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A non-cooperative game approach for power-aware MAC in ad hoc wireless networks

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ABSTRACT

In this paper, the problem of designing a power-aware medium access control (MAC) algorithm for Ad hoc wireless networks is considered. Based on the insights obtained from analyzing the problem in optimization framework, we formulate it as a random scheduling MAC in the game theory framework. Defining a payoff for each link as a function of its persistence probability and power, the objective of the proposed non-cooperative static power-aware MAC game (PAMG) is to find the appropriate strategy for the link in its 2D strategy space. The game theoretic aspects of PAMG including existence, uniqueness, and convergence to the Nash equilibrium are investigated analytically under some mild conditions. Based on PAMG, a message passing totally asynchronous distributed power-aware MAC (PAM) algorithm is presented. In the proposed algorithm, at each active time slot the link broadcasts a message simultaneous to its transmission. At each inactive time slot it listens to the channel to capture the other active links messages and updates its cost factor. Simulation results are provided to evaluate the convergence and performance of the algorithm and are compared to the optimal solution.

1. Introduction

Random multiple access techniques like slotted ALOHA have been widely recognized as suitable candidates to deploy in the medium access control (MAC) layer of wireless Ad Hoc and sensor networks, in which there is no pre-established infrastructure [1]. Extensive research have been performed to analyze and improve these techniques, like the efforts have been made to improve the distributed coordination function (DCF) mode of the IEEE 802.11 standard [2,3]. The goal of the MAC layer design in Ad Hoc networks, is that the persistence probability (or back-off window size) of each active transmitter is adapted according to the changes in network topology and load. On the other hand, improving the throughput of the random multiple access method used in a MAC layer protocol that takes advantage from multi-packet reception is closely related to the power control scheme at the physical layer (PHY layer). The performance improvement of random multiple access techniques that take into account the power control at the PHY layer, has been reported and discussed in [4–6] with different capture models.

1.1. Backgrounds

In a random multiple access wireless network, medium access control based on joint consideration of power and persistence probability, which is refereed to as power-aware MAC (PAM), is important from the standpoints of both information theory and network protocol stack design.

In the former, the focus is to characterize the network capacity in the MAC layer [7] and in the latter, as it is the subject of this paper, the objective is to design an appropriate and efficient MAC layer for wireless networks [4–6]. Specifically, an appropriate PAM for ad hoc wireless networks is one that can be implemented as a totally asynchronous distributed algorithm between network links as well as being efficient. In a totally asynchronous distributed algorithm, large communication delays between nodes can be tolerated [8]. These delays are unavoidable in a random multiple access environment since packets are exposed to collisions. These communications are typically required between a link transmitter and its corresponding receiver or between different links in a message passing environment. An efficient PAM, on the other hand, is one that can minimize the destructive interference effects that each link experienced from other links.

Therefore, the objective of the PAM design is to develop an algorithm that (1) can be implemented as a totally asynchronous distributed algorithm between network links; (2) maximizes the attainable link's data rate by simultaneous adjusting of links' persistence probabilities and powers.

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Game theory provides a nice mathematical framework for analyzing communication networks, where the network agents must work in a distributed fashion [9]. This framework is extensively applied for analyzing different aspects of communication networks in recent years. Specifically, the PAM problem as a non-cooperative static game can be stated as follows. Each active link is considered as a player that should decide on his persistence probability and power, according to his location in the network and the possible received feedback from the channel. The decision at each link is based on the maximization of a payoff function that is defined on the 2D strategy space, i.e., persistence probability and power. The value of this function indicates the link's payoff from the medium. The aim is to find an efficient equilibrium point when each link selfishly maximizes its own payoff.

Obviously, the link utility from the medium is maximized when it sends at all time slots and with the maximum allowable power. However, this selfish behavior leads to excessive mutual interference and degrades the network performance. Therefore, in order to maximize the network utility, the payoff function should be defined such that the selfish maximization by each user leads to an efficient use of network resources.

Introducing an appropriate cost function in the users' payoffs is a technique that is used in game theory to encourage them to behave more sociably. The cost function, which is also defined on the 2D strategy space, reflects the cost of using network resources. In this paper, we use this approach to analyze and develop a totally asynchronous distributed PAM algorithm.

1.2. Related works

Using the capture effect to increase the throughput of the multiple access protocols, is a well-known subject that has been received considerable attentions in the study of wireless networks, see [4] and its references. This effect describes a situation in which some simultaneous successful transmissions are possible provided that the power of the received signal at the corresponding receivers is sufficiently greater than that of the lumped interference of the contenders.

Assuming different capture models, the achievable medium throughput are analyzed in [5,6]. The PAM problem is approached by a cross layer method in [10], where its coupling to the transport layer is considered. Their extracted PAM algorithm, uses message passing between network links that can be implemented as a partially asynchronous distributed algorithm. In a partially asynchronous algorithm, certain bounds are required on the amount of asynchronism that may exist between concurrent local computations at agents due to the message passing. Since the messages are themselves exposed to collision in the random access environment, designing a totally asynchronous PAM algorithm is an interesting task. Game theory provides appropriate tools for the analysis of this type of algorithms. Specifically, it is employed in the analysis of both random access schemes and power control in wireless ad hoc networks.

The Aloha protocol from the perspective of a selfish user is analyzed in [11] by introducing the Aloha game. The Aloha game is a stochastic game in which the number of links who wish to transmit is the state of the game, and each time slot is a stage of it. At each stage, each player decides for transmitting or waiting, and depending on whether his transmission is successful, receives a specified payoff. In the Aloha game, delay and energy parameters are considered in the cost function. The stability of multi-packet slotted Aloha is analyzed in [12] using game theory. In [13] the back off-based random access MAC protocol is reversed engineered and it is shown that the contention resolution algorithm can be considered as a non-cooperative game. The extracted utility function has two terms. The first term is the expected reward that the link obtains by transmitting with a specified persistence probability and the second one is the cost for failure transmission.

Power control as a non-cooperative game in cellular networks is analyzed in [14] by defining an appropriate utility function for each user. This utility function is modified in [15] by introducing a pricing function to improve the network performance. Power control in multi-carrier CDMA systems is also modeled as a game in [16] to determine how much power should be assigned to each carrier to maximize the defined utility. In [17] a Nash game algorithm for SIR-based power control in wireless CDMA networks is developed. Using an appropriate cost function, the game leads to power saving compared to the power balancing algorithms, while the achieved SIR is reduced slightly. In [18] a price based distributed and asynchronous power control algorithm for ad hoc wireless networks is developed and analyzed based on supermodular game theory. A distributed game theoretic algorithm is presented in [19] to jointly solve the rate and power control where the problem is modeled as two distinct games. The first game adjusts the rates and the second one controls the powers to achieve those rates. The Game theory framework is also used to analyze and enhance the performance of WLANs MAC protocols. In [20] an interference-aware channel access game is proposed to mitigate the inter-cluster interference of wireless LANS. Based on this game, a decentralized transmission strategy is extracted that considers network throughput and battery consumption at each transmission. The data rate and energy consumption of IEEE 802.11 WLANs are also studied in the game theory framework in [21].

