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PERTURBATION ANALYSIS OF HEAT TRANSFER AND A NOVEL METHOD FOR CHANGING THE THIRD KIND BOUNDARY CONDITION INTO THE FIRST KIND

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Heat transfer phenomena play a vital role in many problems, such as transport of flow through a porous medium. In this article, a singular perturbation method and Laplace transform are used to solve the one-dimensional heat transfer problem in semi-infinite porous media divided into inner and outer solutions. To prevent the mistakes of other researchers' analyses, a new approach for outer and inner matching boundary conditions is suggested. In addition, the boundary condition of the third kind (Robin boundary condition) at $y = 0$ *is changed into first kind by means of a novel structure. This approach shows a good accuracy with direct use of the first kind of boundary condition. However, when the slop value at* $y = 0$ *requires high accuracy of measurement, applying such an approach is not recommended. On the other hand, by applying the new matching idea, results of an asymptotic solution show good agreement with a numerical solution.*

KEY WORDS: *perturbation method, boundary layer, small thermal conductivity, Robin boundary condition*

1. INTRODUCTION

Packed beds are often used to store heat energy and also in chemical industry. These important applications explain the permanent interest in transport phenomena in packed beds for analytical and numerical investigations. Heat transfer analysis during the past two decades has been developed on numerical solutions. However, approximate analytical methods have continued to develop useful solutions to a variety of problems (Caldwell and Kwan, 2003). Nusselt (1911, 1930) obtained the analytical solution for the problem of heat transfer in plug flow heat exchangers when the axial heat conduction is neglected. The same solution was discovered by Anzelius (1926), Schumann (1929), and Furnas (1930) for the heating or cooling one-dimensional porous media by passing hot or cool fluid. Most analytical studies were concentrated on the Schumann model of a porous medium suggested in Furnas. This model ignores the thermal conduction terms in both fluid and solid phase energy equations. Kuznetsov (1996) presented an analytical solution for heating a rectangular sensible heat storage packed bed with a constant temperature at the walls by a nonthermal equilibrium flow of incompressible fluid.

Cheng (1977) investigated the mixed convection adjacent to inclined surfaces embedded in a porous medium using the boundary layer approximation. Similar solutions have been obtained for the situation where the free stream velocity and the surface temperature distribution vary according to the same power function of the distance along the surface. The separation in mixed convection flow was first discussed by Merkin (1969), who examined the effect of opposing buoyancy forces on the boundary layer flow on a semi-infinite vertical flat plate at a constant temperature in a uniform free stream. Furthermore, this problem was studied by Wilks (1973, 1974) and Hunt and Wilks (1980), who also considered the case of uniform flow over a semi-infinite flat plate but heated at a constant heat flux rate. Merkin

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NOMENCLATURE

A	area
C	specific heat capacity (J/kgK)
C_p	specific heat capacity at constant pressure (J/kgK)
D	diffusivity tensor
F	initial solution function
G	Green's function
H	Heaviside function
h	heat transfer coefficient [W/((m^3/m^2) m^2 K)]
h_b	heat transfer coefficient at the boundary [W/(m^2 K)]
k	coefficient of thermal conductivity (W/mK)
Nu	Nusselt number
O	order
Pe	Peclet number
Pr	Prandtl number
q''	heat generation per unit volume (W/m^2)
Re	Reynolds number
t	time (s)
T	temperature (°C)
u	velocity (m/s)
v	velocity of the fluid phase
V	volume
x	channel length (M)
X	nondimensionl temperature of the solid phase
y	nondimensional length
$\langle\rangle$	average volume
$\hat{}$	Laplace transform

Greek Symbols

α	thermal diffusivity coefficient
ε	porosity
$\in$	nondimensionl temperature of the fluid phase
ν	kinematic viscosity (m^2/s)
τ	nondimensionl time
δ	Dirac delta function

Subscripts

eff	effective characteristic
f	fluid phase
m	mean value
s	solid phase

and Pop (2002) obtained similar equations for mixed convection boundary layer flow over a vertical semi-infinite flat plate in which the free stream velocity is uniform and the wall temperature is inversely proportional to the distance along the plate. Aly et al. (2003) have investigated a vertically flowing fluid in a fluid-saturated porous medium maintained at a constant temperature, T_∞, passing a thin vertical fin, which is modeled as a fixed and semi-infinite vertical surface (Nusselt, 1911).

Javeri (1978) has investigated the influence of the temperature boundary condition of the third kind on the laminar forced convection heat transfer in the thermal entrance region of a rectangular channel; the energy equation was solved by applying the Galerkin–Kantorowich method of variation calculus. Davis and Brenner (1997) have modeled steady state heat conduction through an insulating layer separating the surface of a rough, isothermal body (e.g., a sphere) from an isothermal semi-infinite region bounded by a rough plane by employing the Robin boundary condition with a "slip" coefficient on the smoothed body surface and plane.

