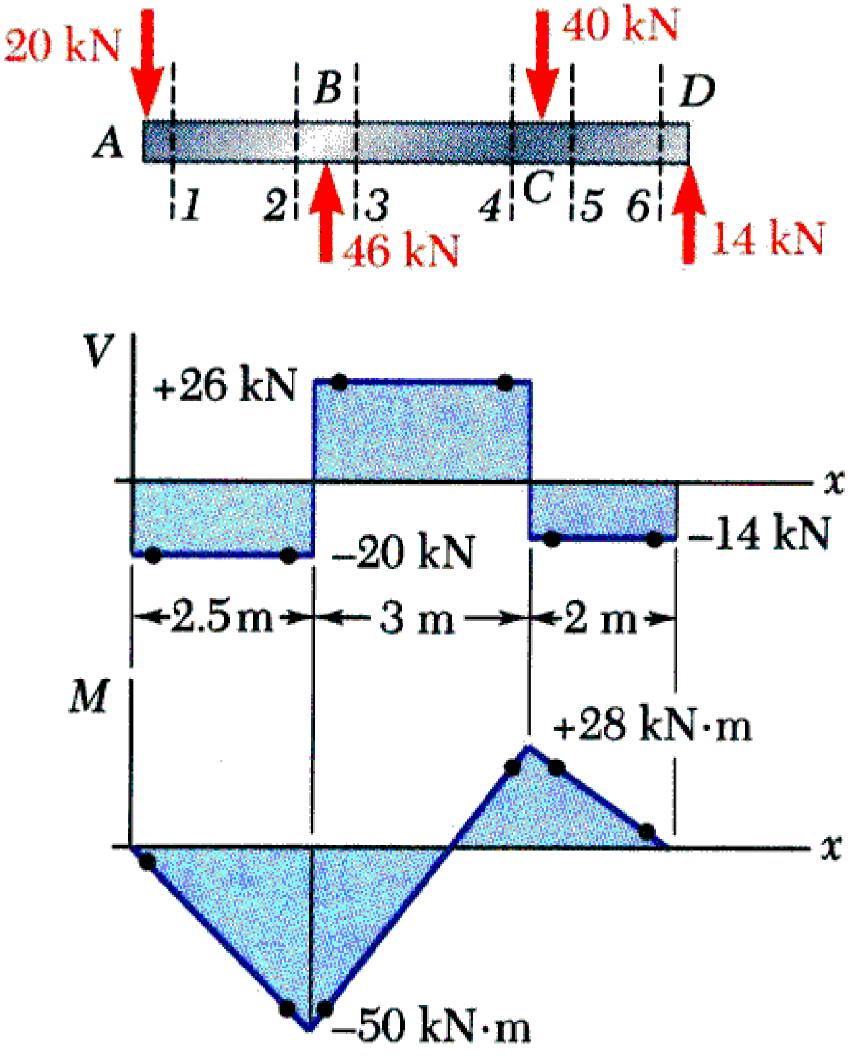
$$\sum F_y = 0: \quad -20\,\text{kN} - V_1 = 0 \qquad \boxed{V_1 = -20\,\text{kN}}$$

$$\sum M_2 = 0: \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \qquad \boxed{M_1 = 0}$$

Similarly,

$$\boxed{\begin{array}{ll} V_3 = 26\,\text{kN} & M_3 = -50\,\text{kN}\cdot\text{m} \\ V_4 = 26\,\text{kN} & M_4 = -50\,\text{kN}\cdot\text{m} \\ V_5 = 26\,\text{kN} & M_5 = -50\,\text{kN}\cdot\text{m} \\ V_6 = 26\,\text{kN} & M_6 = -50\,\text{kN}\cdot\text{m} \end{array}}$$

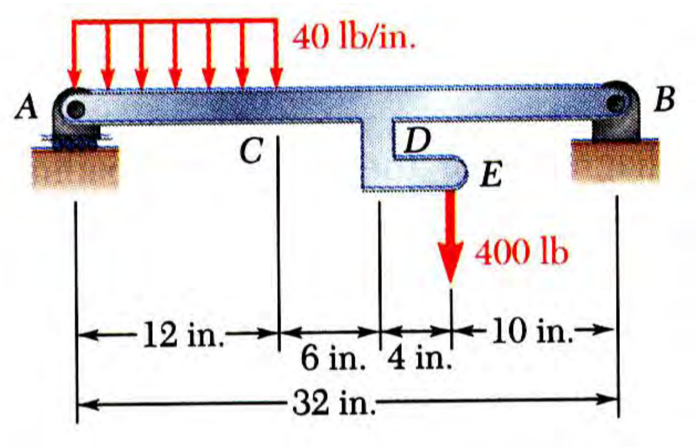
Sample Problem 7.2



- Plot results.