1.3. Motivation, results, and paper organization

To the best of our knowledge, while both persistence probability control and power control for random access wireless ad hoc networks are separately investigated in the game theory framework, as we reviewed in the previous subsection, they are not considered jointly as a PAM game. The main challenge here is that in the 2D strategy space, analyzing the game and specifically proving the uniqueness and efficiency of the game equilibrium – if any one exists – are more complicated compared to 1D strategy space.

The early result of this paper shows that there exists a random scheduling scheme that in conjunction with power control can guarantee an average threshold SINR for all active links in any ad hoc network topology. Then, to find an efficient scheme for persistence probability and power adjustment, the problem is formulated and analyzed in the optimization theory framework. In fact, we use optimization theory to get the required insights to define the utility and cost functions. Game theory is then used to analyze the distributed and asynchronous implementation aspects. In addition, the optimal solution is used as a reference to evaluate the efficiency loss of the designed game. Following this approach, the non-cooperative static power-aware MAC game (PAMG) is presented and its convergence to the unique Nash equilibrium is investigated. Finally, based on PAMG a distributed and totally asynchronous message passing PAM algorithm is designed.

The rest of this paper is organized as follows. The system model and notations are presented in Section 2. In Section 3, the problem statement and challenges in the game theory framework as well as some backgrounds on power control and random scheduling are provided. The game and its analysis are presented in Section 4. In Section 5, the PAM algorithm is presented and evaluated on a sample random network topology. The efficiency loss of PAMG and the possible extensions of it are discussed in Section 6. In Section 7, the network implementation issues and the related simulation results that take into account the practical implementation issues are presented. The conclusion is provided in Section 8.

2. System model

From the MAC layer point of view, a wireless ad hoc network involves N transmitter–receiver pairs in which each transmitter attempts to communicate to its corresponding receiver through a single hop transmission. The transmitter nodes are distributed randomly and uniformly in a square area of size $L \times L$. The corresponding receiver of each transmitter is assumed to be located randomly in a circle around it of radius R, $R \leqslant L$. The parameter R is used to model the operation of the routing layer to find the next neighbor node in an end to end scenario for a real multihop wireless network. The link gain and the distance between the transmitter of link j to the receiver of link l are denoted by G_{lj} and d_{lj}, respectively; where $G_{lj} = \left(\frac{d_0}{d_{lj}}\right)^{\alpha}$, d_0 is the reference distance, and α is the path loss exponent [22]. All distance values are assumed to exceed a minimum threshold of unity, i.e., $d_{lj}^{min} = 1$. It is assumed that the system is time slotted and code division multiple access (CDMA) is deployed at the physical layer. The spreading gain effect is reflected in the links' gains.

Dedicated to each link l, there is a persistence probability q_l, $0 < q_l \leqslant 1$, representing the rate at which the link persists to access the channel, and transmission power $p_l, p_{min} \leqslant p_l \leqslant p_{max}$. $\mathbf{q} = (q_1, q_2, \ldots, q_N)$ and $\mathbf{p} = (p_1, p_2, \ldots, p_N)$ are the vectors of all links' persistence probabilities and powers. In addition, the following notations are used for vectors throughout the paper. For vectors $\mathbf{a}, \mathbf{b}$ and scalar c, the notations $\mathbf{a} \geqslant c$ and $\mathbf{a} \geqslant \mathbf{b}$ indicate that all entries of $\mathbf{a}$ are greater than or equal to c and $\mathbf{a} - \mathbf{b} \geqslant 0$, respectively.

Since the received interference at the receiver of link l in each access to the channel is stochastic, the instantaneous SINR is also stochastic. Let $\mathbf{1}_l(t)$ be an indicator function taking the value of one when the transmitter of link l attempts to access the channel in time slot t. The conditional expectation of SINR at receiver l is given by [10]:

$$\overline{\gamma_l} = \mathbb{E}(\gamma_l | \mathbf{1}_l(t) = 1) \geqslant \frac{G_{ll} p_l}{\sum_{k \neq l} G_{lk} p_k q_k + \eta} \tag{1}$$

where η is the background noise. We use the right-hand side of the inequality in (1) as the achievable average link SINR in each access to the channel. The attainable data rate of link l in a high SINR regime is $c_l = \log(\gamma_l)$ where for its average, denoted by $\overline{c_l}$, we have: $\overline{c_l} \leqslant \log(\overline{\gamma_l})$ [20]. We use $\overline{\gamma_l}$ as a measure of the achievable average link data rate.

3. Problem statement and background

3.1. PAM game problem statement

The objective of a PAM algorithm is to find the links' persistence probabilities and powers in each time slot. Let $\mathcal{G} = [\mathcal{N}, \{Q_j, P_j\}, \{u_j(\cdot)\}]$ denote the non-cooperative static PAM game (PAMG), i.e., a game in which all players make decisions independently and without knowledge of other players strategies; where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the index of active links in the network, $Q_j = [0, 1]$ and $P_j = [p_{min}, p_{max}]$ are the strategy space of link j. The MAC layer payoff function of link j is denoted by $u_j(q_j, p_j, \mathbf{q}_{-j}, \mathbf{p}_{-j})$. This function indicates the link satisfaction when the transmission in link j is accomplished with persistence probability q_j and power p_j; $\mathbf{q}_{-j}$ and $\mathbf{p}_{-j}$ denote the vectors of persistence probabilities and powers of all network links excluding link j. The PAMG can be expressed as

$$PAMG \quad \max_{q_j \in Q_j, p_j \in P_j} u_j(q_j, p_j, \mathbf{q}_{-j}, \mathbf{p}_{-j}), \quad \forall j \in \mathcal{N} \tag{2}$$

According to the network requirements at the MAC layer, different games could be designed by defining appropriate payoff functions. In fact, the contention between links to seize the medium resources in time and space should be reflected in the links' payoff functions. The link can utilize from time and space resources by adjusting its persistence probability and power, respectively.

From the standpoint of game theory, the existence and uniqueness of the designed game is crucial. On the other hand, as a MAC layer protocol, some other aspects should be considered in the utility function including:

- The payoff function should be such that the designed game converges to an efficient equilibrium point.
- Since in a random access scheme all packets are exposed to collisions, the asynchronous convergence of the extracted algorithm is necessary if the receiver feedback to the transmitter is required or if the algorithm uses some kind of message passing between network links.
- The fairness problem at the MAC layer should be considered in the utility function.
- A cost function should be included in the payoff function for efficient use of network resources. The dependency of the cost function to the link location in the network is also important for MAC layer design in Ad hoc wireless networks.
- The interactions with other layers and especially physical layer should be considered in the payoff function. For example, the capture model and the ability of adaptive rate transmission depend on the physical layer.

