Switching Frequency Dependent Averaged Model for STATCOM

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Abstract—**A Static Compensator (STATCOM) is a FACTS controller, which can either absorb or deliver reactive power to power systems. Here it is modelled using an averaging approach, resulting in an equivalent circuit model. To achieve this, the state space equation of STATCOM is averaged to get an average mathematical model, leading to a circuit model. Theoretical considerations show that the averaged model should agree well with the original system, and this is con rmed by MATLAB and PSpice simulations. The solution is expandable as a Fourier series which can be suitably truncated.**

Keywords—**STATCOM, modelling, SSA, FACTS**

I. INTRODUCTION

THE use of FACTS controllers can overcome disadvantages of electromechanically controlled transmission systems. The STATCOM can be employed as a parallel device in ac power systems, generating balanced three-phase sinusoidal voltages at fundamental frequency. The amplitude and angle of these voltages should be rapidly controllable. Different voltage-sourced inverters topologies could be implemented using GTOs and IGBTs for high power utility applications such as voltage regulation. The analysis of STATCOM as a FACTS controller is presented in [1]–[3].

The analysis of a power electronic system is complex, due to its switching behaviour. Therefore there is a need for simpler, approximate models. One common approach to the modelling of power converters is averaging. This approximates the operation of the discontinuous system by a continuous-time model. The model produces waveforms that approximate the time-averaged waveforms of the original system. As well as simplifying analysis and making it easier to understand the system's behaviour under steady state and transient conditions, averaged models have the advantage that they speed up simulation.

In this paper, we start with the time-varying state space equations of STATCOM. We approximate them by averaged equations, giving a mathematical model. Then an equivalent circuit model is introduced, which provide a useful tool for analytical purposes. The average model gives good agreement with the original system, as demonstrated using MATLAB and PSpice.

II. OVERVIEW OF STATCOM

Fig. 1(a) shows a three-phase STATCOM, comprising *a voltage-sourced inverter connected through* an inductance in series with a transformer to a power system. The converter may consist of several six pulse voltage-sourced converters to improve the harmonic performance of STATCOM. The capacitor carries the dc voltage V_C. Now suppose both the ac system voltage v and the converter-composed voltage v0 are in phase. When v0 > v, STATCOM delivers reactive power to the power system. When v0 = v, reactive power is zero. When v0 < v, STATCOM absorbs reactive power from the power system. Thus, by varying v0, reactive power can be controlled to emulate a certain application such as voltage regulation.

However, for stable operation of STATCOM, the converter output has a small phase di erence with the ac system voltage (®). In other words, the current ows through STATCOM contains both reactive and active components. In fact, changing ® will vary the dc voltage V_C, and consequently the converter output v'. In [1]–[3], the explained mode of operation is modelled by transforming the system to a synchronous frame. Then, the resulting state space model is analyzed, showing a stable system with oscillatory dynamic response for STATCOM.

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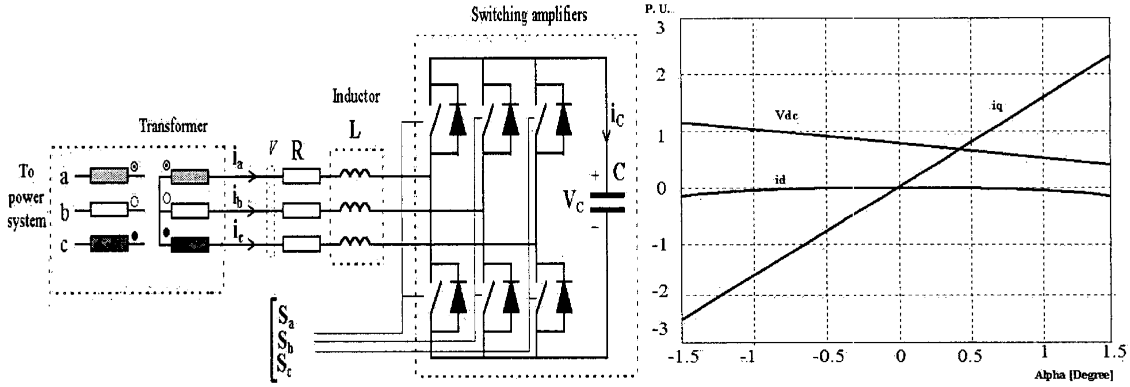

Figure 1: (a) An equivalent three-phase circuit of STATCOM; (b) equivalent reactive current, active current, and capacitor voltage as a function of α.