Exact analytical approaches cannot be applied for many reasons, so either numerical or perturbation methods are required. The perturbation solutions for the planar solidification of a saturated liquid with convection at the wall have been found by Pedroso and Domoto (1973) and Huang and Shih (1975).

Fourier series is the only perturbation solution method which is applied by Kuznetsov (1997) for solving a two-dimensional energy equation in packed beds. Villatoro et al. (2011) obtained an analytical solution for one-dimensional heat transfer between an inert gas and porous medium based on Laplace transforms using the solid thermal conductivity as a small parameter.

In the present analysis, the heat transfer between an inert gas and porous medium, proposing a matching method, is presented. Also, the boundary condition of the third kind is changed into the first kind using a novel approach (Ahmadi, 2013).

Studies show that semi-analytical methods [such as homotopy analysis method (HAM), variational iteration method (VIM), differential transform method (DTM)] can be used in simulation and prediction of thermal performance of solar exchangers filled with a porous medium energy harvesting system (Dehghan et al., 2015). The analytical technique called the Adomian decomposition method is proposed for the solution methodology. Solutions are validated using a numeric scheme called the finite difference method. The results indicate that the numerical data and analytical approach are in agreement with each other. The study is further extended to the porous fin in the stationary condition, and it is found that the porous fin in the moving condition transfers more heat than it does in the stationary condition (Bhanja et al., 2014).

Ma et al.'s (2016) spectral collocation method is presented to predict the thermal performance of a convective-radiative porous fin with temperature-dependent convective heat transfer coefficient, fin surface emissivity, and internal heat generation with an increase in collocation points. The effects of various geometric and thermophysical parameters on the dimensionless fin temperature, fin efficiency, and heat transfer rate are comprehensively analyzed. In addition, an optimum design analysis is also carried out.

2. GOVERNING EQUATIONS

Figure 1 shows the geometry of the problem. The fluid phase is considered an incompressible Newtonian fluid, with negligible viscous dissipation and heat conduction among the fluid particles, and also fluid motion is only in the axial direction of the solid. The condition is quasi-steady. The solid has a constant porosity and negligible radial temperature gradient, with only an axial temperature gradient.

Under the assumptions, the set of energy equations can be used as

$$\varphi\rho_f c_f\left(\frac{\partial T_f}{\partial t}+v_f\frac{\partial T_f}{\partial x}\right)=-h(T_f-T_s) \tag{1}$$

$$(1-\varphi)\rho_s c_s\frac{\partial T_s}{\partial t}=h(T_f-T_s)+(1-\varphi)\lambda_s\frac{\partial^2 T_s}{\partial x^2} \tag{2}$$

where λ_s is the thermal conductivity and φ is the connected void fraction of the solid. The subscripts f and s show the fluid and solid, respectively, with initial conditions and boundary conditions as

$$T_f(0,x)=T_{s0} \tag{3}$$

$$T_s(0,x)=T_{s0} \tag{4}$$

$$T_f(t,0)=T_{f0} \tag{5}$$

$$\lambda_s\frac{\partial T_s}{\partial x}(t,0)=h_b(T_s(t,0)-T_f(t,0)),\quad \lim_{x\to\infty}T_s(t,x)=\lim_{x\to\infty}T_f(t,x)=0, \tag{6}$$

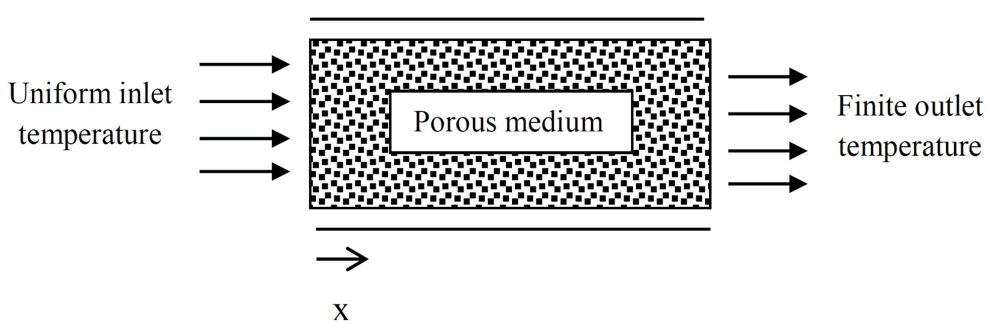

FIG. 1: Geometry of the problem

To apply the perturbation technique, Eqs. (1)–(7) were brought to a dimensionless form by introducing the following variables:

$$\tau = \frac{ht}{(1-\varphi)\rho_s c_s}, \qquad y = \frac{nhx}{\varphi \rho_f c_f v_f} \tag{7}$$

where $n = [\varphi\rho_f c_f]/[(1-\varphi)\rho_s c_s]$ and $X(\tau, y)$ and $\in(\tau, y)$ are defined as follows:

$$T_s = T_{s0} + X(\tau, y)(T_{f0} - T_{s0}), \quad T_f = T_{s0} + \in(\tau, y)(T_{f0} - T_{s0})$$

