K.N.Toosi University of Technology

Machine Element Design I

Fatigue Failure Criteria

By

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Chapter 6 - Outline

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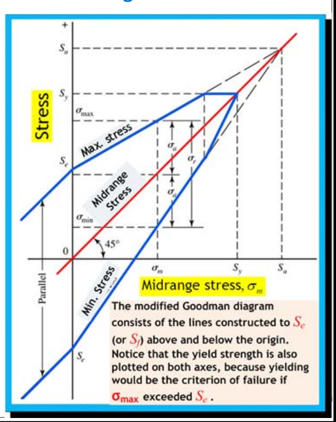
Fatigue failure criteria for fluctuating stress

- It has midrange stress plotted along the abscissa and all other components of stress plotted on the ordinate, with tension in the positive direction.
- The endurance limit, fatigue strength, or finite-life strength whichever applies, is plotted on the ordinate above and below the origin.
- The midrange line is a 45° line from the origin to the tensile strength of the part.

Figure 7-24

Modified Goodman diagram showing all the strengths and the limiting values of all the stress components for a particular midrange stress



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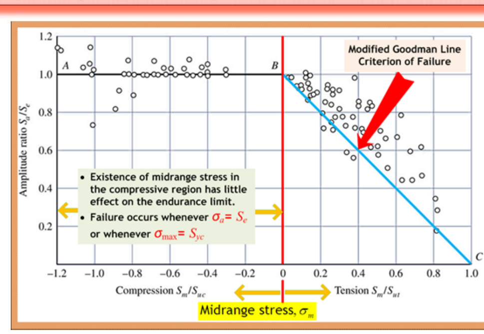
Plot of Fatigue Failures for Midrange Stresses in both Tensile and Compressive Regions.



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Master curve for fatigue

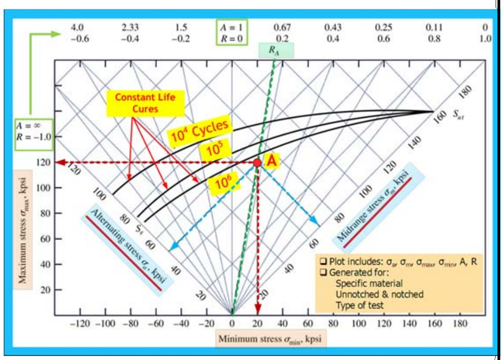
Figure 7-26

Master fatigue diagram for AISI 4340 steel with S_{ut} = 158
S_y = 147 kpsi.

The stress component at A are

σ_{min} = 20,
σ_{max} = 120,
σ_m = 70,
σ_a = 50

all in kpsi



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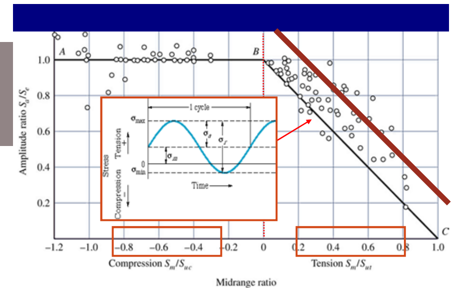
Fluctuating Stresses

Mean Stress Effect (R ≠ -1)

2. Representing mean stress effect using modified Goodman Diagram

S is for strength



Failure data for S_m in tension and in compression

COMPRESSIVE mean stresses are BENEFICIAL (or have no effect) in fatigue
TENSILE mean stresses are DETRIMENTAL for fatigue behavior

Various curve of failure

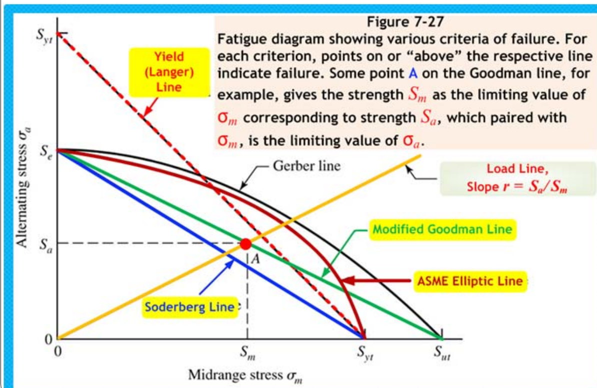


Figure 7-27
Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to strength S_a, which paired with σ_m, is the limiting value of σ_a.