  Note that shear is of constant value between concentrated loads and bending moment varies linearly.

Sample Problem 7.3

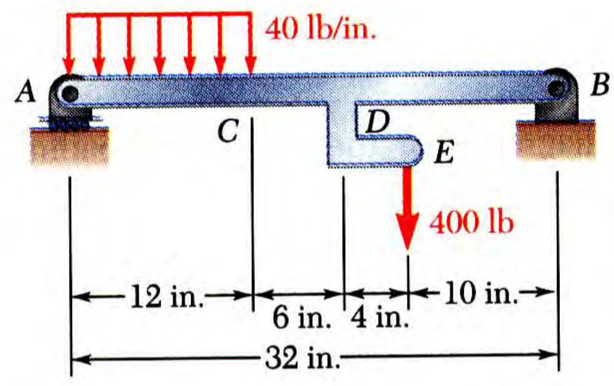


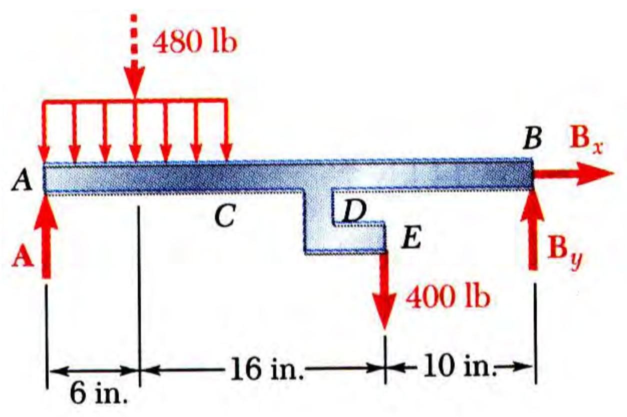
Draw the shear and bending moment diagrams for the beam *AB*. The distributed load of 40 lb/in. extends over 12 in. of the beam, from *A* to *C*, and the 400 lb load is applied at *E*.

SOLUTION:

- Taking entire beam as free-body, calculate reactions at *A* and *B*.
- Determine equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.
- Plot results.

Sample Problem 7.3





SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *A* and *B*.

$$\sum M_A = 0:$$

$$B_y(32\,\text{in.}) - (480\,\text{lb})(6\,\text{in.}) - (400\,\text{lb})(22\,\text{in.}) = 0$$

$$\boxed{B_y = 365\,\text{lb}}$$

$$\sum M_B = 0:$$

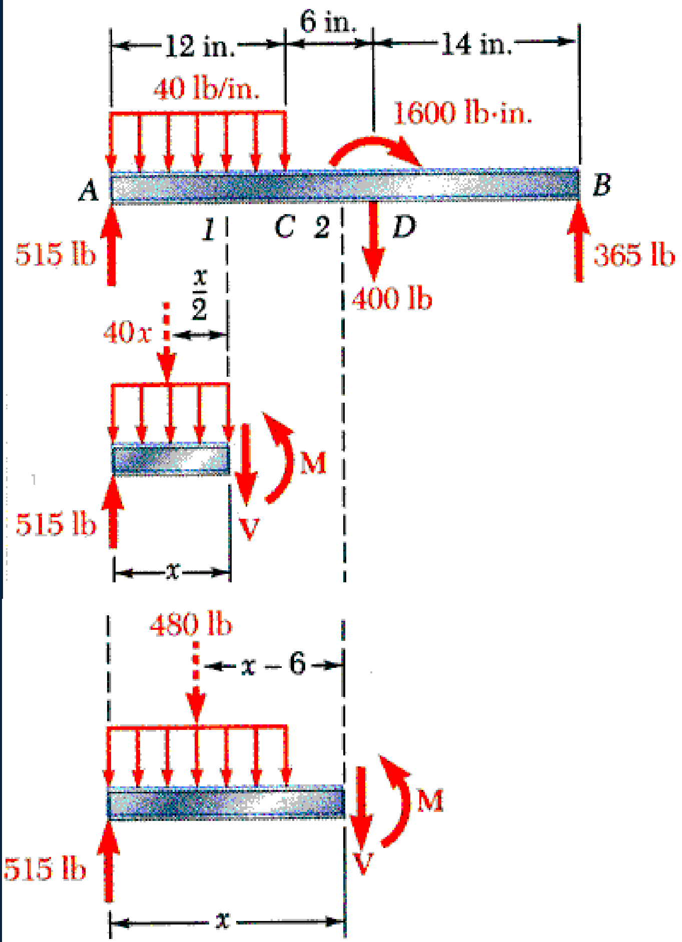
$$(480\,\text{lb})(26\,\text{in.}) + (400\,\text{lb})(10\,\text{in.}) - A(32\,\text{in.}) = 0$$