Now Eqs. (1)–(7) can be written as

$$n\left(\frac{\partial \in}{\partial \tau} + \frac{\partial \in}{\partial y}\right) = X - \in \tag{8}$$

$$\frac{\partial X}{\partial \tau} - \beta^2 \frac{\partial^2 X}{\partial y^2} = \in - X \tag{9}$$

$$\in(0, y) = 0, \quad X(0, y) = 0, \quad y > 01 \tag{10}$$

$$\in(\tau, 0) = 1 \tag{11}$$

$$\frac{\partial X}{\partial y}(\tau, 0) = \gamma(X(\tau, 0) - \in(\tau, 0)) \tag{12}$$

$$\lim_{y\to\infty} X(\tau, y) = \lim_{y\to\infty} \in(\tau, y) = 0 \tag{13}$$

where

$$\beta^2 = (1-\varphi)\lambda_s h\left(\frac{n}{\varphi\rho_f c_f v_f}\right)^2, \quad \gamma = \frac{h_b \varphi \rho_f c_f v_f}{hn\lambda_s}$$

To solve Eqs. (8)–(13), the Laplace transform method can be used. So $\hat{\epsilon}(sy)$ and $\hat{X}(sy)$ are the Laplace transform on τ of $\epsilon(\tau y)$ and $X(\tau y)$, respectively. Then, the governing equations and boundary conditions will be as follows:

$$n\left(s\widehat{\in} + \frac{\partial \widehat{\in}}{\partial y}\right) = \widehat{X} - \widehat{\in} \tag{14}$$

$$s\widehat{X} - \beta^2 \frac{\partial^2 \widehat{X}}{\partial y^2} = \widehat{\in} - \widehat{X} \tag{15}$$

$$\widehat{\in}(s, 0) = \frac{1}{s} \tag{16}$$

$$\frac{\partial \widehat{X}}{\partial y}(s, 0) = \gamma(\widehat{X}(s, 0) - \widehat{\in}(\tau, 0)) \tag{17}$$

The solution and the inverse Laplace transform of the current solution cannot be obtained analytically, hence an approximate asymptotic solution for both gas and solid temperature, considering $\beta^2 \ll 1$, has been developed; also, inner and outer solutions have been achieved.

3. CHANGE OF BOUNDARY CONDITION FROM THIRD INTO FIRST KIND

In this work, the Robin boundary condition is changed into the first kind and then applied to this problem. Considering the change of variables, the boundary condition can be written as follows:

$$\frac{\partial \widehat{X}^{(i)}}{\partial Y}(s,0) = \gamma\beta(\widehat{X}^{(i)}(s,0) - \widehat{\in}^{(i)}(s,0)) \tag{18}$$

$$\widehat{\in}^{(i)}(s,0) = \frac{1}{s} \tag{19}$$

$$\widehat{X}^{(i)}(s,0) - \frac{1}{\gamma\beta}\frac{\partial \widehat{X}^{(i)}}{\partial Y}(s,0) = \frac{1}{s} \tag{20}$$

The above equation is the first two statements of a Taylor series of $\hat{X}^{(i)}\,[s, 0 - 1/(\gamma\beta)]$. Substituting the inner solution in this equation, it can be considered as follows:

$$\tilde{X}_0\Big|_{0-1/(\gamma\beta)} + \beta\ \tilde{X}_{1/2}\Big|_{0-1/(\gamma\beta)} + \beta^2\ \tilde{X}_1\Big|_{0-1/(\gamma\beta)} + ... = \frac{1}{s} \tag{21}$$

In this part, the Taylor series of each part in the preceding equation must be written. So

$$\left(\tilde{X}_0\Big|_0 - \frac{1}{\gamma\beta}\ \tilde{X}_0'\Big|_0 + ...\right) + \beta\left(\tilde{X}_{1/2}\Big|_0 - \frac{1}{\gamma\beta}\ \tilde{X}_{1/2}'\Big|_0 + ...\right) + \beta^2\left(\tilde{X}_1\Big|_0 - \frac{1}{\gamma\beta}\ \tilde{X}_1'\Big|_0 + ...\right) = \frac{1}{s} \tag{22}$$