FAILURE CRITERIA (mean stress)

1- Modified Goodman Theory (Germany, 1899)

For infinite life Failure Occurs When:

$$\frac{S_a}{S_e}+\frac{S_m}{S_u}=1$$

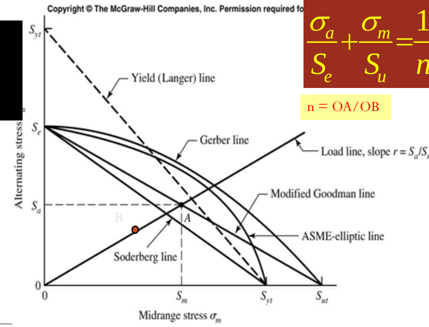
Load Line slope

$$r=\frac{S_a}{S_m}$$

Factor of Safety

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_u}=\frac{1}{n}$$

n = OA/OB



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FAILURE CRITERIA (mean stress)

2- The Soderberg Theory (USA, 1933)

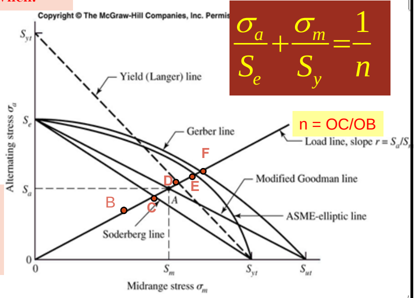
For infinite life Failure Occurs When:

$$\frac{S_a}{S_e}+\frac{S_m}{S_y}=1$$

Factor of Safety

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}=\frac{1}{n}$$

n = OC/OB



For finite life fatigue strength $S_f = \sigma_a$ replaces Se

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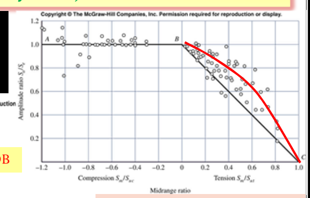
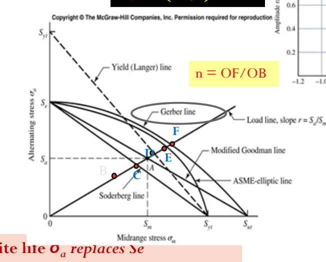
FAILURE CRITERIA (mean stress)

3- The Gerber Theory (Germany, 1874)

Failure Occurs When:

$$\frac{S_a}{S_e}+\left(\frac{S_m}{S_u}\right)^2=1$$





n = OF/OB

Factor of Safety

$$\frac{n\sigma_a}{S_e}+\left(\frac{n\sigma_m}{S_u}\right)^2=1$$

For finite life σ_a replaces Se

FAILURE CRITERIA (mean stress)

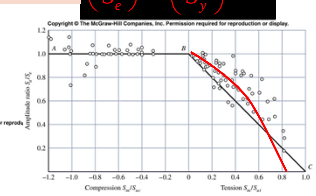
4- The ASME Elliptic

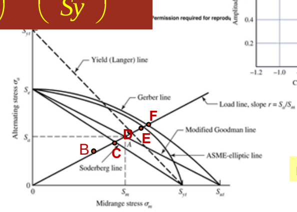
Failure Occurs When:

$$\left(\frac{S_a}{S_e}\right)^2+\left(\frac{S_m}{S_y}\right)^2=1$$

Factor of Safety

$$\left(\frac{n\sigma_a}{S_e}\right)^2+\left(\frac{n\sigma_m}{Sy}\right)^2=1$$





n = OE/OB

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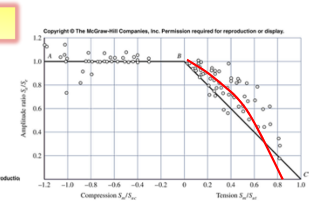
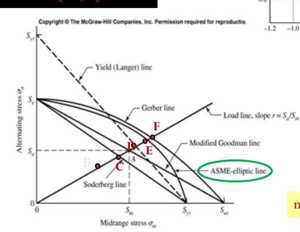
FAILURE CRITERIA (mean stress)

4- The ASME Elliptic

Failure Occurs When

$$\left(\frac{S_a}{S_e}\right)^2+\left(\frac{S_m}{S_y}\right)^2=1$$





Factor of Safety

$$\left(\frac{n\sigma_a}{S_e}\right)^2+\left(\frac{n\sigma_m}{S_y}\right)^2=1$$

n = OE/OB

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FAILURE CRITERIA (mean stress)