3.2. Random scheduling and power control

Assume that the persistence probabilities of links are known and fixed. Therefore, the average interference on the receiver of each link depends on the links' powers. To guarantee a minimum average SINR threshold for each link, one may use the Foschini–Miljanic algorithm to adjust the powers [23]. In each iteration of this algorithm, power is adjusted to hit the SINR threshold using the channel feedback in the previous time slot. Provided that there exists a solution, this algorithm does converge asynchronously [24]. Therefore, to apply this algorithm, we should ensure the existence of the solution. This is done in [25] using a centralized scheme for a time division multiple access (TDMA) ad hoc network, i.e., in the first phase the set of links for which the feasible solution exists are extracted and then the Foschini–Miljanic algorithm is applied to adjust their powers.

In a random access scenario, adjusting the persistence probabilities can be considered as a random distributed scheduling scheme that ensures to hit the average SINR threshold for each link. This is shown in the following proposition for any network topology.

Proposition 1. *For any network topology there exists vectors* $\mathbf{q}$, $\mathbf{p}$, $p_{min} \leqslant \mathbf{p} \leqslant p_{max}$, $0 < \mathbf{q} \leqslant 1$ *such that* $\bar{\gamma}_l \geqslant \beta$, $\forall l$, *where* β *is a given threshold SINR.*

Proof. First assume that there is no constraint on the upper value of the powers. We can write the required $\bar{\gamma}_l \geqslant \beta$, $\forall l$ in matrix form as: $(\mathbf{I} - \mathbf{F_q})\mathbf{P} \geqslant \mathbf{u}$, where $F_q = \mathbf{FQ}, \mathbf{Q} = Diag(\mathbf{q}), \mathbf{u}_i = \beta \frac{\eta}{G_{ii}}$, and

$$\mathbf{F}_{ij} = \begin{cases} 0 & i = j \\ \frac{G_{ij}}{G_{ii}} \beta & i \neq j \end{cases} \tag{3}$$

Matrix $\mathbf{F_q}$ has nonnegative elements and it is reasonable to assume that it is irreducible. Therefore, it is sufficient to prove that $\rho(\mathbf{F_q}) < 1$, where ρ is the spectral radius of the matrix [26]. We need to show that:

$$\rho(\mathbf{F}_q) < \|\mathbf{F}\|_2 \|\mathbf{Q}\|_2 < \|\mathbf{F}\|_2 q_i^{max} < 1$$

where $\|.\|_2$ is the Euclidean norm and q_i^{max} is the maximum element of the vector $\mathbf{q}$. Since $\|\mathbf{F}\|_2 > 0$, it is sufficient to choose $\mathbf{q}$ such that $0 < q_i^{max} < \frac{1}{\|\mathbf{F}\|_2}$. This shows the existence of the solution without considering the constraint, $p_l \leqslant p_{max}, \forall l$.

Let $\mathbf{q}^*, \mathbf{p}^*$ be a solution disregarding this constraint and $p^*_{max} > p_{max}$ is the value of the maximum element of $\mathbf{p}^*$. A simple analysis shows that by scaling the computed vectors in the previous part, we can find a solution which satisfies the required constraint on the maximum allowable transmission power. Specifically, $\hat{\mathbf{p}} = \alpha_1 \mathbf{p}^*, \ \hat{\mathbf{q}} = \alpha_2 \mathbf{q}^*$, are solutions satisfying the constraint, where $\alpha_1, \alpha_2 \leqslant 1$ are scaling factors and $\alpha_1 = \frac{p_{max}}{p^*_{max}}, \ \alpha_2^l \leqslant 1 - \frac{\eta\left(\left(\frac{1}{\alpha_1} - 1\right)\right)}{\sum_{k \neq l} G_{lk} p_k^* q_k^*}$. Intuitively, by scaling the power vector with the ratio of the maximum allowable power to the largest required power, the persistence vector should be also scaled appropriately to find a feasible solution. □

By hitting an average SINR threshold, it is assumed that the links' transmission rates in each access are the same and fixed. In addition, there are many ways to assign the persistence probabilities. Note that the case $\mathbf{q} = \mathbf{1}$ is also possible, i.e., where all links could access the channel in all time slots and just adjust their powers to hit the SINR threshold. On the other hand, it is also possible to define $\beta_l, \forall l$, i.e., different average SINR thresholds for different links. In this case, the links' transmission rates are different. Therefore, assigning the powers and persistence probabilities, depends on the required objectives at the MAC layer. In the game theory framework, these requirements should be reflected in the players' utility functions.

3.3. Optimization formulation

Assume that the objective is to maximize the sum of links' persistence probabilities while the links' powers are adjusted to hit the SINR threshold. In fact, the objective is to maximize the average long term links' rate by adjusting their persistence probabilities and by employing a fairness criterion under the assumption that the transmission rate of each link is fixed during each access to the channel. In the following, we formulate this problem in the optimization framework and in the next section in the game theory framework.

Consider this optimization problem:

$$\max \quad \sum_{j=1}^{N} u_j(q_j) \tag{4}$$

$$\text{subject to} \quad \bar{\gamma}_j \geqslant \beta \qquad j = 1, \ldots, N \tag{5}$$

$$0 \leqslant q_j \leqslant 1 \qquad j = 1, \ldots, N \tag{6}$$

$$p_{min} \leqslant p_j \leqslant p_{max} \qquad j = 1, \ldots, N \tag{7}$$

$u_j(q_j)$ is the MAC layer utility function of link j defined on $[0, 1]$ and is assumed to be strictly increasing and concave. Let $u_j(q_j) = \log(q_i)$. Using this utility function, the proportional fairness among links are guaranteed as we discuss in the following. By taking logarithm of (4) and using the transformation $\tilde{q} = \log(q)$ and $\tilde{p} = \log(p)$ the problem is turned into a convex optimization problem and has a unique optimal solution [10]. Applying KKT optimality conditions [27], link j should update its variables as follows, to reach the optimal solution.

$$q_j(t+1) = \left[q_j(t) + \xi \left(\frac{1}{q_j} - M_j p_j \right) \right]_0^1 \tag{8}$$

$$p_j(t+1) = \left[p_j(t) + \kappa \left(\frac{\lambda_j(t)}{p_j(t)} - M_j q_j(t) \right) \right]_{p_{min}}^{p_{max}} \tag{9}$$

$$\lambda_j(t+1) = \left[\lambda_j(t) + \delta(\beta - \bar{\gamma}_j(t)) \right]^+ \tag{10}$$

where $[x]_b^a = \max\{\min\{x, a\}, b\}$ and $[x]^+ = \max\{0, x\}$. Also, $(\lambda_1, \ldots, \lambda_N)$ are the Lagrange multipliers for the constraints in (5), $0 < \xi, \ \kappa, \ \delta < 1$ are some small enough constants, and M_j is given by:

$$M_j = \sum_{k \neq j} \frac{\lambda_k G_{kj}}{\sum_{l \neq k} G_{kl} p_l q_l + \eta} = \sum_{k \neq j} G_{kj} m_k \tag{11}$$

M_j can be interpreted as the sum of other links' messages, i.e., $k \neq j$, when the link messages are defined as $m_k = \frac{\lambda_k}{\sum_{l \neq k} G_{kl} p_l q_l + \eta}$. It is assumed that M_j is available at link j as the channel feedback.