$$\boxed{A = 515\,\text{lb}}$$

$$\sum F_x = 0: \qquad \boxed{B_x = 0}$$

- Note: The 400 lb load at *E* may be replaced by a 400 lb force and 1600 lb-in. couple at *D*.

Sample Problem 7.3



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From A to *C*:

$$\sum F_y = 0: \qquad 515 - 40x - V = 0$$

$$\boxed{V = 515 - 40x}$$

$$\sum M_1 = 0: \qquad -515x - 40x\left(\tfrac{1}{2}x\right) + M = 0$$

$$\boxed{M = 515x - 20x^2}$$

From *C* to *D*:

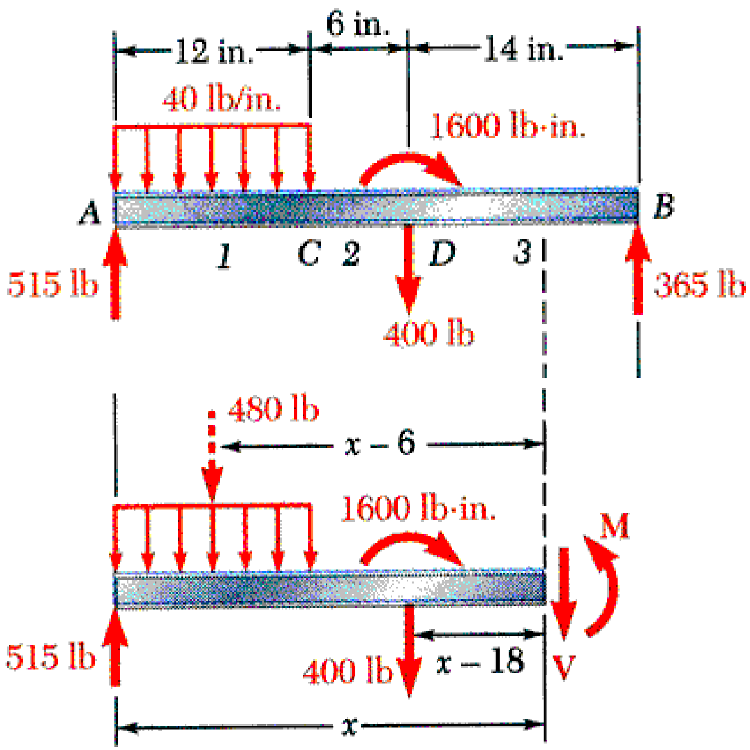
$$\sum F_y = 0: \qquad 515 - 480 - V = 0$$

$$\boxed{V = 35\,\text{lb}}$$

$$\sum M_2 = 0: \qquad -515x + 480(x - 6) + M = 0$$

$$\boxed{M = (2880 + 35x)\,\text{lb} \cdot \text{in.}}$$

Sample Problem 7.3



- Evaluate equivalent internal force-couple systems at sections cut within segments *AC*, *CD*, and *DB*.

