A Robust Approach to Frequency and Security Constrained Unit Commitment with Virtual Inertia Support from Wind Farms (Supplementary Document)

Mehdi Mahmoudnia, Turaj Amraee, *Senior Member, IEEE*, and Mohammadreza Toulabi

This supplementary document is established to express the linear form of nonlinear terms of the main manuscript.

A. *UC Linearization Part*

In (6a) and (6b) the multiplication of two binary variables $U_{j,h}$ and $U_{j,h-1}$ is a nonlinear term. Constraints including (6a) and (6b) are linearized as follows:

$$\zeta_{j,h} \leq U_{j,h} \qquad \forall j \in \Omega_G, h \in \Omega_T \tag{S-1}$$

$$\zeta_{j,h} \leq U_{j,h-1} \qquad \forall j \in \Omega_G, h \in \Omega_T \tag{S-2}$$

$$\zeta_{j,h} \geq U_{j,h} + U_{j,h-1} - 1 \qquad \forall j \in \Omega_G, h \in \Omega_T \tag{S-3}$$

$$P_{j,s,h} - P_{j,s,h-1} \leq (U_{j,h} - \zeta_{j,h}) \times P_j^{min} + RU_j \times (\zeta_{j,h} - U_{j,h}) \quad \forall j \in \Omega_G, h > 1, s \in \Omega_s \tag{S-4}$$

$$P_{j,s,h-1} - P_{j,s,h} \leq (U_{j,h-1} - \zeta_{j,h}) \times P_j^{min} + RD_j \times (\zeta_{j,h} - U_{j,h-1}) \quad \forall j \in \Omega_G, h > 1, s \in \Omega_s \tag{S-5}$$

B. *MAFR Linearization Part*

The swing equation contains a binary variable that has made it nonlinear. In this section, the multiplication of binary and continuous variables needs to be linearized. By placing the (9a) and (9b) in (10b), (10c) and by multiplying both sides in $\sum_{j\in\Omega_G} S_j \times U^F_{j,h}$ the following relations are obtained.

$$\sum_{j\in\Omega_G} 2 \times B_{h,m} \times H_j \times S_j \times U^F_{j,h} = \sum_{j\in\Omega_G} \Delta P^{gov}_{h,m} \times S_j \times U^F_{j,h} - \frac{D}{3} \times \Delta f_{h,m} \times S_j \times U^F_{j,h} + \Delta P^{wind}_{h,m} - \Delta P_h \tag{S-6}$$

$$\sum_{j\in\Omega^{G_0}} -\frac{\Delta t}{T_g} \times \Delta f_{h,m-1} \times \frac{S_j \times U^F_{j,h}}{R_j} = \sum_{j\in\Omega_G} \Delta P^{gov}_{h,m} \times S_j \times U^F_{j,h} + (\frac{\Delta t}{T_g} - 1) \times \Delta P^{gov}_{h,m-1} \times S_j \times U^F_{j,h} \tag{S-7}$$

By using the auxiliary variable, the above equations are rewritten as below.

$$\sum_{j\in\Omega^G} B^b_{j,h,m} = \sum_{j\in\Omega_G} B^d_{j,h,m} - B^c_{j,h,m} + \Delta P^{wind}_{h,m} - \Delta P_h \tag{S-8}$$

$$\sum_{j\in\Omega^{G_0}} B^a_{j,h,m} = \sum_{j\in\Omega_G} B^d_{j,h,m} - (\frac{\Delta t}{T_g} - 1) \times B^d_{j,h,m-1} \tag{S-9}$$

Auxiliary variables are also defined as follows.

$$-M \times U^F_{q,j,h} \leq B^b_{j,h,m} \leq M \times U^F_{q,j,h} \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-10}$$

$$-M \times (1 - U^F_{j,h}) \leq B^b_{j,h,m} - (\sum_{j\in\Omega_G} 2 \times B_{h,m} \times H_j \times S_j) \leq M \times (1 - U^F_{j,h}) \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-11}$$

$$-M \times U^F_{j,h} \leq B^c_{j,h,m} \leq M \times U^F_{j,h} \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-12}$$

$$-M \times (1 - U^F_{j,h}) \leq B^c_{j,h,m} - (\frac{D}{3} \times \Delta f_{h,m} \times S_j) \leq M \times (1 - U^F_{j,h}) \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-13}$$

$$-M \times U^F_{j,h} \leq B^d_{j,h,m} \leq M \times U^F_{j,h} \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-14}$$

$$-M \times (1 - U^F_{j,h}) \leq B^d_{j,h,m} - (\Delta P^{gov}_{h,m} \times S_j) \leq M \times (1 - U^F_{j,h}) \tag{S-15}$$

$$-M \times U^F_{j,h} \leq B^a_{j,h,m} \leq M \times U^F_{j,h} \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-16}$$

$$-M \times (1 - U^F_{j,h}) \leq B^a_{j,h,m} - (-\frac{\Delta t}{T_g} \times \Delta f_{h,m-1} \times \frac{S_j}{R_j}) \leq M \times (1 - U^F_{j,h}) \quad \forall j \in \Omega_G, h \in \Omega_T, m \in \Omega_m \tag{S-17}$$

Like previous part, by placing (9b) in the (16h) and and by multiplying both sides in $\sum_{j\in\Omega_G} S_j \times U^F_{j,h}$ the following relations are obtained.