Note that the vector of optimal solution for persistence probabilities and hence the long term links' data rates, are proportionally fair. Specifically, denoting the optimal persistence probability vector by $\mathbf{q}^*$, it can be shown that for any other feasible vector $\mathbf{q}$, the sum of proportional changes is zero or negative [28], i.e.,

$$\sum_{l \in L} \frac{q_l - q_l^*}{q_l^*} \leqslant 0 \tag{12}$$

Eq. (12) can be derived by applying the optimality condition for the differentiable objective function in (3) [27]. According to this condition, $\mathbf{q}^*$ is the optimal solution if:

$$\left(\nabla \sum_{l \in L} u_l(q_l^*) \right)^T (\mathbf{q} - \mathbf{q}^*) \leqslant 0 \tag{13}$$

where using $u_l(q_l) = \log(q_l)$, (13) leads to (12). Also, we have the following proposition about the optimal solution.

Proposition 2. *Let $\mathbf{q}^*, \mathbf{p}^*$ be the optimal solution of (4)–(7).*

(a) *If there exist some j for which $q_j^* < 1$, then $\gamma_i^* = \beta$ for all i, i.e., the inequality constraints in (5) are active and the optimal power vector can be derived using the Foschini–Miljanic algorithm.*
(b) *At the equilibrium we have:*

$$\lambda_j = 1 \qquad \text{if} \ \ p_{min} < p_j < p_{max}, \ q_j < 1 \tag{14}$$

$$\lambda_j > 1 \qquad \text{if} \ \ p_j = p_{max}, \ q_j < 1 \tag{15}$$

$$\lambda_j < 1 \qquad \text{if} \ \ p_j = p_{min}, \ q_j \leqslant 1 \tag{16}$$

Proof.

(a) By contradiction, assume there exists a link i for which $p_i^* > p_i^{min}$ and $\bar{\gamma}_i > \beta$, where p_i^{min} is the minimum required power to hit β. Since u_j is a strictly decreasing function of p_i and strictly increasing of q_j, by changing the i element of $\mathbf{p}^*$ from p_i^* to p_i^{min}, q_j^* would be increased due to the produced slack. This has contradiction with the optimality of the q^*.
(b) By joint considering 8, 9 at equilibrium, (14)–(16) are derived easily. □

4. PAMG: power-aware MAC game

4.1. Game theory formulation

Following the insights from the optimization formulation, in this section, we formulate the PAM algorithm as a non-cooperative static game. Using the same notations as in Section 3.1, the MAC layer payoff function for link j, $u_j(\cdot)$, is defined as

$$u_j(q_j, p_j, \mathbf{q}_{-j}, \mathbf{p}_{-j}) = \log(q_j) + \delta_j \log\left(\bar{\gamma}_j - \beta\right) - \widetilde{M}_j q_j p_j \tag{17}$$

where δ_j is a constant and

$$\widetilde{M}_j = \sum_{k\neq j} \frac{G_{kj}}{\sum_{l\neq k,j} G_{kl} p_l q_l + \eta} = \sum_{k\neq j} G_{kj} \tilde{m}_k \quad (18)$$

where $\tilde{m}_k = \frac{1}{\sum_{l\neq k,j} G_{kl} p_l q_l + \eta}$ is the message of link k. It is assumed that $\widetilde{M}_j$ parameters can be computed at the transmitter of link j by message passing; where the path gains G_{kj} can be computed by the training sequences.

Comparing with m_k in (11) which is derived from the optimization formulation, we see that $\tilde{m}_k$ is an approximation of m_k with two simplifications. First, it is assumed that $\lambda_k = 1$ for all k. Second, the effect of link j in the normalizing factor at the denominator of m_k is neglected. These simplifications, as we will see, lead to a non-optimal equilibrium solution of the PAMG. However, using these simplifications, the game analysis is simplified and especially the asynchronous convergence of the game to the suboptimal equilibrium point is guaranteed. In a message passing environment, totally asynchronous convergence is an important issue at the MAC layer design because messages are exposed to collisions and there is no guarantee for the reliable delivery of messages.

According to Proposition 2, $\lambda_k \leqslant 1$ except for the links that use the maximum allowable power at equilibrium. Therefore, the first simplification tends to decrease a few links messages. On the other hand, since the denominator of all links messages is decreased, the second simplification tends to increase their messages.

In the following and to analyze the PAMG, we assume that $\widetilde{M}_j > M_j, \forall j \in \mathcal{N}$. That is we assume that the outcome of these two simplifications leads to an increase in the sum of all received messages compared to the optimal solution. This assumption is used to guarantee the existence of Nash equilibrium as it is stated in Section 4.3.

This assumption is validated in all simulated topologies. On the other hand, it does not restrict the algorithm that is developed based on the PAMG. The reason is that in a practical scenario it is possible to gradually increase the messages of the links which are reach their maximum allowable power but could not hit the SINR threshold.

4.2. Interpretation of payoff function in PAMG

The medium is utilized by a link in time and space when the link adjusts its persistence probability and power, respectively. In the PAMG, each link selfishly decides on its persistence probability and power to maximize its payoff from the medium.

The defined payoff function in (17), which exactly follows the optimization approach, can be interpreted in terms of utilization and cost of network resources usage. The first term in (17) indicates that the higher the link persistence probability, the more is the link utility from the medium. The second term reflects the effect of the link power on the utility function. It acts as a barrier function to ensure the minimum required threshold, β, where parameter δ_j shows the weight of using higher power and hence higher SINR in the utility function. Therefore, each link tends to increase its persistence probability while adjusting its power to ensure that the required SINR at its receiver is achieved. Finally, the term $\widetilde{M}_j q_j p_j$ is the cost function where the constant $\widetilde{M}_j$ reflects the dependency of the cost to the location in which the link transmits. In locations with dense active links, according to the $\widetilde{M}_j$ formula, the cost is high. The cost function is introduced in the link payoff function to coordinate the selfish decisions of links to have better network utilization of medium. We now analyze the PAMG.