A typical steady state operation of STATCOM as a function of α is depicted in Fig. 1(b). Three state variables i_d, i_q, and V_C give the equivalent active current, reactive current, and dc voltage respectively. This gure shows almost a linear relationship for the i_q as a function of α over [¡1.5 °, 1.5°], although the state equations represent a nonlinear system. When α is negative, STATCOM works in capacitive mode, and positive ® corresponds to inductive mode. This suggests a way of controlling STATCOM, mainly by α.

III. OVERVIEW OF AVERAGING

We rst review the theoretical foundations of average modelling, then explain how it can be applied to STATCOM. State-space averaging (SSA) was established by Middlebrook and Cuk [5], and has been widely used for modelling dc-dc converters. The time-piecewise state equations are averaged over a switching cycle to give a time-continuous description. Essentially SSA assumes that the averaged state equations will give waveforms that are close to the averaged exact waveforms. This is true only at zero perturbation frequency, and when perturbations approach the switching frequency the error is ill de ned. In classical SSA, the switching frequency is absent from the average model, while it is clearly an important parameter of the real system. However, in [6] a switching-frequency dependent average model for dc-dc converters was proposed, giving more accurate results than standard SSA.

A. Application to STATCOM

Assume the converter voltage is synthesized using a PWM control to improve the harmonic performance. Note that the control loop focuses on ® to get the required operation point (see Fig. 1(b)), while the modulation index is xed close to one. Thus, there are two main periods involved in STATCOM: the reference period T_R (16.7ms or 20ms) and the switching period T_C, the period of the PWM carrier, typically a few kilohertz. The open-loop average equations are obtained in a standard form that can later be modi ed for closed-loop control:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{s}(t), \mathbf{u}(t)) \tag{1}$$

where x (t) is the state vector $[i_a, i_b, V_C]^T$, u (t) is the input vector and $\mathrm{s}(t) = [s_a(t), s_b(t), s_c(t)]^T$ is the vector of switching function. Note that at this stage we do not consider D to be a continuous function of time, but rather a discrete value associated with an individual switching period, adopting this convention: '1' means the upper switch of STATCOM is closed, and '−1' means the lower switch is closed. We now apply the averaging operator

$$\xi_a(t) = \text{average}\, \xi(t) \triangleq \frac{1}{T_C} \int_{t-T_C}^{t} \xi(\tau)\, d\tau \tag{2}$$

to (1) over $[t - T_C, t]$, to get an average model described by

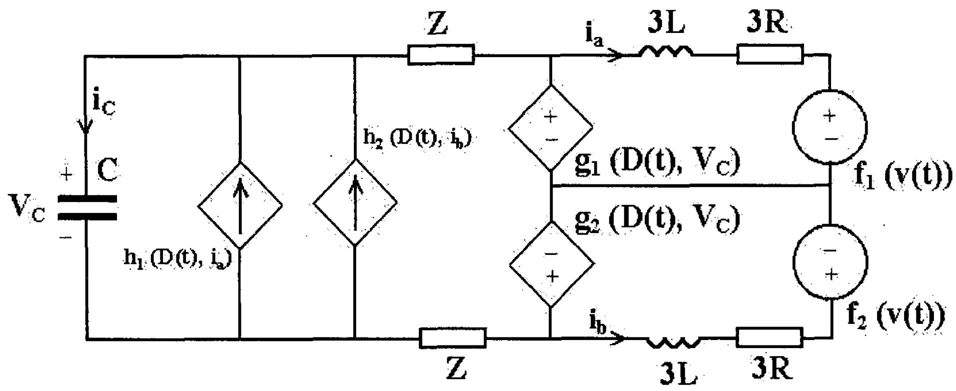

Figure 2: Equivalent circuit average model of STATCOM, suitable for circuit simulators such as SPICE.

$$\dot{\mathbf{x}}_a(t) = \mathbf{g}(\mathbf{x}_a(t), \mathbf{D}(t), \mathbf{u}(t)) \tag{3}$$

where $x_a(t)$ is the average state vector, and D(t) is the approximate continuous duty ratio. This new vector D (t) is a continuous function of time that is controlled by α. These issues as well as the closeness of $x(t)$ and $x_a(t)$ are detailed in the full paper, validating the averaging approach to modelling STATCOM.