The order of $1/(\gamma\beta)$ is β, so the preceding equation may be written as

$$\left(\tilde{X}_0\Big|_0 - \beta\ \tilde{X}_0'\Big|_0 + ...\right) + \beta\left(\tilde{X}_{1/2}\Big|_0 - \beta\ \tilde{X}_{1/2}'\Big|_0 + ...\right) + \beta^2\left(\tilde{X}_1\Big|_0 - \beta\ \tilde{X}_1'\Big|_0 + ...\right) = \frac{1}{s} \tag{23}$$

4. OUTER SOLUTION

It is necessary to consider both inner and outer solutions. A singular perturbation method has been applied to equations in Laplace space. The outer solution is

$$\widehat{\in}^{(o)}(s,y) = \widehat{\in}_0(s,y) + \beta^2\widehat{\in}_1(s,y) + \beta^4\widehat{\in}_2(s,y) + O(\beta^6) \tag{24}$$

$$\widehat{X}^{(o)}(s,y) = \widehat{X}_0(s,y) + \beta^2\widehat{X}_1(s,y) + \beta^4\widehat{X}_2(s,y) + O(\beta^6) \tag{25}$$

Substituting the outer solution into governing equations,

$$\beta^0 \to n\left(s\widehat{\in}_0 + \frac{\partial\widehat{\in}_0}{\partial y}\right) = \widehat{X}_0 - \widehat{\in}_0, \qquad s\widehat{X}_0 = \widehat{\in}_0 - \widehat{X}_0 \tag{26}$$

$$\beta^2 \to n\left(s\widehat{\in}_1 + \frac{\partial\widehat{\in}_1}{\partial y}\right) = \widehat{X}_1 - \widehat{\in}_1, \qquad s\widehat{X}_1 - \frac{\partial^2\widehat{X}_0}{\partial y^2} = \widehat{\in}_0 - \widehat{X}_0 \tag{27}$$

and high-order terms also can be directly determined. The solutions of Eqs. (26) are

$$\widehat{\in}_0 = ce^{-sy}\exp\left(\frac{-sy}{n(1+s)}\right), \quad \widehat{X}_0 = \frac{c}{1+s}e^{-sy}\exp\left(\frac{-sy}{n(1+s)}\right) \tag{28}$$

and the solutions of Eqs. (27) yield

$$\widehat{\in}_1 = A_1 \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right] + \frac{c}{n(1+s)^2}\left(s+\frac{s}{n(1+s)}\right)^2 y \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right]$$
$$\widehat{X}_1 = \frac{A_1}{1+s}\exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right] + \frac{c}{n(1+s)^3}\left(s+\frac{s}{n(1+s)}\right)^2 y \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right] \quad (29)$$
$$+\frac{c}{n(1+s)^2}\left(s+\frac{s}{n(1+s)}\right)^2 \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right]$$

5. INNER SOLUTION

The inner approximations $\hat{\varepsilon}^{(i)}(sY)$ and $\hat{X}^{(i)}\,(s,Y)$, where $Y = y/\beta$ and $\partial/(\partial y) = \partial/(\beta\partial Y)$, have been determined. The results of the numerical simulations suggested the appearance of a boundary layer at $y = 0$ with a width of $O\,(\beta)$ (Villatoro et al., 2011):

$$\widehat{\in}^{(i)}(s,Y) = \tilde{\in}_0(s,Y) + \beta\tilde{\in}_{1/2}(s,Y) + \beta^2\tilde{\in}_1(s,Y) + O(\beta^3) \quad (30)$$

$$\widehat{X}^{(i)}(s,Y) = \tilde{X}_0(s,Y) + \beta\tilde{X}_{1/2}(s,Y) + \beta^2\tilde{X}_1(s,Y) + O(\beta^3) \quad (31)$$

For the inner solution, regarding $Y = y/\beta$, governing equations and boundary conditions will become

$$\beta^0 \to n\left(s\beta\widehat{\in}^{(i)} + \frac{\partial\widehat{\in}^{(i)}}{\partial Y}\right) = \beta\left(\widehat{X}^{(i)} - \widehat{\in}^{(i)}\right), \qquad s\widehat{X}^{(i)} - \frac{\partial^2\widehat{X}^{(i)}}{\partial Y^2} = \widehat{\in}^{(i)} - \widehat{X}^{(i)} \quad (32)$$

$$\text{B.C}\begin{cases} \widehat{\in}^{(i)}(s,0) = 1/s \\ \widehat{X}^{(i)}\left(s,-\dfrac{1}{\gamma\beta}\right) = 1/s \\ \lim\limits_{Y\to\infty}\widehat{X}^{(i)}(s,Y) = \lim\limits_{Y\to\infty}\widehat{\in}^{(i)}(s,Y) = 0 \end{cases}$$