4.3. PAMG analysis

The Nash equilibrium study of a game can be used to predict the outcome of the game where each player decisions are based on self-optimization. Briefly, at the Nash equilibrium, no player can improve his payoff by making individual changes in his decisions [29]. Therefore, by self-optimizing of (17), the decision of link j which is denoted by $\tilde{q}_j, \tilde{p}_j$, is computed by solving: $\frac{\partial u_j(\cdot)}{\partial q_j} = 0$, $\frac{\partial u_j(\cdot)}{\partial p_j} = 0$. Taking the differentiation and doing some simplification, we have:

$$\tilde{q}_j = \frac{1}{\widetilde{M}_j \tilde{p}_j} \quad (19)$$

$$\tilde{p}_j = \beta I_j + \frac{\delta_j}{\widetilde{M}_j \tilde{q}_j} \quad (20)$$

where $I_j = \frac{p_j}{\bar{\gamma}_j} = \frac{\sum_{k\neq j} G_{jk} p_k q_k + \eta}{G_{jj}}$ is the effective interference at the receiver of link j.

We note that adjusting the power to satisfy (20) is equivalent to hit the γ_j^{tar}, given by:

$$\bar{\gamma}_j^{tar} = \beta + \frac{\delta_j}{\widetilde{M}_j q_j I_j} \quad (21)$$

Note that under the assumption $\widetilde{M}_j > M_j, \forall j \in \mathcal{N}$, all links can hit β.

Proposition 3. *There exists Nash equilibrium for PAMG provided that $\delta_j, \forall j \in \mathcal{N}$ is sufficiently small.*

Proof. The existence of the Nash equilibrium is proved by showing that the strategy space of each user is a non-empty compact and convex set of $\mathbb{R}^2$ and $u_j(\mathbf{q},\mathbf{p})$ is continuous in $(\mathbf{q},\mathbf{p})$ and quasi-concave in (q_j, p_j) [29]. By Proposition 1, the strategy space of each user is a non-empty subset of the box $[0,1] \times [p_{min}, p_{max}]$ in $\mathbb{R}^2$ which is convex and compact. Also, the utility function is continuous in $(\mathbf{q},\mathbf{p})$ and provided that $\delta_j, \forall j \in \mathcal{N}$ is sufficiently small, $u_j(\cdot)$ is strictly concave and hence quasi-concave on the link strategy space. To show this, we note that the Hessian of the payoff function is given by:

$$\nabla^2_{(q_j,p_j)} u_j(\cdot) = \begin{pmatrix} \frac{-1}{q_j^2} & -\widetilde{M}_j \\ -\widetilde{M}_j & -\frac{\delta_j \bar{\gamma}_j^2}{(\bar{\gamma}_j - \beta)^2 p_j^2} \end{pmatrix}$$

Since $\frac{-1}{q_j^2} < 0$, the sufficient condition for the Hessian to be negative definite is [30]:

$$\frac{\delta_j \bar{\gamma}_j^2}{(\bar{\gamma}_j - \beta)^2 p_j^2 q_j^2} - \widetilde{M}_j^2 > 0 \quad (22)$$

Noting that if $\delta_j \to 0$ then according to (21), $\bar{\gamma}_j \to \beta$ and the denominator of (22) tends to zero in second order. Therefore, in limit the left-hand side of (22) tends to infinity and the inequality is satisfied. □

Proposition 4. *The best response of link j, in the PAMG is given by: $p_j^* = \max\{p_{min}, \min\{p_{max}, \tilde{p}_j\}\}$, $q_j^* = \min\left\{1, \frac{1}{\widetilde{M}_j p_j^*}\right\}$; where $\tilde{p}_j$, $\tilde{q}_j$ are computed using (19) and (20).*

Proof. The proof is clear by noting that 19, 20 are obtained by unconstrained optimization of a strictly concave function, when $p_j = p_{max}$ and $q_j = 1$ are the largest power and persistent probability that can be assigned to a link. □

Therefore, according to (21) in the PAMG the power control scheme is reduced to a simple SINR based target hitting algorithm and by adjusting the power, the persistence probability is adjusted by a simple formula in (19). In other words, the link decision for its persistence probability is decoupled from its decision on the power. A special case is when for all active links the persistence probability is equal to one and each player just adjusts his power by a simple SINR target tracking algorithm. The following proposition shows that the Nash equilibrium point of PAMG is also unique.

Proposition 5. *The Nash equilibrium for PAMG is unique.*

Proof. To prove that the Nash equilibrium is unique, it is sufficient to prove that the best response function of each player for the power and persistence probability is a standard function [24]. Let $\mathbf{v} = (\mathbf{p},\mathbf{q})$ denotes the network power and persistence probability vectors and $\mathcal{T}(\mathbf{v}) = (\mathcal{T}_1(\mathbf{v}),\ldots,\mathcal{T}_N(\mathbf{v}))$ denote the links' best response function vector; where

$$\mathcal{T}_j(\mathbf{v}) = \begin{cases} \max\left\{p_{min}, \min\left\{p_{max}, \frac{\gamma_j^{tar}}{\bar{\gamma}_j}p_j\right\}\right\} & \text{to update } p_j \\ \min\left\{1, \frac{1}{\widetilde{M}_j p_j}\right\} & \text{to update } q_j \end{cases}$$

That is the power and persistence of link j in iteration t is given by: $p_j(t) = \max\left\{p_{min}, \min\left\{p_{max}, \frac{\gamma_j^{tar}}{\bar{\gamma}_j(t-1)}p_j(t-1)\right\}\right\}$ and $q_j(t) = \min\{1, \frac{1}{\widetilde{M}_j(t-1)p_j(t-1)}\}$. To show that $\mathcal{T}$ is a standard vector function the following properties should be satisfied.

- positivity: $\mathcal{T}(\mathbf{v}) > 0$
- monotonicity: if $\mathbf{v} \geqslant \mathbf{v}'$ then $\mathcal{T}(\mathbf{v}) \geqslant \mathbf{T}(\mathbf{v}')$
- scalability: for all $\mu > 1, \mu\mathcal{T}(\mathbf{v}) > \mathbf{T}(\mu\mathbf{v})$

Since the minimum and maximum of two standard functions are standard and the fixed function does converge [24], we could neglect the constraints in the updating function. It is easy to show that the power update function, $p_j(t) = \gamma_j^{tar} I_j(t-1) = \gamma_j^{tar}\left(\sum_{k\neq j} G_{jk}p_k(t-1)q_k(t-1)\right)$, is a standard function of $\mathbf{v}$. On the other hand, according to the relation of $\widetilde{M}_j$ to vector $\mathbf{v}$ in (18), the above properties could be verified for the persistence probability update function. □

It should be noted that the assumption $\lambda_j = 1, \forall j \in \mathcal{N}$ which cause to a suboptimal equilibrium, is necessary in proving the uniqueness. In summary, PAMG has a unique Nash equilibrium provided that the pricing factors, $\widetilde{M}_j$, can be computed at the links. In addition, since the best response vector is standard, totally asynchronous convergence of the algorithm is guaranteed. Therefore, the convergence is independent from the frequency at which the link broadcasts its message and will not be affected by messages collisions.

5. Asynchronous distributed PAM algorithm

Based on the PAMG, each link broadcasts a message at each probable successful access to the channel and adjusts its pricing factor, power, and persistence probability according to the received messages.