From *D* to *B*:

$$\sum F_y = 0: \qquad 515 - 480 - 400 - V = 0$$

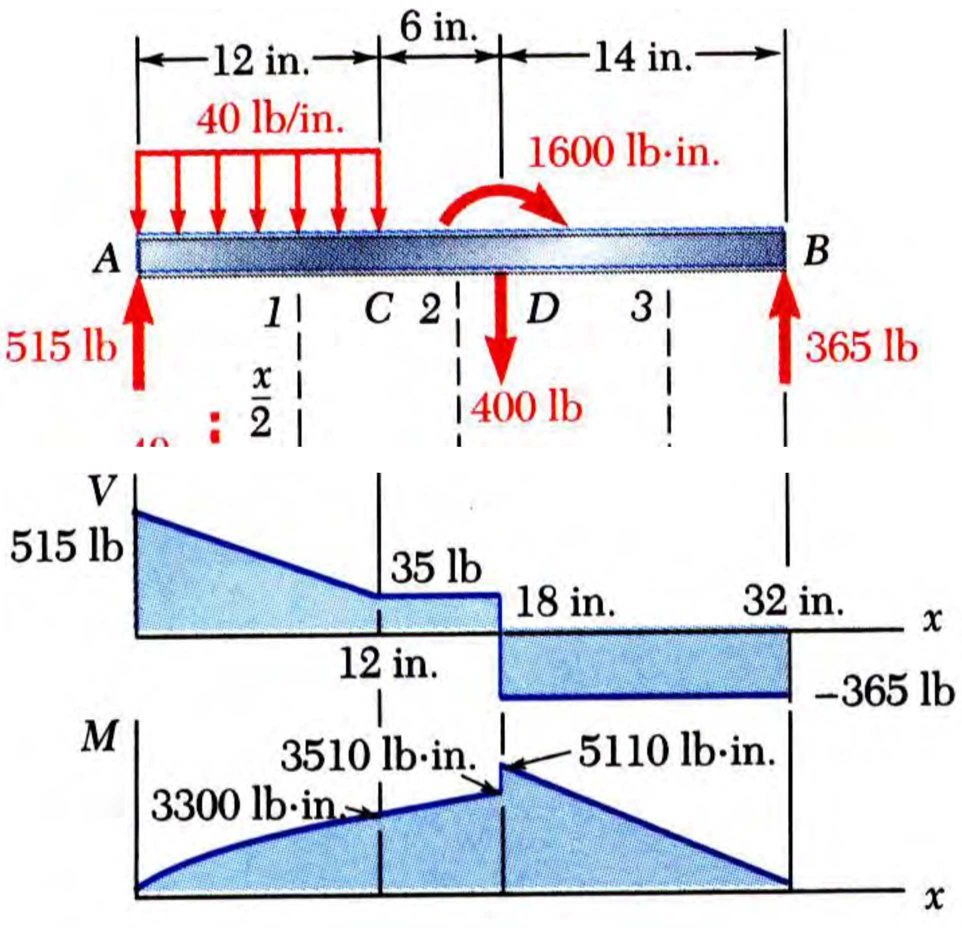
$$\boxed{V = -365\,\text{lb}}$$

$$\sum M_2 = 0:$$

$$-515x + 480(x-6) - 1600 + 400(x-18) + M = 0$$

$$\boxed{M = (11{,}680 - 365x)\,\text{lb}\cdot\text{in.}}$$

Sample Problem 7.3



- Plot results.

From A to C:

$$V = 515 - 40x$$

$$M = 515x - 20x^2$$

From C to D:

$$V = 35\ \text{lb}$$

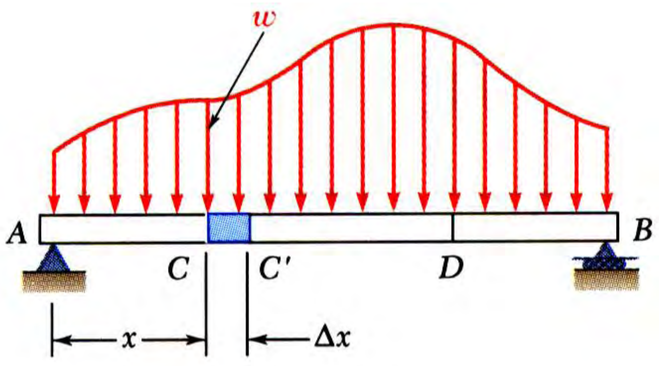
$$M = (2880 + 35x)\ \text{lb} \cdot \text{in.}$$