IV. AVERAGE MODEL OF STATCOM

Consider STATCOM of Fig. 1(a). There are two topological modes for every leg. In the rst, the upper switch is closed and the lower switch is open, and in second, the upper switch is open and the lower switch is closed. The state equations for the two modes can be obtained separately then, introducing the switching function s $(t) \in \{-1, 1\}$, combined into a single state equation:

$$\dot{\mathbf{x}}(t) = (\mathbf{A}_a s_a(t) + \mathbf{A}_b s_b(t) + \mathbf{A}_c s_c(t))\mathbf{x}(t) + \mathbf{b}\mathbf{u}(t) \tag{4}$$

Then (4) is averaged over a switching period to develop a time-continuous model, in the form of (3). Applying the averaging operator of (2), let $x_a(t)$, the averaged state vector, be de ned as

$$\mathbf{x}_a(t) = \frac{1}{T_C} \int_{t-T_C}^{t} \mathbf{x}(\tau)\, d\tau \tag{5}$$

Integrating (4) over $[t - T_C, t]$ and applying (5), we get

$$\dot{\mathbf{x}}_a(t) = \mathbf{A}_a \frac{1}{T_C} \int_{t-T_C}^{t} s_a(\tau)\mathbf{x}(\tau) d\tau + \mathbf{A}_b \frac{1}{T_C} \int_{t-T_C}^{t} s_b(\tau)\mathbf{x}(\tau) d\tau + \mathbf{A}_c \frac{1}{T_C} \int_{t-T_C}^{t} s_c(\tau)\mathbf{x}(\tau) d\tau + \mathbf{b} \frac{1}{T_C} \int_{t-T_C}^{t} \mathbf{u}(\tau)\, d\tau \tag{6}$$

Now, by doing a detailed analysis, three points are shown. First, integrating the switching functions over $[t - T_C, t]$ de nes the continuous duty-ratio function D (t). Second, the resultant averaged state equation is

$$\dot{\mathbf{x}}_a(t) = (\mathbf{A}_a(2D_a(t) - 1) + \mathbf{A}_b(2D_b(t) - 1) + \mathbf{A}_c(2D_c(t) - 1))\mathbf{x}_a(t) + \mathbf{b}\mathbf{u}(t) \tag{7}$$

Third, the error term is obtained as a function of x (t) and T_C. This implies that the slower the variation of $x(t)$ and the higher the switching frequency, the smaller the error. Using Fourier analysis and truncating higher harmonics other than the fundamental, duty ratio for three phases are obtained for a typical ramp carrier

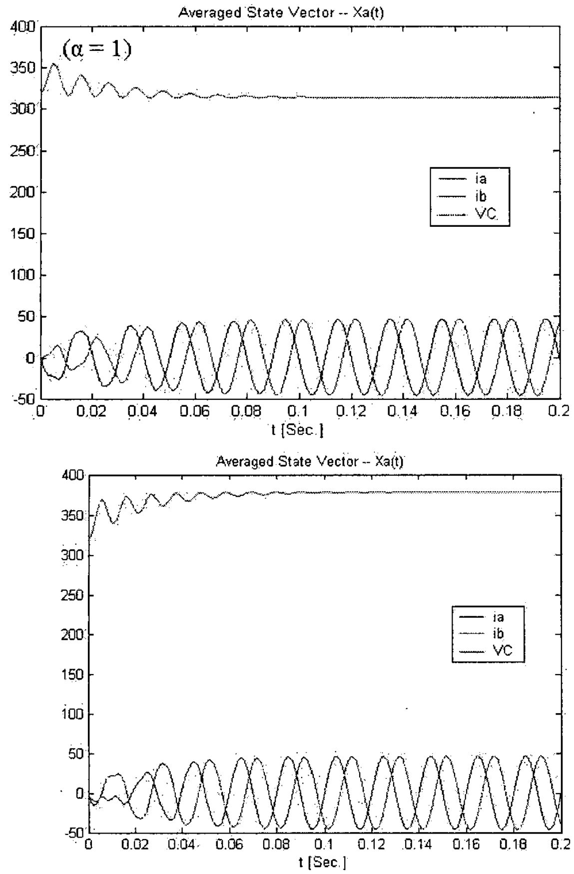

Figure 3: MATLAB simulation results for the average model, two cases: (a) $\alpha = 1°$ (inductive); (b) $\alpha = -1°$ (capacitive).