5.1. The PAM algorithm

For $j \in \mathcal{N}$, let T_j denote the set of time slots at which link j, accesses to the channel. These random time slots show the possible time instances at which the link could update its power and persistence. In fact, link j can get feedback from its corresponding receiver to compute the current average SINR, with a frequency not greater than $\frac{1}{q_j}$. The following PAM algorithm is employed to adjust the links persistence probabilities and powers.

Algorithm 1. The PAM algorithm

(1) Initialization: For each link $j \in \mathcal{N}$ choose some power $p_j(t_0) \in P_j$ and persistence probability $q_j(t_0) \in Q_j$

(2) At each $t_k \in T_j$, link j:

(2-a) Transmitting and message broadcasting: Transmits and simultaneously broadcasts its message.

(2-b) Power Update: Updates its power using:

$$p_j(t_k) = \max\left\{p_{min}, \min\left\{p_{max}, \frac{\gamma_j^{tar}}{\bar{\gamma}_j(t_{k-1})}p_j(t_{k-1})\right\}\right\} \tag{23}$$

where γ_j^{tar} is computed by (21).

(2-c) Persistence Probability Update: Updates its persistence probability to:

$$q_j(t_k) = \min\left\{1, \frac{1}{\widetilde{M}_j(t_{k-1})p_j(t_{k-1})}\right\} \tag{24}$$

where $\widetilde{M}_j$ comes from (18).

5.2. The PAM algorithm evaluation

In this subsection, we evaluate the PAM algorithm and discuss on its results. For numerical study, a random network topology is generated with $N = 10, L = 200$ m, and $R = 40$ m according to the system model in Section 2. This network is shown in Fig. 1. Each transmitter is connected to its corresponding receiver by a bold line. In order to show the high level interference regions of the medium, each transmitter–receiver pair is surrounded by a circle. For simulation, we assume that $\delta_j, \forall j \in \mathcal{N}$ are sufficiently small, i.e., $\gamma_j^{tar} = \beta, \forall j \in \mathcal{N}$. In addition, we set $\beta = 10$, $\alpha = 4$, $d_0 = 10$ m, $\eta = 5 \times 10^{-12}$ mW, $p_{max} = 500$ mW, $p_{min} = 0.1$ mW.

Fig. 2 shows the variations and convergence of links average SINR when PAM algorithm is deployed in the network. In simulation, link j parameters are updated with frequency $\frac{1}{q_j}$. As it can be seen from this figure, links' average SINR converge to β. The

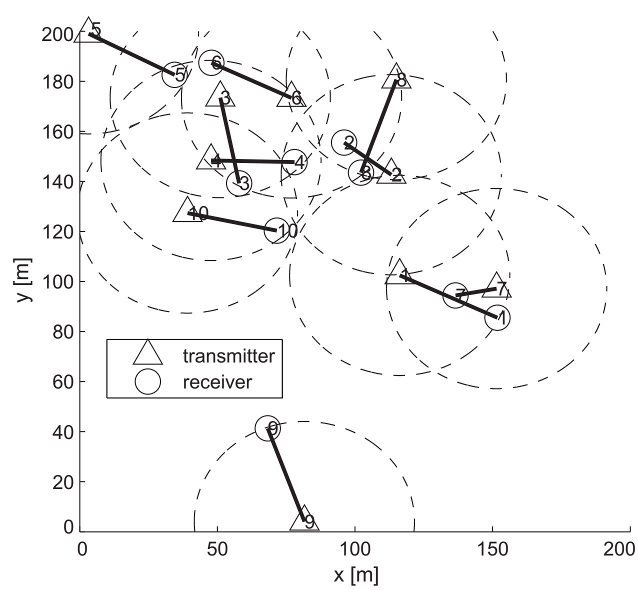

Fig. 1. Random network topology.

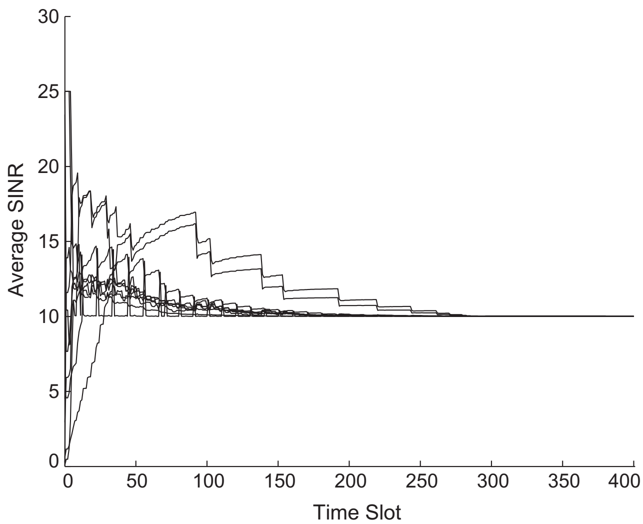

Fig. 2. Variations of the links' SINR until the PAM algorithm convergence.

convergence speed depends on the links' update frequencies and is ultimately determined by the link that has the least persistence probability. The final values of links persistence probabilities and powers, in mW, after the algorithm convergence, are computed and depicted in Fig. 3. In this figure, the optimal power and persistence probabilities using (8)–(10) are also shown for comparison.

From this figure and by comparing the results of the PAM algorithm with the optimal ones, we see that:

- By deploying the PAM algorithm, the links powers and persistence probabilities are adjusted according to their locations in the network as in the optimal solution. This justifies the reasonability of the assumptions that are used in deriving the PAM algorithm.
- Considering the sum of the logarithm of all links as network utility, we have: $U_{opt} = -10.50$, $U_{PAM} = -11.70$. That is the game solution is a suboptimal solution with an approximately 11.4% efficiency loss.
- From the energy efficiency viewpoint, the PAM solution is more efficient. This is reasonable since the persistence probability of each link is smaller compared to the optimal solution.
- Fig. 3 also shows the relationship of the final links' parameters to their locations in the network after the algorithm convergence. We can see from this figure that some links, e.g., link numbers 5,7,9,10, utilize higher persistence probabilities and lower powers while some others, e.g., link numbers 1,3,6,8, utilize higher powers and lower persistence probabilities. According to the network topology in Fig. 1, we see the latter links are those which their receivers are exposed to a severe interference from the neighboring links. This result is reasonable because in a dense region with high level of interference, theses links should decrease their persistence to mitigate the mutual interference. On the other hand, they should use higher power to hit the minimum average SINR in each access. This result emphasizes that the link persistence and power should be adjusted simultaneously according to the link location in the network and is consistent with the result of [10].
- The values of the links' messages which are more exposed to others interference are greater than those which are less exposed to others interference. The values of these messages in the above experiment are: $\widetilde{\mathbf{M}} = (0.76, 2.48, 5.53, 1.54, 0.05, 0.75, 4.28, 0.16, 0.01, 0.28) \times 10^3$ In fact, the awareness of the link location in the network is achieved by interchanging the messages.