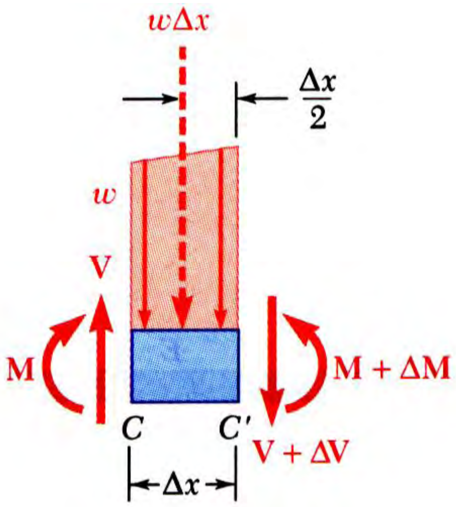
From D to B:

$$V = -365\ \text{lb}$$

$$M = (11{,}680 - 365x)\ \text{lb} \cdot \text{in.}$$

Relations Among Load, Shear, and Bending Moment





- Relations between load and shear:

$$V-(V+\Delta V)-w\Delta x=0$$

$$\frac{dV}{dx}=\lim_{\Delta x\to 0}\frac{\Delta V}{\Delta x}=-w$$

$$V_D-V_C=-\int_{x_C}^{x_D} w\,dx=-(\text{area under load curve})$$

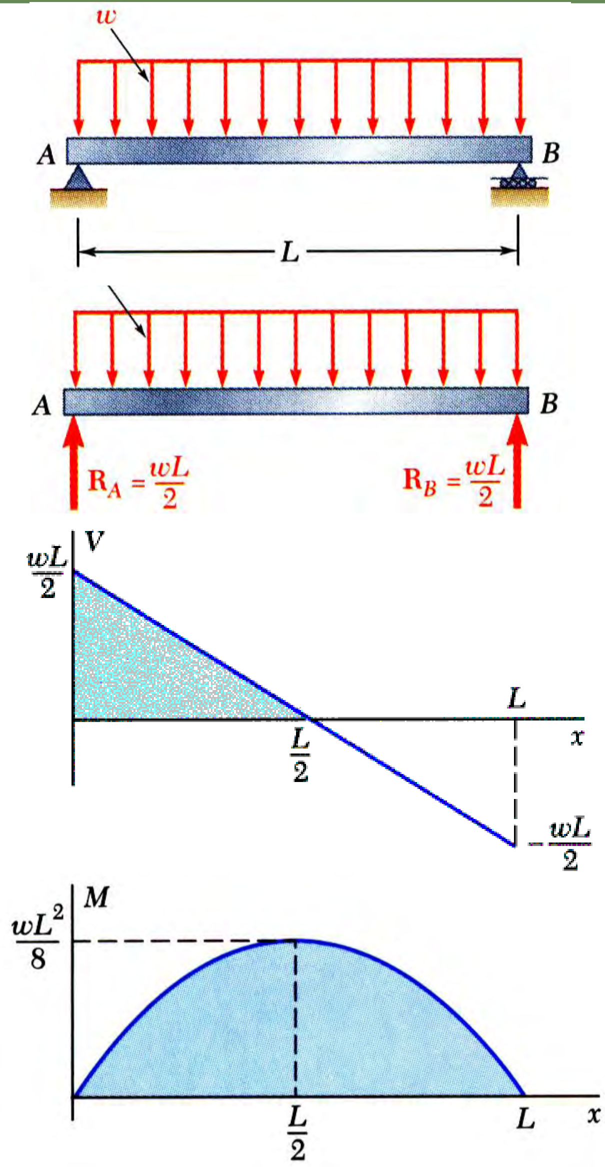
- Relations between shear and bending moment:

$$(M+\Delta M)-M-V\Delta x+w\Delta x\frac{\Delta x}{2}=0$$

$$\frac{dM}{dx}=\lim_{\Delta x\to 0}\frac{\Delta M}{\Delta x}=\lim_{\Delta x\to 0}\left(V-\tfrac{1}{2}w\Delta x\right)=V$$

$$M_D-M_C=\int_{x_C}^{x_D} V\,dx=(\text{area under shear curve})$$

Relations Among Load, Shear, and Bending Moment



- Reactions at supports, $R_A = R_B = \frac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

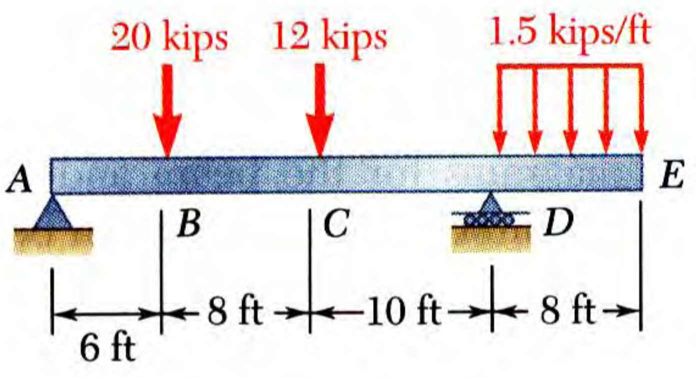
- Moment curve,

$$M - M_A = \int_0^x V dx$$

$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}\left(Lx - x^2\right)$$

$$M_{\max} = \frac{wL^2}{8} \quad \left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Sample Problem 7.4