By substituting Eqs. (30) and (31) with (32) and using Eq. (22), equations and boundary conditions of $O(1)$, $O(\beta)$, and $O(\beta_2)$ can be obtained, and the results can be written as follows:

$$\tilde{\in}_0(s,Y) = \frac{1}{s} \quad (33)$$

$$\tilde{X}_0(s,Y) = \frac{1}{s(1+s)}\exp(-\sqrt{1+s}Y) + \frac{1}{s(1+s)} \quad (34)$$

$$\tilde{\in}_{1/2}(s,Y) = 0 \quad (35)$$

$$\tilde{X}_{1/2} = \frac{-1}{\sqrt{1+s}}\exp(-\sqrt{1+s}Y) \quad (36)$$

$$\tilde{\in}_1(s,Y) = 0 \quad (37)$$

$$\tilde{X}_1(s,Y) = 2\exp(-\sqrt{1+s}Y) \quad (38)$$

These exponential solutions seem to be true due to the zero value of the solution, when $Y \to \infty$.

6. SUGGESTED MATCHING METHOD

To determine the outer solution, the following relation is used as the matching condition:

$$\left(\widehat{\in}^{(o)}\right)^{i}_{y\to 0} = (\tilde{\in}^{(i)})^{o}_{Y\to\infty} \tag{39}$$

$$\left(\widehat{X}^{(o)}\right)^{i}_{y\to 0} = (\tilde{X}^{(i)})^{o}_{Y\to\infty} \tag{40}$$

According to the preceding equations, the inner solution when $Y \to \infty$ is equal to the outer solution when $y \to 0$, hence the unknown coefficient in the outer solution can be obtained:

$$c = \frac{1}{s} \tag{41}$$

$$A_1 = 0 \tag{42}$$

The values of $(\hat{X}_1)_{y\to 0}$ and $(\tilde{X}_1)_{Y\to\infty}$ are not equal, which proves the accuracy of the current method. The solutions to Eqs. (28) and (29) are

$$\widehat{\in}_0 = \frac{1}{s} e^{-sy} \exp\left(\frac{-sy}{n(1+s)}\right) \tag{43}$$

$$\widehat{X}_0 = \frac{1}{s(1+s)} e^{-sy} \exp\left(\frac{-sy}{n(1+s)}\right) \tag{44}$$

$$\widehat{\in}_1 = \frac{s}{n(1+s)^2}\left(1+\frac{1}{n(1+s)}\right)^2 y \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right] \tag{45}$$

$$\begin{aligned}\widehat{X}_1 = {} & \frac{s}{n(1+s)^3}\left(1+\frac{1}{n(1+s)}\right)^2 y \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right] \\ & + \frac{s}{n(1+s)^2}\left(1+\frac{1}{n(1+s)}\right)^2 \exp\left[-sy\left(1+\frac{1}{n(1+s)}\right)\right]\end{aligned} \tag{46}$$

The solution of Eqs. (43) and (46) can be achieved using the Bromwich integral for the inverse Laplace transform [Javeri, 1978, Eqs. (3) and (5)]; the final solutions is written in Table 1. Furthermore, the Laplace inversion of Eqs. (45) and (46) can proceed as mentioned by Villatoro et al. (2011); the solution is written in Table 1. The inverse Laplace transform of solutions of inner approximations are mentioned in Table 2. The main solution is suggested as follows:

$$\in_0 = \tilde{\in}_0^{(i)} + \in_0^{(o)} - \frac{1}{2}\left[(\tilde{\in}_0^{(i)})^{(o)}_{Y\to\infty} + (\in_0^{(o)})^{(i)}_{y\to 0}\right] \tag{47}$$

$$\in_1 = \tilde{\in}_1^{(i)} + \in_1^{(o)} - \frac{1}{2}\left[(\tilde{\in}_1^{(i)})^{(o)}_{Y\to\infty} + (\in_1^{(o)})^{(i)}_{y\to 0}\right] \tag{48}$$

$$X_0 = \tilde{X}_0^{(i)} + X_0^{(o)} - \frac{1}{2}\left[(\tilde{X}_0^{(i)})^{(o)}_{Y\to\infty} + (X_0^{(o)})^{(i)}_{y\to 0}\right] \tag{49}$$

$$X_1 = \tilde{X}_1^{(i)} + X_1^{(o)} - \frac{1}{2}\left[(\tilde{X}_1^{(i)})^{(o)}_{Y\to\infty} + (X_1^{(o)})^{(i)}_{y\to 0}\right] \tag{50}$$

TABLE 1: Solutions of outer solution for the solid $[X(\tau, y)]$ and gas $[\in(\tau, y)]$ temperatures

