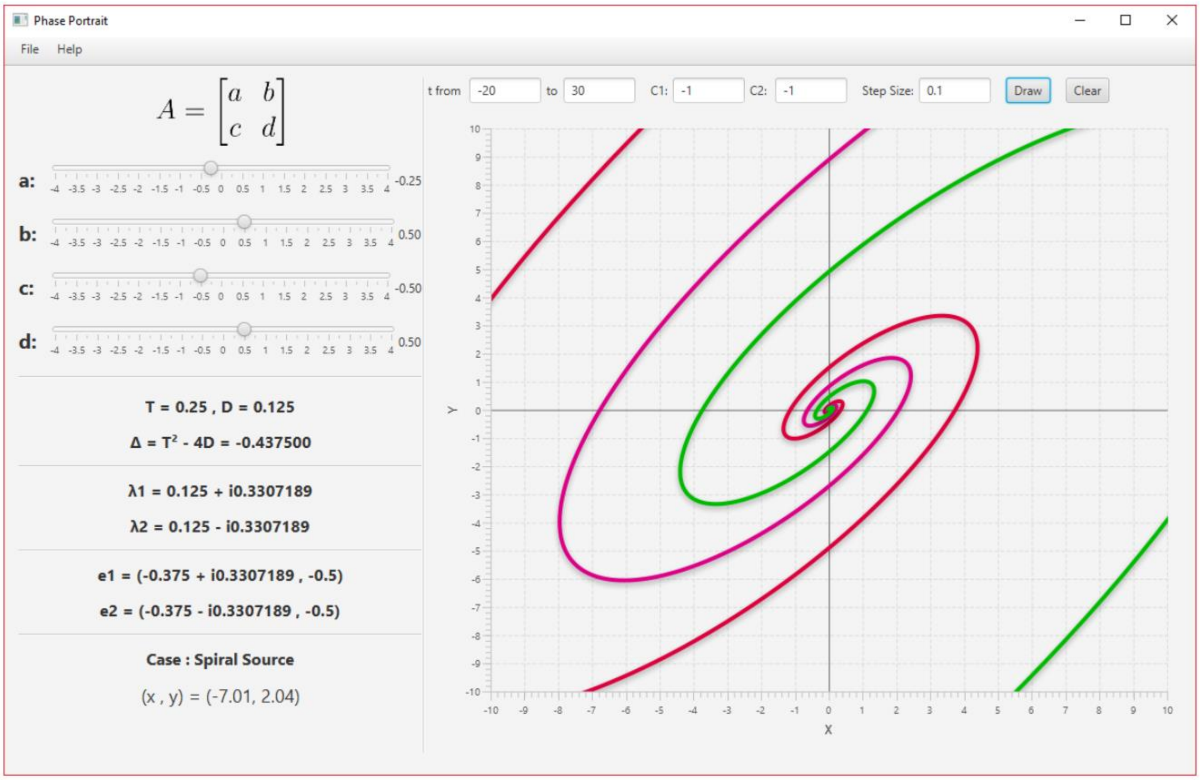
Phase Portrait User Guide



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I. How To Install JRE

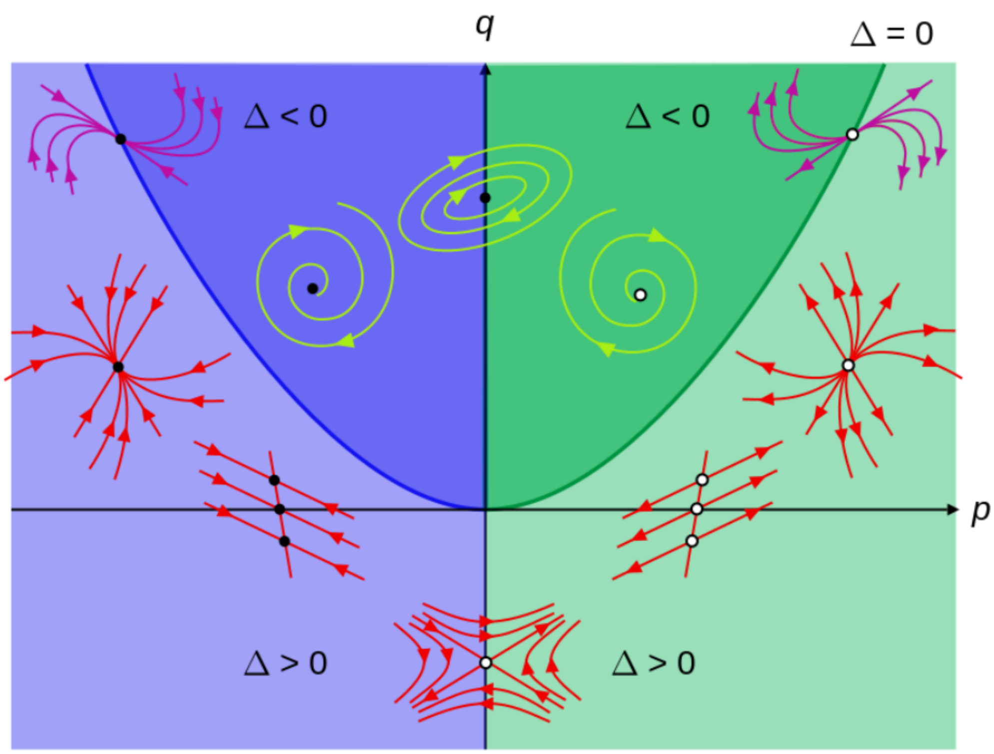
For running this application on your desktop computer, you must install JRE (Java Runtime Environment) on your computer.