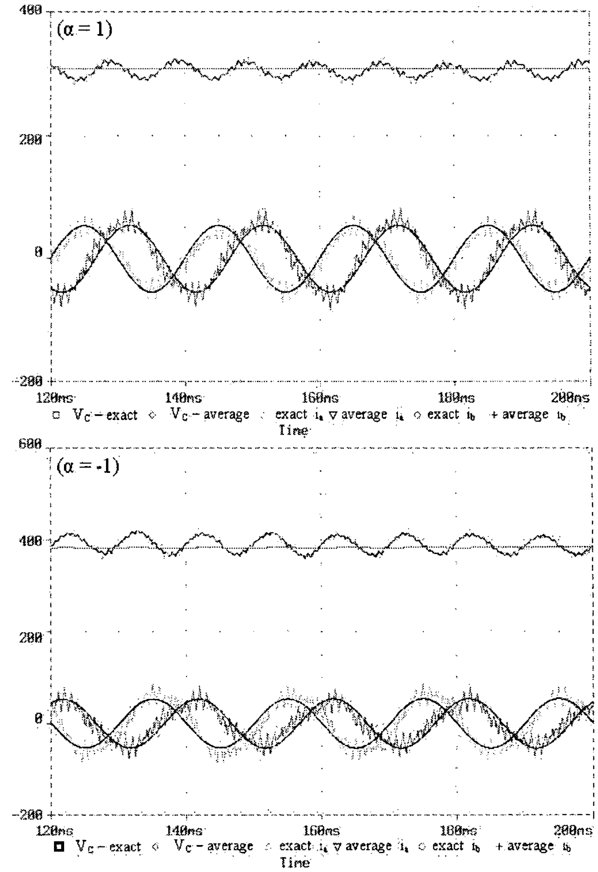

Figure 4: Results of the average equivalent circuit model simulated with PSpice: (a) $\alpha = 1°$ (inductive); (b) $\alpha = -1°$ (capacitive).

$$D_a(t) \approx \frac{1}{2}[1 + m\frac{\sin(\pi/M)}{\pi/M}\sin(\omega t - \frac{\pi}{M} + \alpha)] \quad (8)$$

where *m* is the xed modulation index. A MATLAB program was written to calculate the error between the exact duty ratio (at $t = nT_C$, $n = 1, 2, \cdots, M$) and the continuous approximation in (8). For $M = 40$ (e.g. 2kHz carrier, 50Hz reference), the worst-case error is less than 2%.

V. AVERAGE CIRCUIT MODEL

Using the mathematical model (7), an equivalent circuit is introduced by the aid of voltage-controlled voltage sources (VCVS) $g(\mathbf{D}(t), V_C)$ and current-controlled current sources (CCCS) shown in Fig. 2.

VI. SIMULATION RESULTS

In this section we compare various simulation results for STATCOM and our models, performed with MATLAB and PSpice. The parameters used here are based on those of a practical STATCOM is currently being designed for a distribution substation. First, the average model of STATCOM was simulated. The input voltage was $\mathbf{u}(t) = 155.6[sin(\omega t + \pi/2), sin(\omega t + \pi/2 - 2\pi/3), sin(\omega t + \pi/2 + 2\pi/3)]^T$, and $L = 1.0$mH, $C = 1.2$mF, $R = 0.06\omega$. The initial state vectors were $\mathrm{x}\ \ (0) = \mathbf{x}_a(0) = [0, -10, 320]^T$, and modulation index was xed to 0.9. Figs. 3(a) and (b) show the state variables of STATCOM for two cases: $\alpha = 1^\circ$ (inductive mode) and $\alpha = -1^\circ$ (capacitive mode).

Second, the equivalent circuit model of Fig. 2 together with the exact switched-system model of Fig. 1 were simulated with PSpice. The parameters are the same as for the MATLAB simulations of Fig. 3. Figs. 4 (a) and (b) depict the state variables for both exact and average models. Apart from the ripples, the agreement is good, demonstrating the compatibility of average model for the involved perturbation frequency. The PSpice simulation results can also be compared with those of MATLAB, presented by Fig. 3. Again the agreement is good, validating the equivalent circuit as well as the mathematical average model. But the average model ran much faster than the exact model, o ering useful savings in situations where accurate waveforms are not important (e.g. investigating system transients).

VII. CONCLUSION

A theoretically sound averaging method has been applied to approximate the behaviour of STATCOM. Starting with the exact state equations, an average model was developed. An equivalent circuit model was derived from the resulting equations. The exact system (simulated with PSpice) and the approximate model (simulated with MATLAB and PSpice) are all in good agreement, verifying that the necessary and su cient conditions of the averaging theorems in [8] are satis ed by the proposed models.

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