Zeroth order	$\in_0(\tau, y) = H(\tau - y)e^{-y/n}\left\{1 + \int_0^{\tau-y} \exp(-u)I_1\left(2\sqrt{\frac{yu}{n}}\right)\sqrt{\frac{y}{un}}du\right\}$ $X_0(\tau, y) = H(\tau - y)e^{-y/n}\int_0^{\tau-y} \exp(-u)I_0\left(2\sqrt{\frac{yu}{n}}\right)du$
First order	$\in_1(\tau, y) = \frac{y}{n}H(\tau - y)e^{y-\tau-y/n}\left\{I_0(z) + \frac{2-n}{gn}I_1(z) - \frac{2n-1}{g^2n^2}I_2(z) - \frac{1}{g^3n^2}I_3(z)\right\}$ $X_1(\tau, y) = H(\tau - y)e^{y-\tau-y/n}\left\{I_0(z) + \frac{2-n+y}{gn}I_1(z) - \frac{y}{g^4n^3}I_4(z)\right.$ $\left. + \frac{1+2y-n(2+y)}{g^2n^2}I_2(z) + \frac{y-n(1+2y)}{g^3n^3}I_3(z)\right\}$

TABLE 2: Solutions of inner solution for the solid $[\tilde{X}^{(i)}(\tau Y)]$ and gas $[\tilde{\epsilon}^{(i)}(\tau Y)]$ temperatures (adapted from Ahmadi, 2013)

Zeroth order	$\tilde{\epsilon}_0^{(i)}(\tau, Y) = 1$ $\tilde{X}_0^{(i)}(\tau, Y) = e^{-\tau}erfc\left(Y/2\sqrt{\tau}\right) + 1 - e^{-\tau}$
Middle order	$\tilde{\epsilon}_{1/2}^{(i)}(\tau, Y) = 0$ $\tilde{X}_{1/2}^{(i)}(\tau, Y) = \frac{-e^{-\tau-(Y^2/4\tau)}}{\sqrt{\pi}\sqrt{\tau}}$
First order	$\tilde{\epsilon}_1^{(i)}(\tau, Y) = 0$ $\tilde{X}_1^{(i)}(\tau, Y) = \left(e^{-\tau-(Y^2/4\tau)}Y\right)/\sqrt{\pi}\tau$

7. RESULTS

According to the calculations, the final approximate analytical solution for the one-dimensional problem of heat transfer between an inert gas and a porous semi-infinite medium, including two and three terms for outer and inner solutions, respectively, is presented in Tables 1 and 2.

Some solutions are presented as functions of τ, which shows that this approach is an approximate one, but for $\beta^2 \ll 1$, it has a good accuracy. Furthermore, these results show a good agreement with numerical results of the research done by Villatoro et al. (2011).

Figures 2 and 3 show temperatures of fluid and solid phases. As shown in Fig. 2, the value of gas temperature $\epsilon(\tau y)$ at $(y = 0)$ is 1, which satisfies the boundary condition at $y = 0$. The change of boundary condition at $y = 0$ did not make any difference in temperature due to not using this condition to solve for the gas temperature.

Gas temperature is null when $y > \tau$, as it was considered in solving the outer solution. For $\tau = 1$, the curve of temperature enfolds the smaller range of y than at other times (Villatoro et al., 2011). Figure 3 shows the solid temperature. As shown in Fig. 3, a break in slope is occurring in the area near the wall, where $y = 0$, due to changing boundary conditions. Transferring the value of temperature at $y = 0 - 1/(\gamma\beta)$ into $y = 0$ results in such a difference.

To compare the results of substituting the first kind boundary condition with the third kind, using the Robin boundary condition directly, Fig. 4 has been drawn. As is shown, the solutions for the two states are completely the same, except near the wall, which is because of changing boundary conditions.