1. Follow the following steps to install JRE:
2. download the JRE installer.
3. choose appropriate installer for your operating system.
   Then install JRE.

II. About Application

We use [Drawing Phase Portrait]() of 2-Dimensial Systems of Equations.

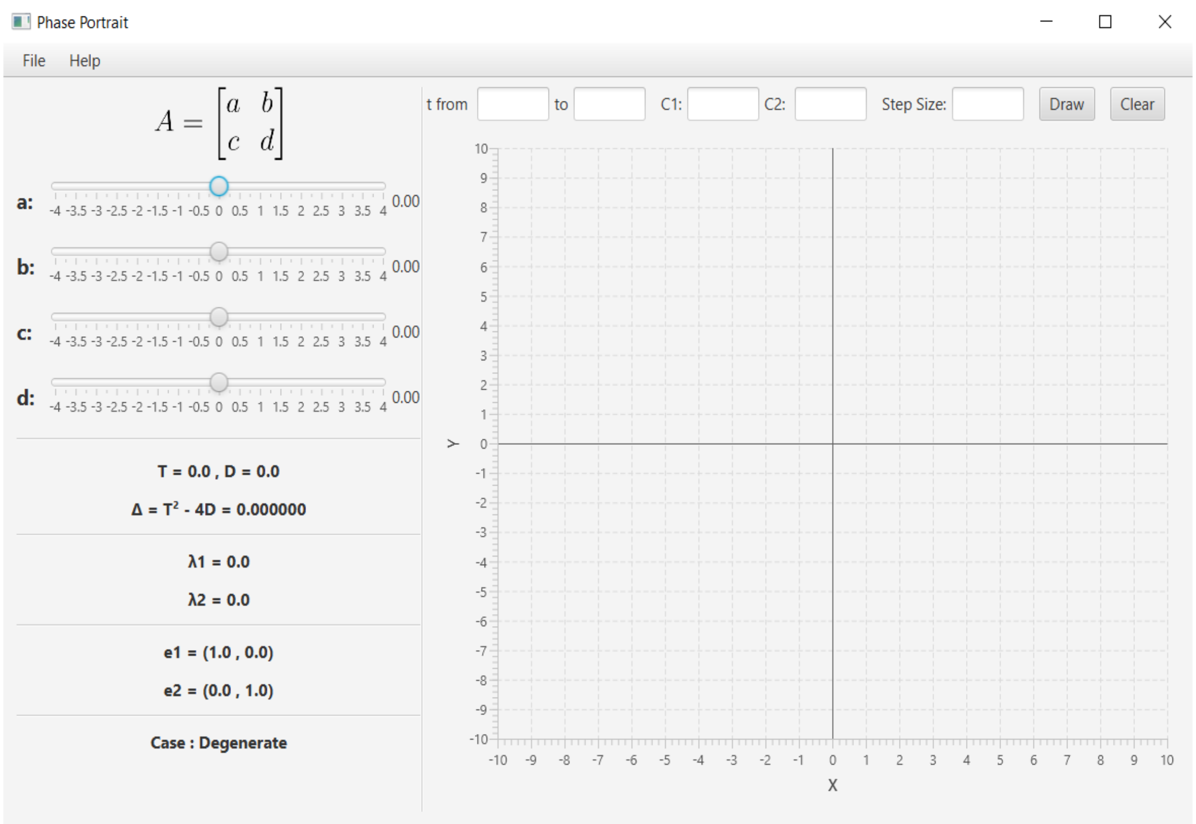
A phase portrait is a plot of multiple phase curves corresponding to different initial conditions in the same phase plane.
The solutions to the differential equation are a family of functions. Graphically, this can be plotted in the phase plane like a two-dimensional vector field. Vectors representing the derivatives of the points with respect to a parameter (say time t), that is $(\frac{dx}{dt}, \frac{dy}{dt})$, at representative points are drawn. With enough of these arrows in place the system behavior over the regions of plane in analysis can be visualized and limit cycles can be easily identified. The entire field is the phase portrait, a particular path taken along a flow line (i.e. a path always tangent to the vectors) is a phase path. The flows in the vector field indicate the time-evolution of the system the differential equation describes.