$$\Delta P_h + D \times \sum_{j\in\Omega_G} \Delta f^{ss}_h \times S_j \times U^f_{j,h} = -\sum_{j\in\Omega^{Go}} \Delta f^{ss}_h \frac{S_j \times U^f_{j,h}}{R_j} \tag{S-18}$$

Now, by using the auxiliary variable to linearize the non-linear coefficients, the above equations are rewritten as below.

$$\Delta P_h + D \times \sum_{j\in\Omega_G} B^e_{j,h} = -\sum_{j\in\Omega^{Go}} B^f_{j,h} \tag{S-19}$$

Auxiliary variables for linearization are also defined as follows.

$$-M \times U^{F}_{j,h} \leq B^{e}_{j,h} \leq M \times U^{F}_{j,h} \quad \forall j \in \Omega_G, h \in \Omega_T \tag{S-20}$$

$$-M \times (1 - U^{F}_{j,h}) \leq B^{e}_{j,h} - (\Delta f^{ss}_{h} \times S_j) \leq M \times (1 - U^{F}_{j,h}) \quad \forall j \in \Omega_G, h \in \Omega_T \tag{S-21}$$

$$-M \times U^{F}_{j,h} \leq B^{f}_{j,h} \leq M \times U^{F}_{j,h} \quad \forall j \in \Omega_G, h \in \Omega_T \tag{S-22}$$

$$-M \times (1 - U^{F}_{j,h}) \leq B^{f}_{j,h} - (\Delta f^{ss}_{h} \times \frac{S_j}{R_j}) \leq M \times (1 - U^{F}_{j,h}) \quad \forall j \in \Omega_G, h \in \Omega_T \tag{S-23}$$

C. WT Linearization Part

The linearization of (11)-(12) is done using a PWL and per-unit form as below:

$$\omega_{h,m} = \Psi^{\omega}_{min} + \sum_{kl=1}^{N_{kl}} \omega^{lin}_{h,m,kl} \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-24}$$

$$u^{\omega}_{h,m,kl+1} \times \mu^{\omega} \leq \omega^{lin}_{h,m,kl} \leq u^{\omega}_{h,m,kl} \times \mu^{\omega} \quad \forall h \in \Omega_T, m \in \Omega_m, kl \in \Omega_{kl} \tag{S-25}$$

$$P^{t}_{h,m} = \Psi^{P_t}_{0} + \sum_{kl=1}^{N_{kl}} (SK^{torqe}_{h,kl} \omega^{lin}_{h,m,kl}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-26}$$

$$P^{MPPT}_{h,m} = \Psi^{P^{MPPT}}_{min} + \sum_{kl=1}^{N_{kl}} (SK^{gen}_{kl} \omega^{lin}_{h,m,kl}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-27}$$

D. Virtual Inertia Controller Linearization Part

Equations of (16) and (17a) are linearized by using logarithms. According to the property of the logarithm, the (S-28) are obtained.

$$log(K^{VIC}_{h,m}) = 3(log(\omega_{ro}) - log(AUX_{h,m})) + log(k_{opt}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-28}$$

$$AUX_{h,m} = \omega_{ro} + 2\pi\gamma\Delta f^{w}_{h,m} \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-29}$$

The logarithmic model of (S-28) is completely linear and it is required to linearize just the logarithmic and exponential functions. For this purpose, PWL is used for both functions. Equation of (17a) is also linearized using the linearization technique around the WP as (S-30).

$$P^{VIC}_{h,m} - P^{MPPT^-}_{h} = P^{MPPT^-}_{h}(\frac{K^{VIC}_{h,m}}{K_{opt}} - 1) + (P^{MPPT}_{h,m} - P^{MPPT^-}_{h}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-30}$$

The linearized model of the virtual inertia controller is expressed as (S-31)-(S-33).

$$log(K^{VIC}_{h,m}) = \Psi^{log}_{min} + \sum_{sl=1}^{SL} K^{VIC-log}_{h,m,sl} \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-31}$$

$$u^{exp(K)}_{h,m,sl+1} \times \mu^{exp} \leq K^{VIC-log}_{h,m,sl} \leq u^{exp(K)}_{h,m,sl} \times \mu^{exp} \quad \forall h \in \Omega_T, m \in \Omega_n, sl \in \Omega_{sl} \tag{S-32}$$

$$K^{VIC}_{h,m} = \Psi^{exp}_{min} + \sum_{sl=1}^{SL} (SK^{exp}_{sl} \times K^{VIC-log}_{h,m,sl}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-33}$$

Similarly, linearized logarithm of the other auxiliary variables is obtained.

$$AUX_{h,m} = 1 + \sum_{sd=1}^{SD} AUX^{log}_{h,m,sd} \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-34}$$

$$u^{log(AUX)}_{h,m,sd+1} \times \mu^{log} \leq AUX^{log}_{h,m,sd} \leq u^{log(AUX)}_{h,m,sd} \times \mu^{log} \quad \forall h \in \Omega_T, m \in \Omega_n, sd \in \Omega_{sd} \tag{S-35}$$

$$log^{AUX}_{h,m} = \Psi^{log}_{min} + \sum_{sd=1}^{SD} (SK^{log}_{sd} \times AUX^{log}_{h,m,sd}) \quad \forall h \in \Omega_T, m \in \Omega_m \tag{S-36}$$