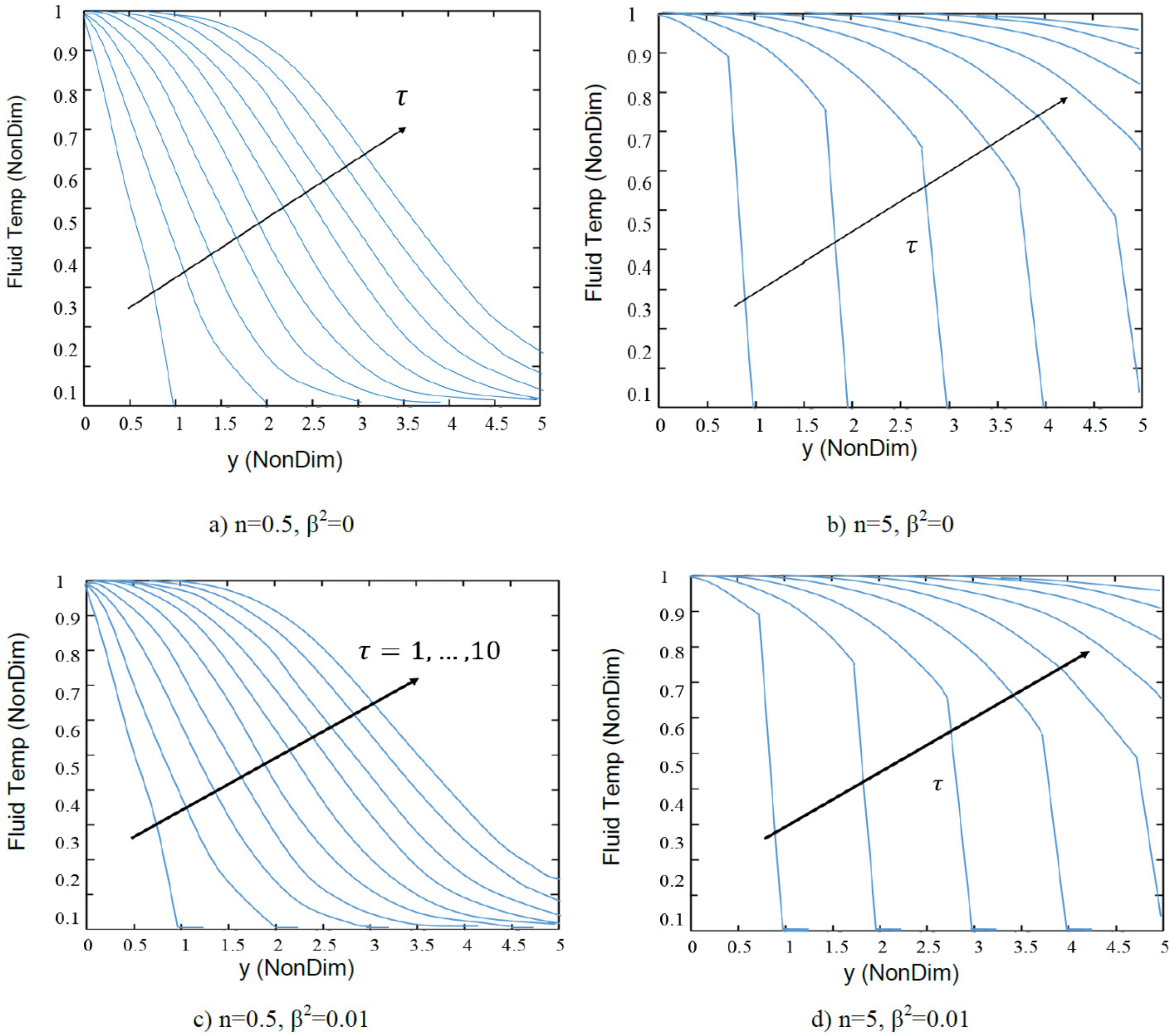

FIG. 2: Plots of the analytical solutions $\in (\tau, y)$ for (a, b) $\beta_2 = 0$ and (c, d) $\beta_2 = 0.01$ for (a, c) $n = 0.5$ and (b, d) $n = 5$. The abscissa in every plot is y.

According to the expansion of Eq. (22), the small parameter for transferring the third kind of boundary condition into the first kind is $1/(\gamma\beta)$. On the other hand, the small parameter for the inner perturbation solution was β. So the range of solution accuracy near the wall will depend on $1/\gamma$, and if $1/\gamma$ is so much smaller than β, the region of accuracy near the wall will expand more. In other words, if the purpose of solving a problem is to obtain a precise profile of temperature, applying this approach will not be reasonable.

8. CONCLUSION

In this article, a novel asymptotic solution for heat transfer between gas and solid phases in a porous medium with small thermal conductivity is presented. The main goal of this article is to develop an approximate asymptotic solution for both the gas and the solid temperatures when $\beta^2 \ll 1$. According to the singular perturbation method, the solution is divided into inner and outer solutions. A novel suggested matching method is proposed for matching inner and