5- The Langer (1st Cycle) Yield Line

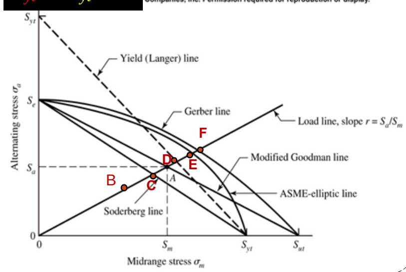
Failure Occurs When

$$\frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1$$

Factor of Safety

$$\frac{\sigma_a}{S_{yt}} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

n = OD/OB



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Criteria Equations

Solderberg Line $\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$ (7-43)

Modified Goodman Relation $\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$ (7-44)

Gerber Failure Criterion $\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$ (7-45)

ASME Elliptic Line $\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ (7-46)

Langer First cycle yielding criterion $\frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1$ (7-47)

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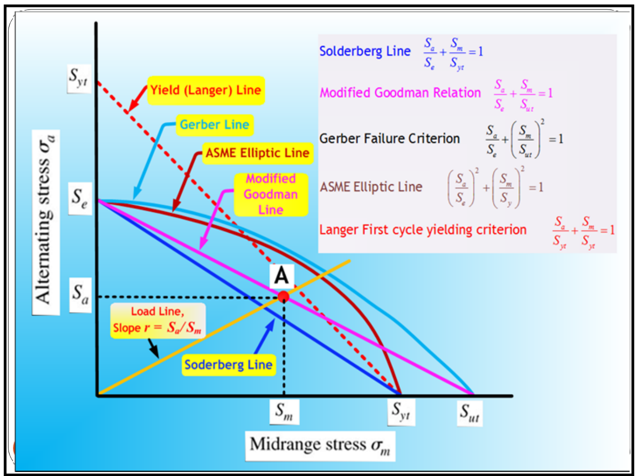

- The stresses $n\sigma_a$ and $n\sigma_m$ can replace S_a and S_m, where n is the design factor or factor of safety. Then, Eqs. (7-43) to (7-46) become:

Solderberg Line $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$ (7-48)

Modified Goodman Relation $\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$ (7-49)

Gerber Failure Criterion $\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$ (7-50)

ASME Elliptic Line $\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$ (7-51)

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We will emphasize the Gerber and ASME-elliptic for fatigue failure criterion and the Langer for first-cycle yielding. However, conservative designers often use the modified Goodman criterion. The design equation for the Langer first -cycle-yielding is

$$\text{Langer static yield} \quad \sigma_a + \sigma_m = \frac{S_y}{n} \qquad (7\,*)$$

The failure criteria are used in conjunction with a load line, $r = S_a/S_m$. Principal intersections are tabulated in Tables 7-9 to 7-11. Formal expressions for fatigue factor of safety are given in the lower panel of Tables 7-9 to 7-11. The first row of each table corresponds to the fatigue criterion, the second row is the static Langer criterion, and the third row corresponds to the intersection of the static and fatigue criteria.

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TABLE (7-9) Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria.

	Intersecting Equations	Intersection Coordinates
Fatigue Criterion	$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$
	Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
Static Langer Criterion	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
	Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
Intersection of the Static and Fatigue Criteria	$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$
	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

TABLE (7-11) Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME Elliptic and Langer Failure Criteria.

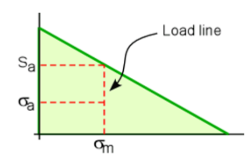
	Intersecting Equations	Intersection Coordinates
ASME Elliptic	$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
	Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
Langer	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
	Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1 + r}$
Intersection of ASME Elliptic and Langer	$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0,\ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$
	$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

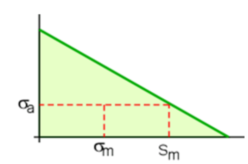
$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

Special Cases of Fluctuating Stresses

- **Case 1: σ_m fixed**



- **Case 2: σ_a fixed**



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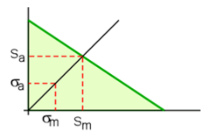
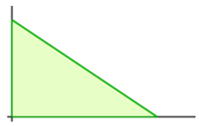
Special Cases of Fluctuating Stresses

- **Case 3: σ_a / σ_m fixed**



- **Case 4: both vary arbitrarily**



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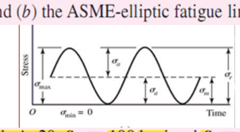
EXAMPLE 7-11 (Textbook)

A 1.5-in-diameter bar has been machined from an AISI 1050 cold-drawn bar. This part is to withstand a fluctuating tensile load varying from 0 to 16 kip. Because of the ends, and the fillet radius, a fatigue stress-concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety guarding against fatigue and first-cycle yielding, using (*a*) the Gerber fatigue line and (*b*) the ASME-elliptic fatigue line.