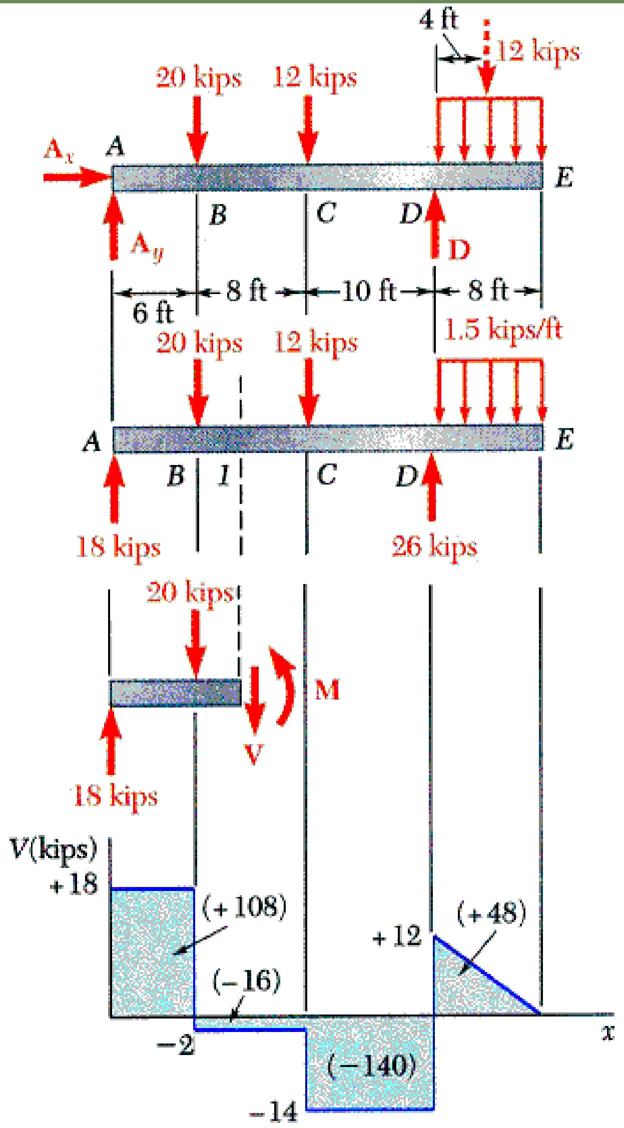


Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.
- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.
- With uniform loading between *D* and *E*, the shear variation is linear.
- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under shear curve between points.
- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

Sample Problem 7.4



SOLUTION:

- Taking entire beam as a free-body, determine reactions at supports.

$$\sum M_A = 0:$$

$$D(24\ \text{ft}) - (20\ \text{kips})(6\ \text{ft}) - (12\ \text{kips})(14\ \text{ft}) - (12\ \text{kips})(28\ \text{ft}) = 0$$