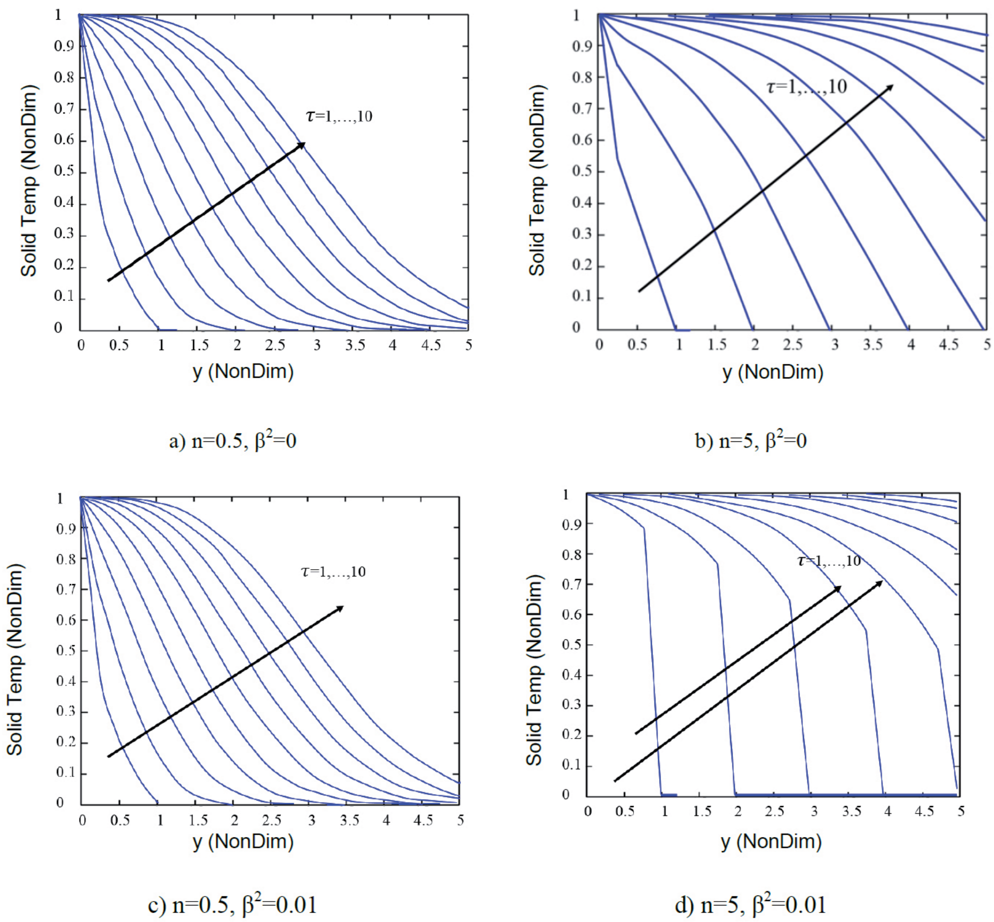

FIG. 3: Plots of the analytical solutions $X(\tau, y)$ for (a, b) $\beta_2 = 0$ and (c, d) $\beta_2 = 0.01$ for (a, c) $n = 0.5$ and (b, d) $n = 5$. The abscissa in every plot is y.

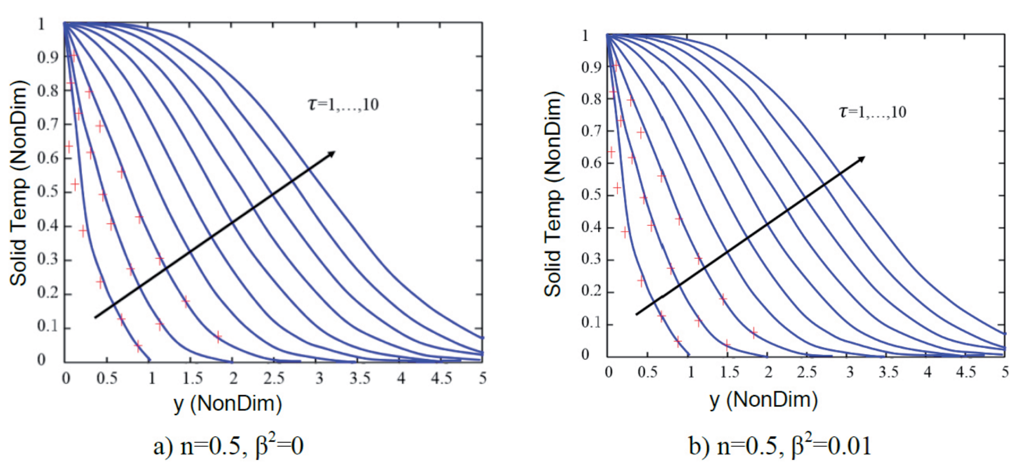

FIG. 4: Plots of the analytical solutions $X(\tau, y)$ for (a) $\beta^2 = 0$ and (b) $\beta^2 = 0.01$ and for $n = 0.5$ and comparing with the solution using Robin B.C $(+)$. The abscissa in every plot is y.

outer solutions. According to the new idea, the Robin boundary condition at $y = 0$ changed into the first kind. This method has good agreement with the solution obtained using the third kind boundary condition. In cases that the slope near the wall is important, using this method does not give an exact solution near the wall; however, this method is suitable for making problems easier.

The singular perturbation method used in this article is a superior method compared to the other methods already in use in the analysis of heat transfer in a porous medium because of applying the first kind of boundary condition instead of the third kind and also because of the suggested novel form of inner and outer condition matching. These contributions decrease computational time and give a closed-form solution.

This method has good agreement with solutions obtained using the third kind boundary condition. In cases that slope near the wall is important, using this method loses accuracy near the wall, but this method is suitable for making problems easier.

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