It is also interesting to compare the results of the PAM algorithm with a scenario in which only powers are adjusted. In this scenario, the links persistence probabilities are equal and increased gradually until the point at which all links could hit the minimum required SINR threshold, β, by adjusting their powers. In other words, the links persistence probabilities are set to the maximum possible value for which there exists a power vector that all links could hit β. The powers are adjusted using the target hitting algorithm in [23]. Simulation results show that the maximum possible achievable persistence for all links is 0.15. Therefore, using this scheme the efficiency would be $\widehat{U} = -18.97$. By comparing the results, we conclude that by simultaneous persistence and power control using the PAM algorithm the efficiency of resource assignment is increased about 62%. This gain is achieved because the PAM algorithm considers the links location in the network when adjusting the links parameters.

In Fig. 4, the utility function of link number 6 is depicted as an example where it is assumed that all links settle at their optimal points. As the figure shows, there is a constraint on the link power

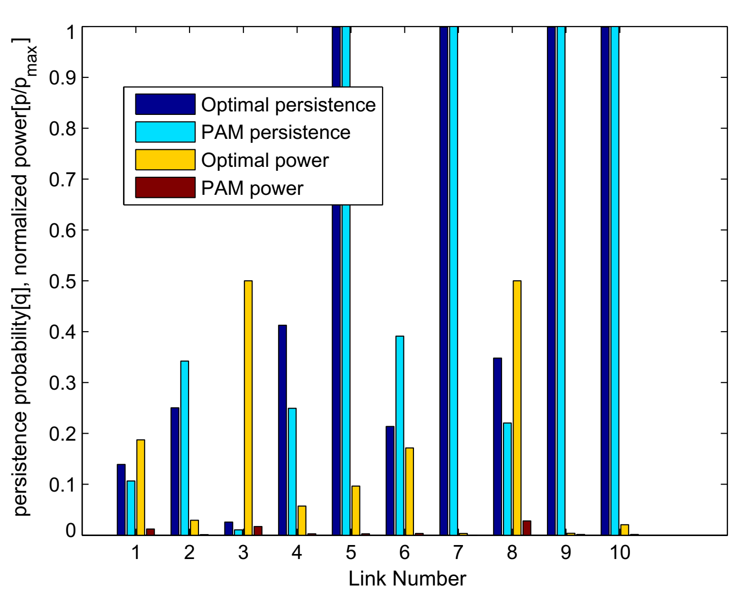

Fig. 3. Links' persistence probabilities and normalized powers at equilibrium using the PAM algorithm and by optimization.

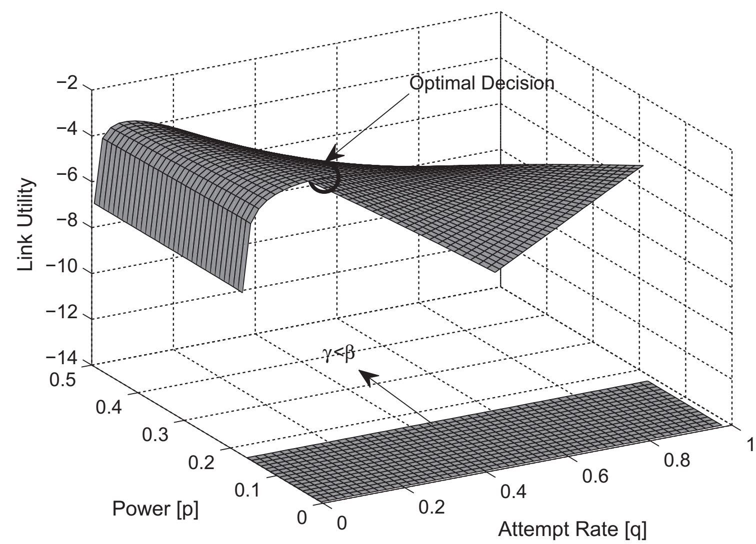

Fig. 4. The shape of the link utility function.

to ensure $\bar{\gamma} > \beta$. After satisfying this constraint, the optimal link's persistence probability should be determined.

6. Efficiency loss and extensions of the PAMG

We already showed that under some assumptions, there exists unique Nash equilibrium point for PAMG and presented the PAM algorithm to find that equilibrium. Another important question is about the efficiency of the equilibrium. This analysis for PAMG can be accomplished in two levels. First, it can be performed by comparing the game solution with the optimal one, resulted from the optimization framework, to find the efficiency loss of the considered simplifying assumptions for distributed implementation. At the second level, one could compare the efficiency of the game equilibrium with other possible strategies. This can be performed by showing that the game equilibrium is Pareto optimal [29].

6.1. Efficiency loss compared to the optimal solution

We first investigate the effect of approximating the cost factors on the efficiency of the equilibrium point using a sensitivity analysis. According to (20), the solution for the power of link j is independent of $\widetilde{M}_j$ provided that δ_j is small enough. Also, from (19) the optimal persistence probability variations is related to the cost factor variations by:

$$\Delta q_j = \frac{\partial q_j}{\partial \widetilde{M}_j}\Delta\widetilde{M}_j = \frac{-1}{p_j\widetilde{M}_j^2}\Delta\widetilde{M}_j \tag{25}$$

Eq. (25) shows that using higher values for cost factors leads to lower equilibrium persistence. However, the persistence probability variations has inverse relation to the square of the cost factor. Therefore, the approximate cost factor, as we have seen in the simulation in Section 6.2, would not lead to a serious efficiency loss of PAMG equilibrium compared to the optimal solution.

6.2. Pareto optimality of PAMG equilibrium

The efficiency of a Nash equilibrium in comparison with other possible solutions can be investigated by examining its Pareto optimality [29]. Briefly, Nash equilibrium is Pareto optimal if it is not possible for a subset of players to increase their utility without hurting any other players.

Proposition 6. *The Nash equilibrium point of PAMG is Pareto optimal.*

Proof. Assume that the Nash equilibrium point of PAMG is denoted by $(\mathbf{q}^*, \mathbf{p}^*)$, where dropping the prime from the M_j,

$$q_j^* = \min\left\{1, \frac{1}{M_j^* p_j^*}\right\} \tag{26}$$

$$p_j^* = \frac{\beta\left(\sum_{i\neq j} G_{ji} p_i^* q_i^* + \eta\right)}{G_{jj}} \tag{27}$$

$$M_j^* = \sum_{i\neq j} \frac{G_{ij}}{\sum_{h\neq i,j} G_{ih} p_h^* q_h^* + \eta} \tag{28}$$

It is enough to show that $\nexists\ (\mathbf{q^o}, \mathbf{p^o}) : u_j\left(q_j^o, p_j^o\right) \geqslant u_j\left(q_j^*, p_j^*\right)$, $\forall j \in \mathcal{N}$ and $u_k\left(q_k^o, p_k^o\right) > u_k\left(q_k^*, p_k^*\right)$ for some $k \in \mathcal{N}$, and $(\mathbf{q^o}, \mathbf{p^o})$ is feasible. The proof is obvious for the special case that $q_j^* = 1$, $\forall j \in \mathcal{N}$, hence we assume $q_j^* < 1$ for some $j \in \mathcal{N}$.