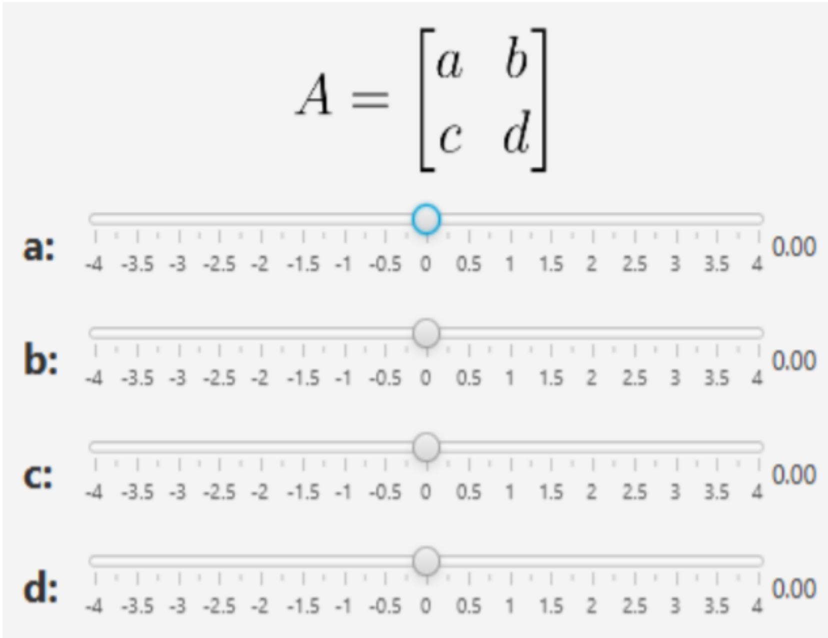
$$\frac{dx}{dt} = Ax + By \qquad p = A + D$$
$$q = AD - BC$$
$$\frac{dy}{dt} = Cx + Dy \qquad \Delta = p^2 - 4q$$

III. How To Use The Application

When you run the application, the following figure will be showed to you on the screen.

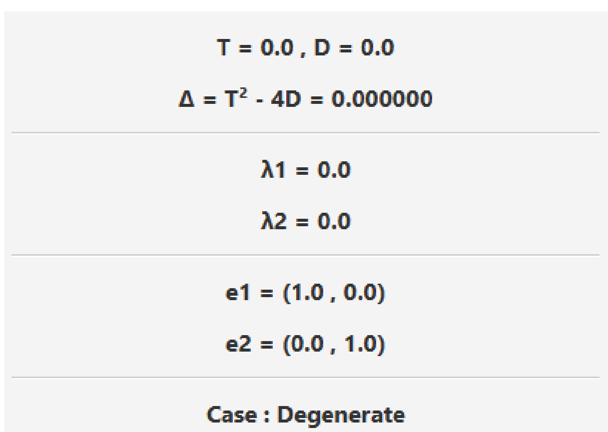


Program takes matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ in $\vec{x'} = A\vec{x}$ by sliders as input.

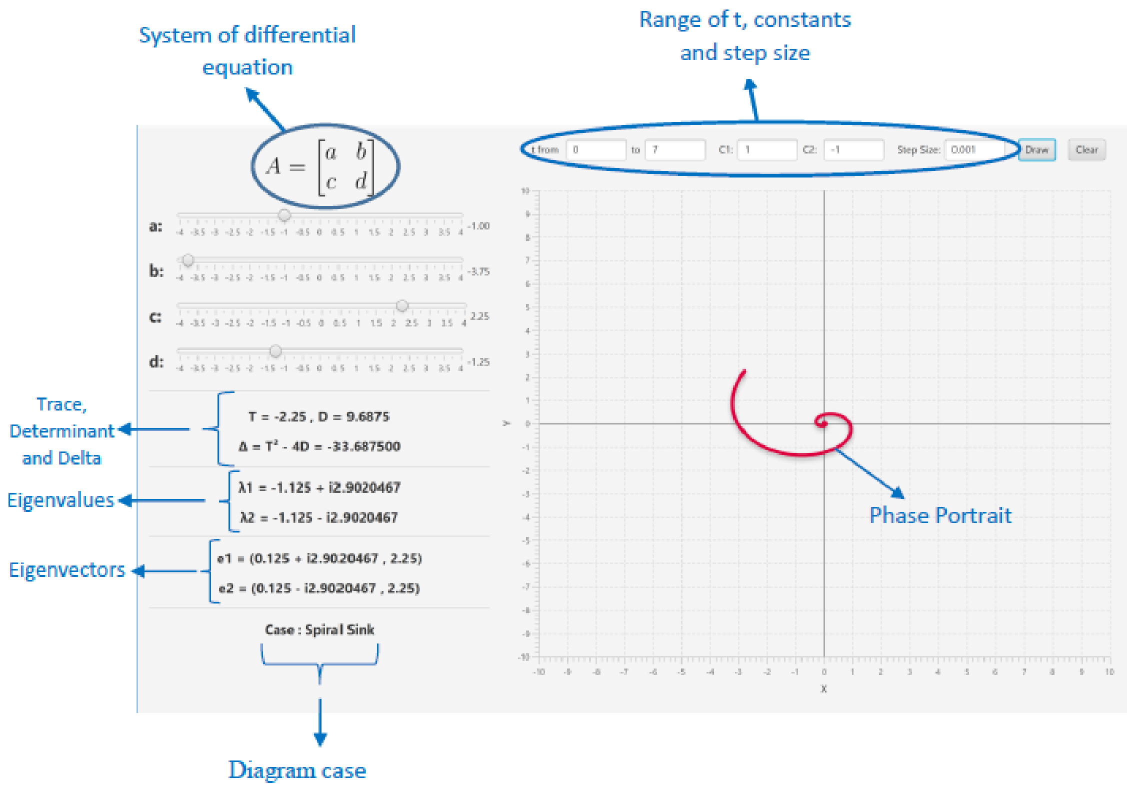


We also determine range of t, constants C1 & C2 and Step size.