Solution



We begin with some preliminaries. From Table A–20, $S_{ut} = 100$ kpsi and $S_y = 84$ kpsi. Note that $F_a = F_m = 8$ kip. The Marin factors are, deterministically,

$k_a = 2.70(100)^{-0.265} = 0.797$: Eq. (7-18), Table (7-4), p. 329

$k_b = 1$ (axial loading, see k_c)

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EXAMPLE 7-11 (Textbook)

$k_c = 0.85$: Eq. (7-25), p. 331

$k_d = k_e = k_f = 1$

$S_e = 0.797(1)0.850(1)(1)(1)0.5(100) = 33.9\,\text{kpsi}$: Eqs. (7-8), (7-17), p. 325, p. 328

The nominal axial stress components σ_{ao} and σ_{mo} are

$$\sigma_{ao} = \frac{4F_a}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53\text{ kpsi} \qquad \sigma_{mo} = \frac{4F_m}{\pi d^2} = \frac{4(8)}{\pi 1.5^2} = 4.53\text{ kpsi}$$

Applying K_f to both components σ_{ao} and σ_{mo} constitutes a prescription of no notch yielding:

$$\sigma_a = K_f \sigma_{ao} = 1.85(4.53) = 8.38\text{ kpsi} = \sigma_m$$

(*a*) Let us calculate the factors of safety first. From the bottom panel from Table (7-10) factor of safety for fatigue is

$$n_f = \frac{1}{2}\left(\frac{100}{8.38}\right)^2\left(\frac{8.38}{33.9}\right)\left\{-1 + \sqrt{1 + \left[\frac{2(8.38)33.9}{100(8.38)}\right]^2}\right\} = 3.66$$

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From Eq. (7-*) the factor of safety guarding against first-cycle yield is

$$n_y = \frac{S_y}{\sigma_a + \sigma_m} = \frac{84}{8.38 + 8.38} = 5.01$$

Thus, we see that fatigue will occur first and the factor of safety is 3.68. This can be seen in Fig (7-28) where the load line intersects the Gerber fatigue curve first at point B. If the plots are created to true scale it would be seen that $n_f = OB/OA$.

From the first panel of Table (7-10) $r = \sigma_a/\sigma_m = 1$,

$$S_a = \frac{(1)^2 100^2}{2(33.9)}\left\{-1 + \sqrt{1 + \left[\frac{2(33.9)}{(1)100}\right]^2}\right\} = 30.7\text{ kpsi}$$

$$S_m = \frac{S_a}{r} = \frac{30.7}{1} = 30.7\text{ kpsi}$$

As a check on the previous result, $n_f = OB/OA = S_a/\sigma_a = S_m/\sigma_m = 30.7/8.38 = 3.66$ and we see total agreement.

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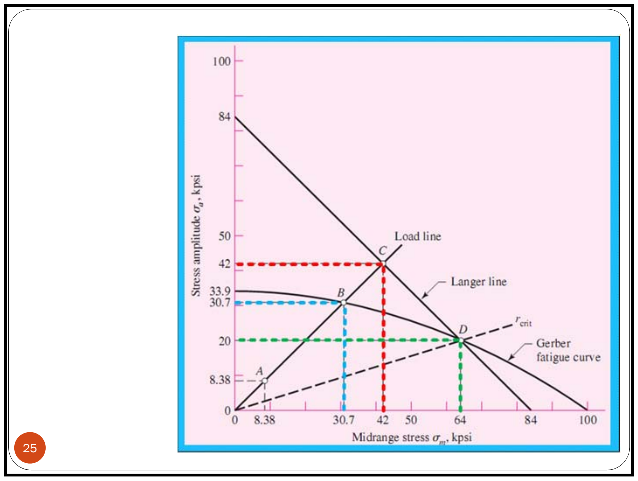
100
84
50
42
33.9
30.7
20
8.38
0
Stress amplitude σ_a, kpsi
Load line
C
Langer line
B
D
r_{crit}
Gerber fatigue curve
A
0
8.38
30.7
42
50
64
84
100
Midrange stress σ_m, kpsi
25