$$\boxed{D = 26\ \text{kips}}$$

$$\sum F_y = 0:$$

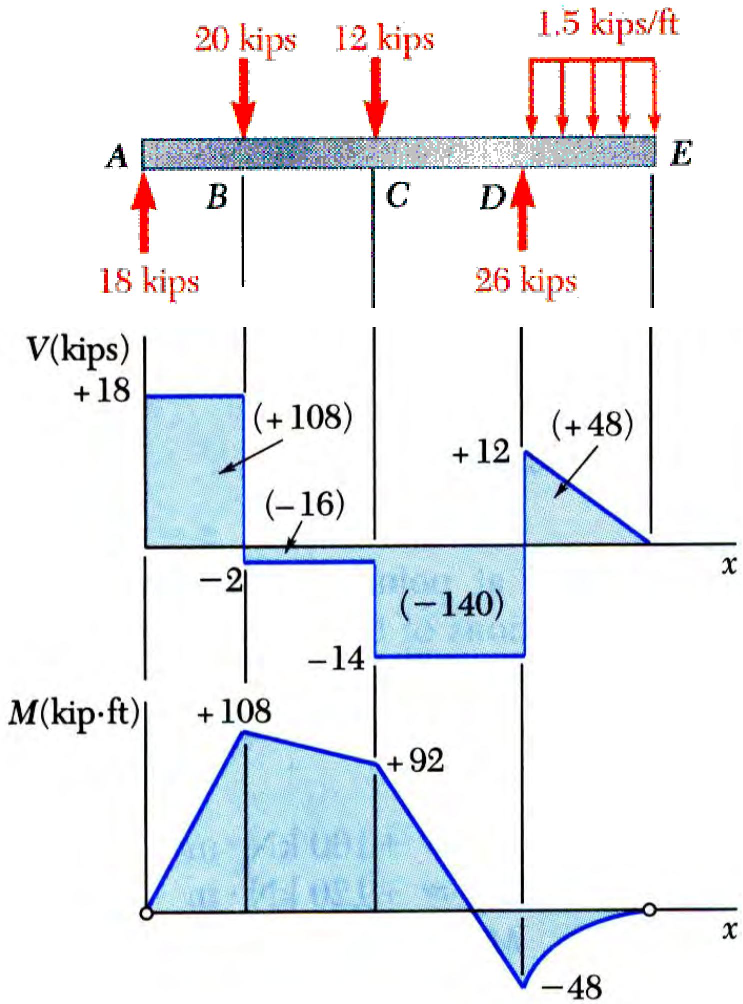
$$A_y - 20\ \text{kips} - 12\ \text{kips} + 26\ \text{kips} - 12\ \text{kips} = 0$$

$$\boxed{A_y = 18\ \text{kips}}$$

- Between concentrated load application points, $dV/dx = -w = 0$ and shear is constant.

- With uniform loading between *D* and *E*, the shear variation is linear.

Sample Problem 7.4

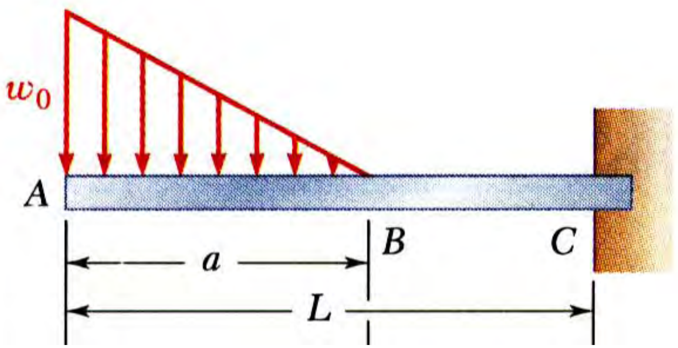


- Between concentrated load application points, $dM/dx = V = \text{constant}$. The change in moment between load application points is equal to area under the shear curve between points.

$$M_B - M_A = +108 \qquad M_B = +108\ \text{kip}\cdot\text{ft}$$
$$M_C - M_B = -16 \qquad M_C = +92\ \text{kip}\cdot\text{ft}$$
$$M_D - M_C = -140 \qquad M_D = -48\ \text{kip}\cdot\text{ft}$$
$$M_E - M_D = +48 \qquad M_E = 0$$

- With a linear shear variation between *D* and *E*, the bending moment diagram is a parabola.

Sample Problem 7.6

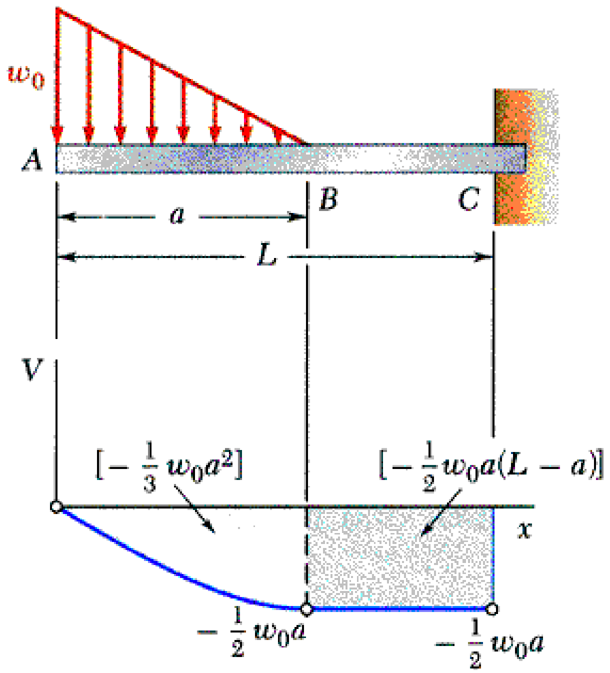


Sketch the shear and bending-moment diagrams for the cantilever beam and loading shown.