By calculating Trace(T) & Determinant(D) of Matrix, we get Δ of $\lambda^2 - T\lambda + D$, then by solving the equation we get Eigenvalues $\lambda 1$ & $\lambda 2$. Having $\lambda 1$ & $\lambda 2$, we get Eigenvectors $e1$ & $e2$ effortlessly.

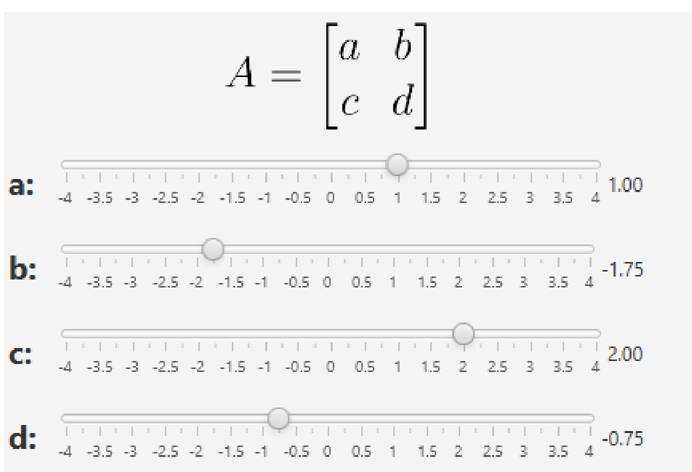


Then by clicking on Draw button, program draws **Phase Portrait** of the system. You can click on Clear button to clear diagrams.



Example

We set the $A = \begin{bmatrix} 1 & -1.75 \\ 2 & -0.75 \end{bmatrix}$



By calculating trace($T = 0.5$) and determinant($D = 2.75$) and delta($\Delta = -10.93$),we get Eigenvalues and Eigenvectors. We can also determine case of **phase portrait**.

T = 0.25 , D = 2.75

$\Delta = T^2 - 4D = -10.937500$

λ1 = 0.125 + i1.6535946

λ2 = 0.125 - i1.6535946

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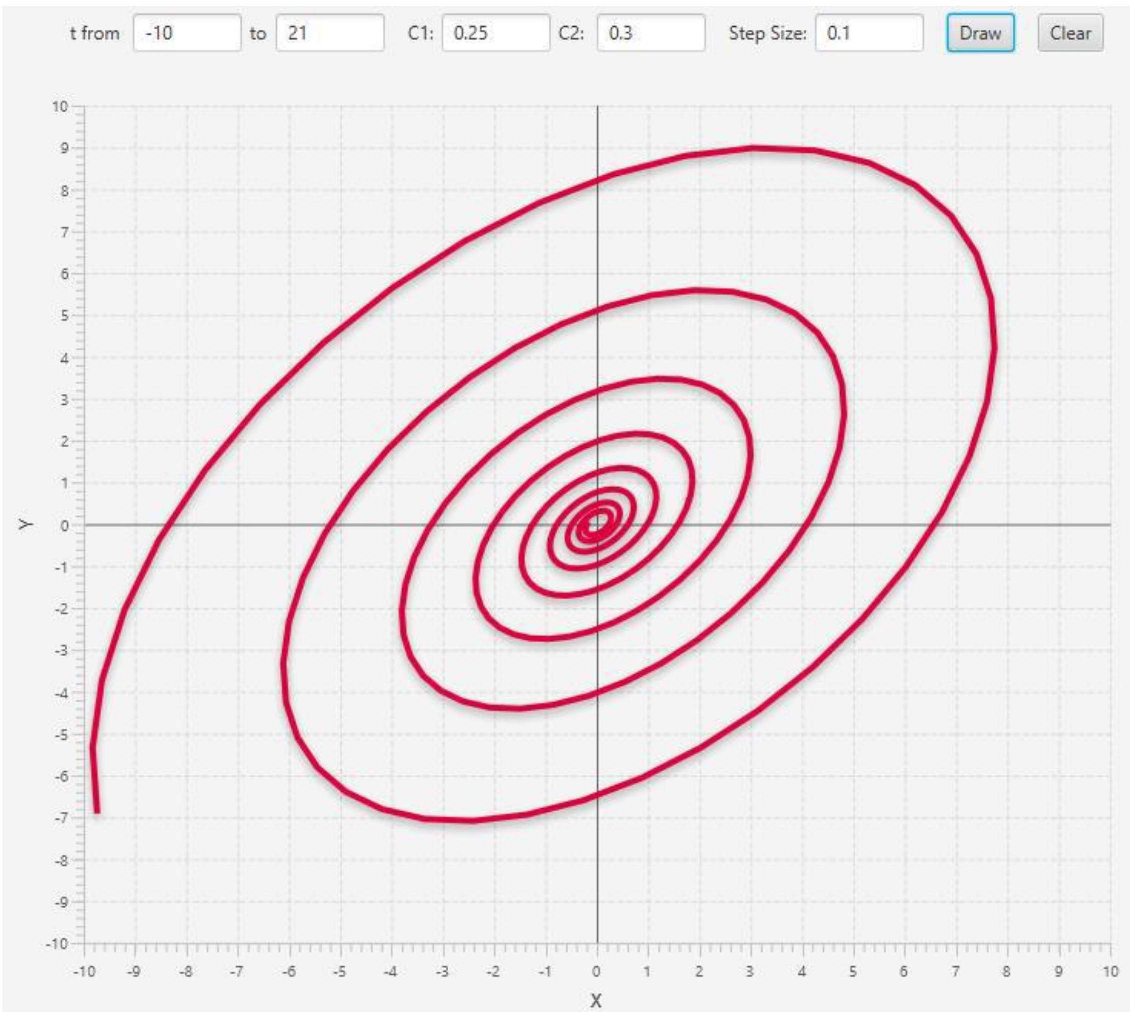
e1 = (0.875 + i1.6535946 , 2.0)

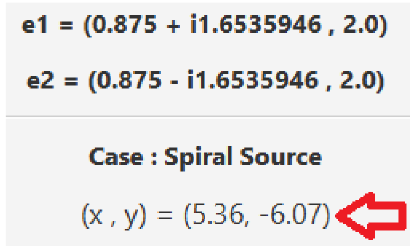
e2 = (0.875 - i1.6535946 , 2.0)

Case : Spiral Source

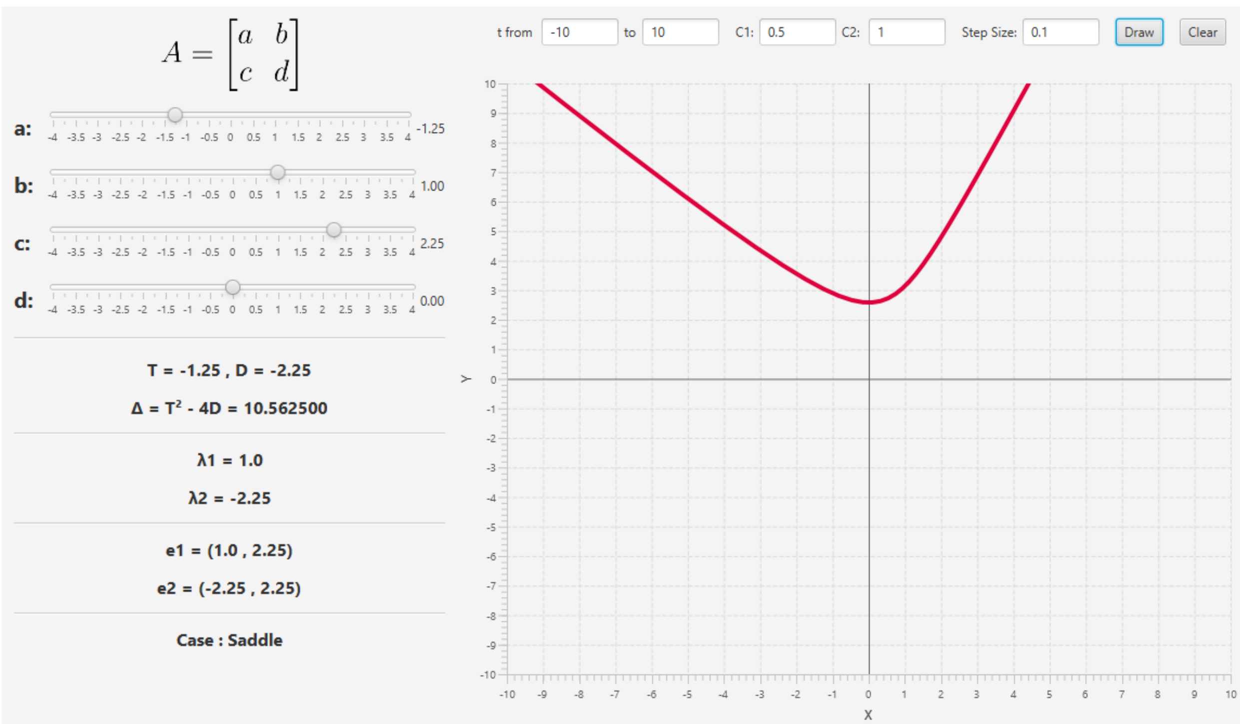
After determining range of t, C1 & C2 and step size click on Draw button to draw **Phase Portrait**.