SOLUTION:

- The change in shear between *A* and *B* is equal to the negative of area under load curve between points. The linear load curve results in a parabolic shear curve.
- With zero load, change in shear between *B* and *C* is zero.
- The change in moment between *A* and *B* is equal to area under shear curve between points. The parabolic shear curve results in a cubic moment curve.
- The change in moment between *B* and *C* is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.

Sample Problem 7.6



SOLUTION:

- The change in shear between *A* and *B* is equal to negative of area under load curve between points. The linear load curve results in a parabolic shear curve.

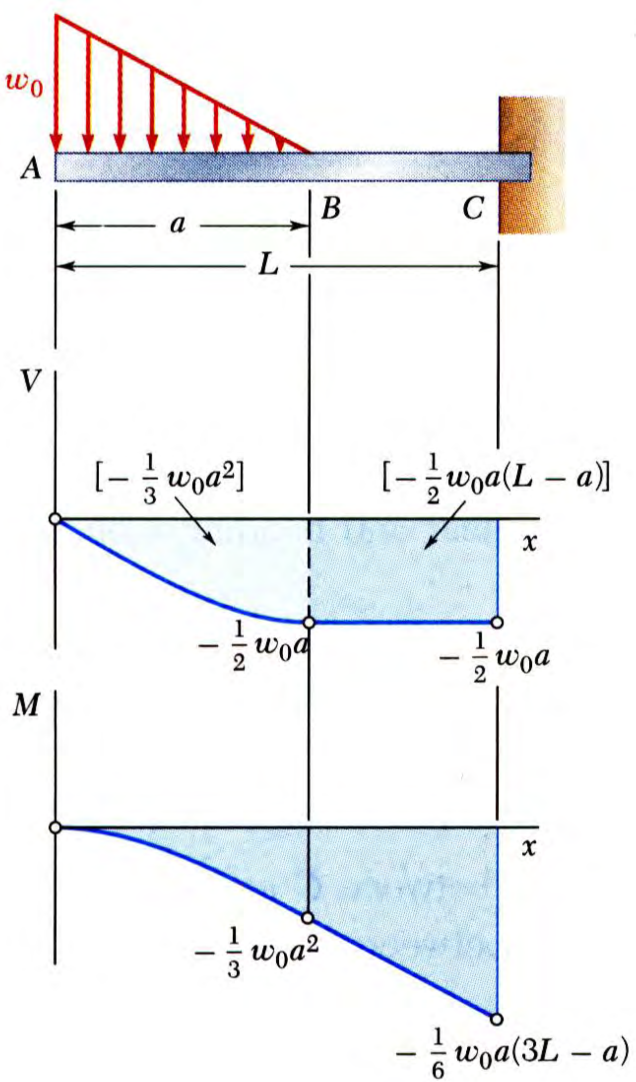
$$\text{at } A, \quad V_A = 0, \quad \frac{dV}{dx} = -w = -w_0$$

$$V_B - V_A = -\tfrac{1}{2}w_0 a \qquad\qquad V_B = -\tfrac{1}{2}w_0 a$$

$$\text{at } B, \quad \frac{dV}{dx} = -w = 0$$

- With zero load, change in shear between *B* and *C* is zero.

Sample Problem 7.6



- The change in moment between A and B is equal to area under shear curve between the points. The parabolic shear curve results in a cubic moment curve.

$$\text{at } A, \quad M_A = 0, \quad \frac{dM}{dx} = V = 0$$

$$M_B - M_A = -\tfrac{1}{3}w_0a^2 \qquad M_B = -\tfrac{1}{3}w_0a^2$$

$$M_C - M_B = -\tfrac{1}{2}w_0a(L-a) \qquad M_C = -\tfrac{1}{6}w_0a(3L-a)$$

- The change in moment between B and C is equal to area under shear curve between points. The constant shear curve results in a linear moment curve.