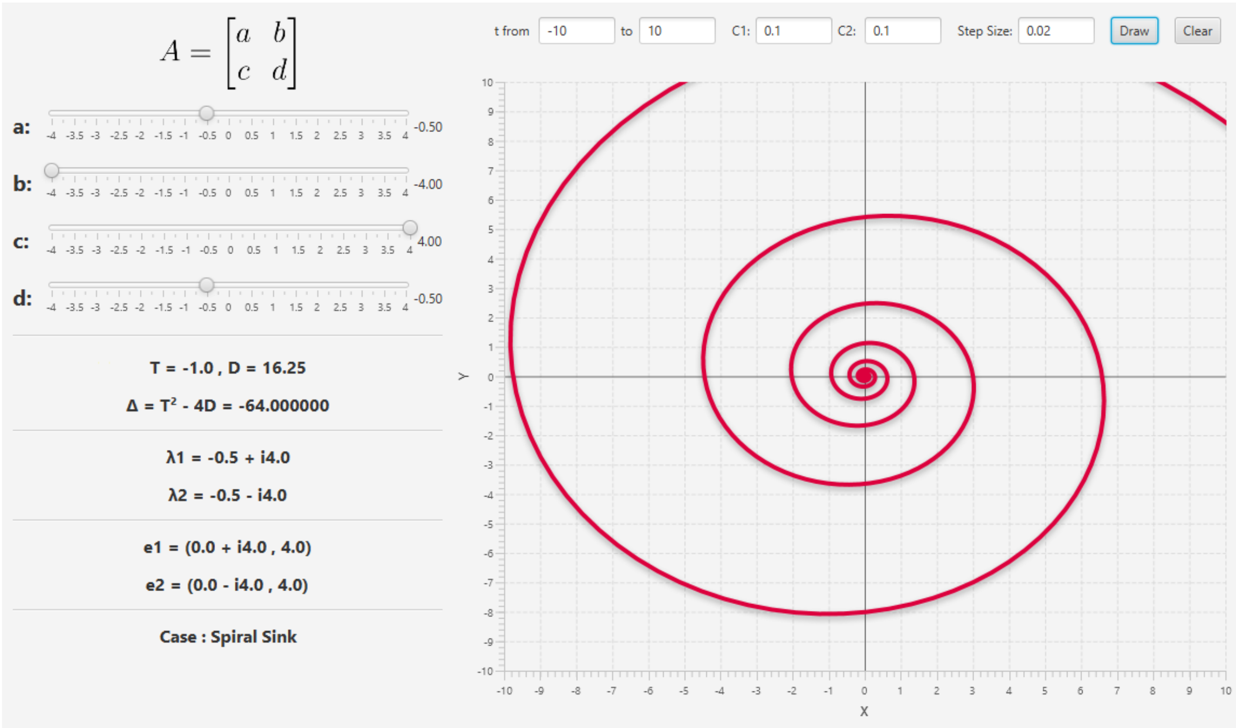


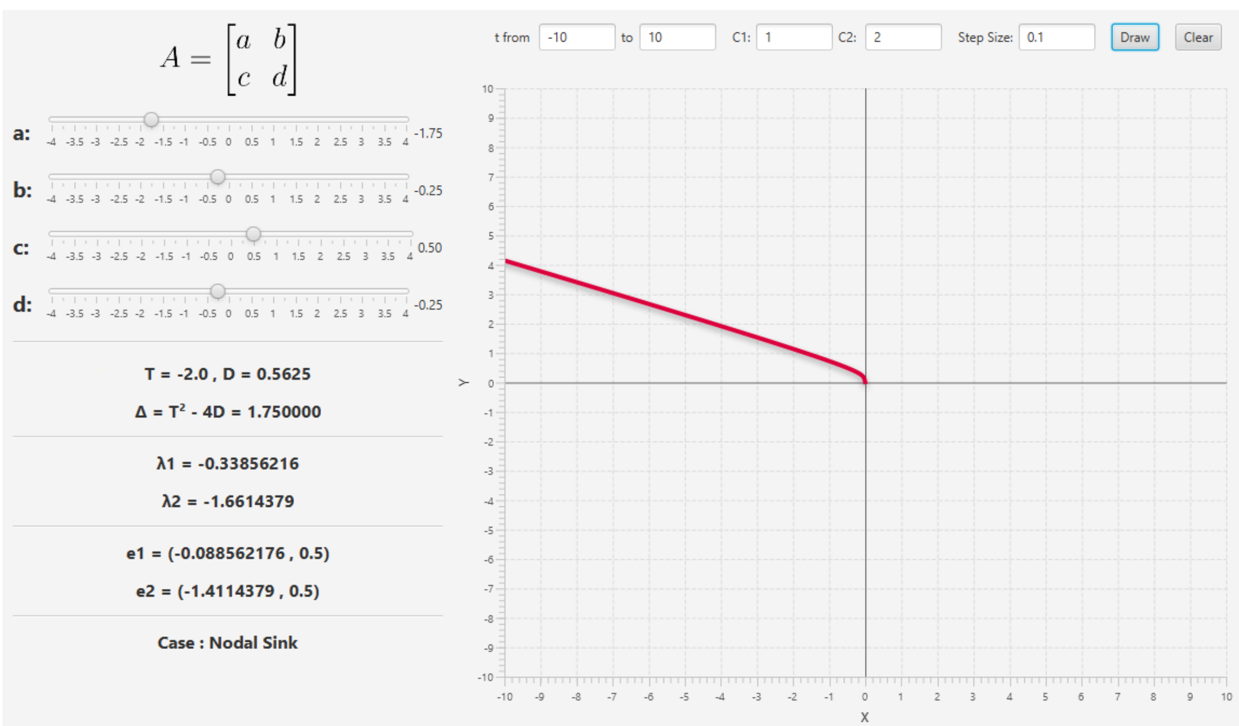
Tip: for more precision, pay attention to coordinates on the left panel.

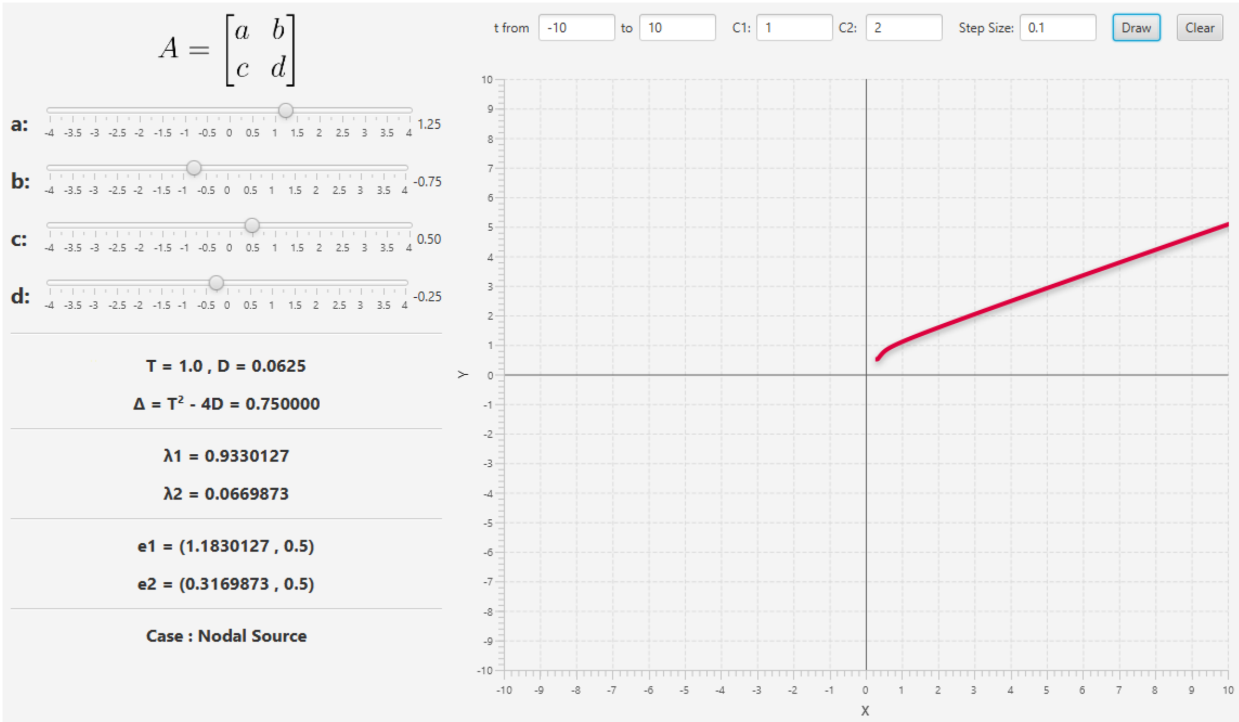


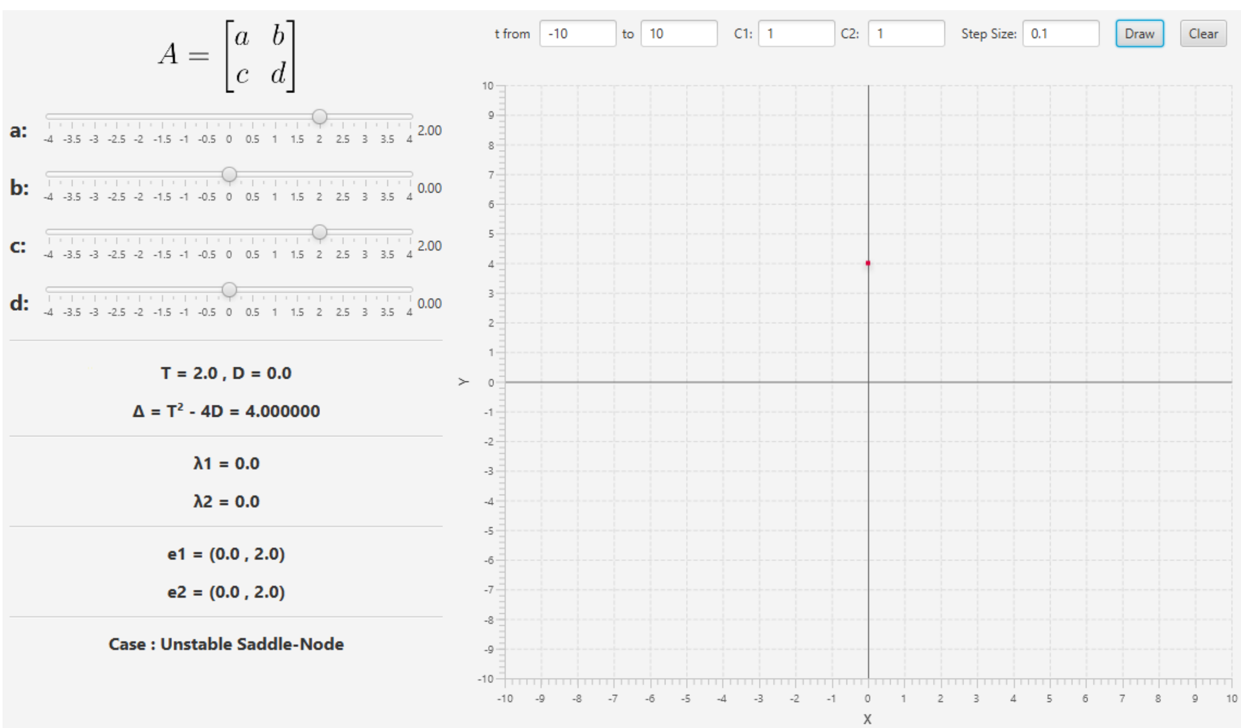
More Examples

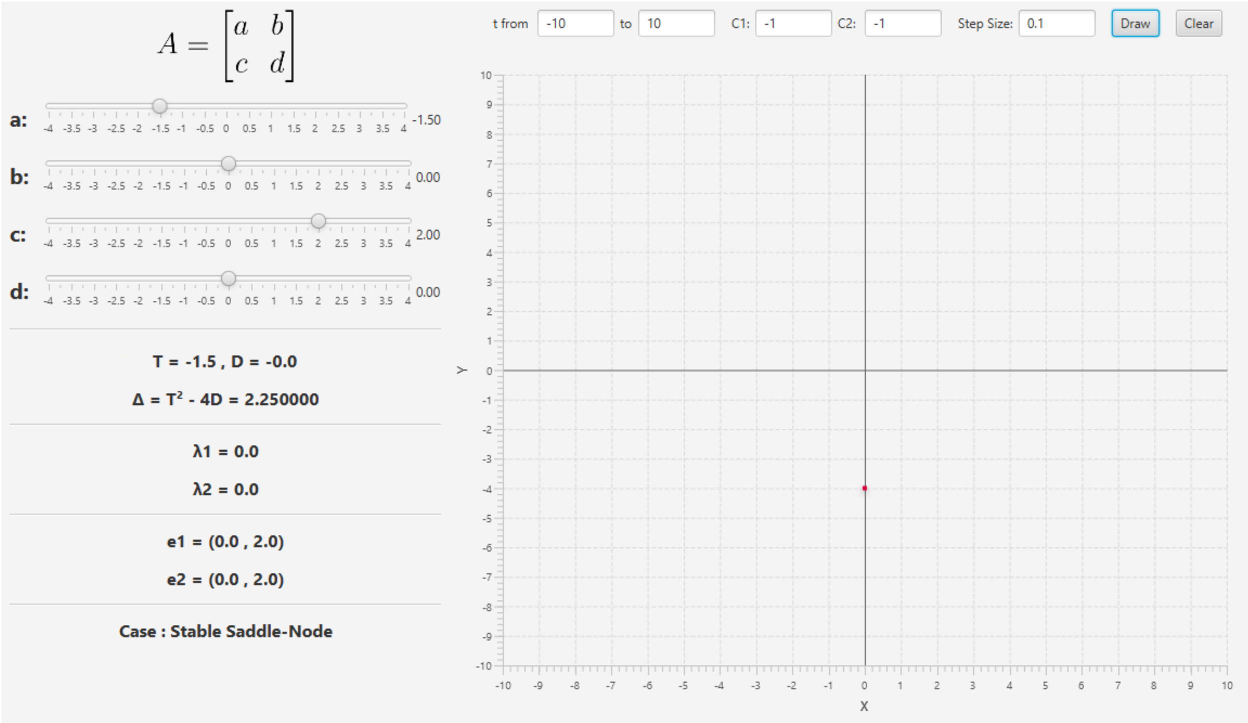



$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
a: -0.50
b: -4.00
c: 4.00
d: -0.50
T = -1.0 , D = 16.25
Δ = T² - 4D = -64.000000
λ1 = -0.5 + i4.0
λ2 = -0.5 - i4.0
e1 = (0.0 + i4.0 , 4.0)
e2 = (0.0 - i4.0 , 4.0)
Case : Spiral Sink
t from -10 to 10 C1: 0.1 C2: 0.1 Step Size: 0.02 Draw Clear
Y
X


$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
a: -1.75
b: -0.25
c: 0.50
d: -0.25
T = -2.0 , D = 0.5625
Δ = T² - 4D = 1.750000
λ1 = -0.33856216
λ2 = -1.6614379
e1 = (-0.088562176 , 0.5)
e2 = (-1.4114379 , 0.5)
Case : Nodal Sink
t from -10 to 10 C1: 1 C2: 2 Step Size: 0.1 Draw Clear
Y
X


$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
a: 1.25
b: -0.75
c: 0.50
d: -0.25
T = 1.0 , D = 0.0625
Δ = T² - 4D = 0.750000
λ1 = 0.9330127
λ2 = 0.0669873
e1 = (1.1830127 , 0.5)
e2 = (0.3169873 , 0.5)
Case : Nodal Source
t from -10 to 10 C1: 1 C2: 2 Step Size: 0.1
Draw
Clear
Y
X


$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
a: 2.00
b: 0.00
c: 2.00
d: 0.00
T = 2.0 , D = 0.0
Δ = T² - 4D = 4.000000
λ1 = 0.0
λ2 = 0.0
e1 = (0.0 , 2.0)
e2 = (0.0 , 2.0)
Case : Unstable Saddle-Node
t from -10 to 10 C1: 1 C2: 1 Step Size: 0.1
Draw
Clear
Y
X



IV. About us

User guide written by M. M. Kheirmand, Ramin Vasseghi & Danial Khoshkholg(updated July 15, 2019)

Application developed by Mohammad Hossein Rimaz on 17/6/2016

Under supervision of Dr. Hadi Aliakbarian

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