By contradiction, let there exist $(\mathbf{q^o}, \mathbf{p^o})$ with the above-mentioned properties. Without loss of generality, let $q_j^o = \phi_j q_j^*$, $\phi_j \geqslant 1$, $\forall j \in \mathcal{N}$ and $\phi_k > 1$ for some $k \in \mathcal{N}$. First, we note that if such a $\mathbf{q^o}$ exists, then for the corresponding $\mathbf{p^o}$, we have: $p_j^o = \psi_j p_j^*$, $\psi_j > 1$, $\forall j \in \mathcal{N}$. The reason is that for a given link j, using the new persistence probability vector, the interference at the receiver of j is strictly increased and to hit the minimum required threshold, β, the link must increase its power. Therefore, $\phi_j\psi_j > 1$, $\forall j \in \mathcal{N}$.

Let $\mathcal{A}, \mathcal{B}$ denote the subsets of $\mathcal{N}$ for them we have $q_j^* = 1$, $\forall j \in \mathcal{A}$ and $q_j^* < 1$, $\forall j \in \mathcal{B}$. We have $\phi_j = 1$, $\forall j \in \mathcal{A}$ and $\phi_j \geqslant 1$, $\forall j \in \mathcal{B}$. Now we find a necessary condition for the existence of $(\mathbf{q}^o, \mathbf{p}^o)$ for each link in subsets $\mathcal{A}, \mathcal{B}$.

(a) for all links $j \in \mathcal{A}$, $u_j\left(q_j^o, p_j^o\right) \geqslant u_j\left(q_j^*, p_j^*\right)$ leads to $-\psi_j p_j^* M_j^o \geqslant -p_j^* M_j^*$. Therefore, using $\phi_j = 1$ the necessary condition is:

$$\phi_j\psi_j M_j^o \leqslant M_j^* \tag{29}$$

(b) for all links $j \in \mathcal{B}$, according to (26) we have $q_j^* p_j^* M_j^* = 1$ and $u_j\left(q_j^o, p_j^o\right) = \log\left(\phi_j q_j^*\right) - \phi_j\psi_j q_j^* p_j^* M_j^o$. The necessary condition to have increase in the utility function with the new persistence probability is: $\frac{\partial u_j\left(q_j^o, p_j^o\right)}{\partial \phi_j} = \frac{1}{\phi_j} - \psi_j q_j^*$

$p_j^* M_j^o \geqslant 0$, that leads to: $1 - \phi_j \psi_j q_j^* p_j^* M_j^o \geqslant 0$. Since $q_j^* p_j^* M_j^* = 1$, this condition is also reduced to (29). Therefore (29) is the necessary condition for all links in $\mathcal{N}$. Now let $k \in \mathcal{N}$ be the index for which $\phi_k \psi_k \geqslant \phi_h \psi_h,\ \forall h \in \mathcal{N}$. Since $\phi_k \psi_k > 1$ we have:

$$\begin{aligned} M_k^o &= \sum_{i \neq k} \frac{G_{ik}}{\sum_{h \neq i,k} G_{ih} p_h^* q_h^* \phi_h \psi_h + \eta} \\ &> \sum_{i \neq k} \frac{G_{ik}}{\phi_k \psi_k \left(\sum_{h \neq i,k} G_{ih} p_h^* q_h^* + \eta \right)} = \frac{M_k^*}{\phi_k \psi_k} \end{aligned} \tag{30}$$

Therefore, $\phi_k \psi_k M_k^o < M_k^*$, indicating (29) could not be satisfied for link k. This completes the proof. □

6.3. Extensions of the PAM

As a special case, the PAM algorithm can be deployed by assuming the same cost factors for all links [31]. This scheme can be used when there exists prior knowledge about the network topology or for simple implementation without message passing. For example, in networks with almost symmetric structure, e.g., the mesh networks, the cost factor of links are close to each other and can be pre-assigned according to the mesh size.

On the other hand, until now by assuming $\delta_j \to 0$, the links' target SINR and therefore their rates are assumed to be fixed in each access. That is, increasing the link target SINR above β, does not increase its utility.

However, the link data rate could be adjusted, for example by adaptive modulation, based on the attainable SINR at the receiver in each access. As it is explained before, persistence and power are two degrees of freedom for the network resource utilization. By relaxing this constraint, the utilization of the power could be enhanced where a link can have higher SINR and hence higher data rate in each access to the channel.

This can be achieved by choosing a small enough value for $\delta_j,\ \forall j \in \mathcal{N}$. According to (21), this leads to higher target SINR for links like j which have lower $I_j \widetilde{M}_j$. That is the link which has small interference at its receiver, small I_j, as well as causing small interference for others, small $\widetilde{M}_j$. Again, this is emphasizing the location awareness requirement for better power utilization. We should note that in this scheme the definition of the link utility is changed and depends on both its persistence to access the channel and its power which determine the achievable SINR in each access.

In Fig. 5, the convergence of links' SINR in the network topology of Fig. 1 is shown assuming $\beta = 10$, $\delta_j = 0.05$, $\forall j \in \mathcal{N}$, and bounding the links' SINR to 5β. Fig. 5, shows that by using the modified utility function, link numbers 5,9,10 converge to higher SINR, i.e., 14.1, 50, 11.4, respectively, while other links SINR converge to about 10.5. According to the locations of these links in Fig. 1, we find that their transmissions would not lead to a severe interference while their receivers are not exposed to a severe interference from other links.

Another important problem is deploying the PAM algorithm in an end-to-end scenario considering the effects of the MAC upper layers, i.e., the network and transport layers. These layers determine the number of packets in the buffer of each link for transmission. Comparing with the cross layer approach [10], the effects of the upper layers can be reflected by some weights in the utility function, i.e.,

$$u_j(\cdot) = \lambda_j \log(q_j) + \delta_j \log\left(\bar{\gamma}_j - \beta\right) - M_j q_j p_j$$

where λ_j is a measure of the average queue length of the transmitter of link j.

7. Network implementation issues and simulations

To implement the PAM algorithm in a practical scenario, each link should broadcast a message simultaneous to its transmission to the corresponding receiver. These messages are then used by neighboring links to adjust their cost factors. On the other hand, the link should estimate its average SINR and cost factor based on the outcome of the transmitted packets and received messages. In this section, the PAM algorithm is investigated where the links decisions are updated based on the measurements.

First, we assume that the MAC frame consists of two parts. The first part that includes the actual transmitter payload, intends for the corresponding receiver. The second part includes the message content and should be captured by all inactive links that are not transmitting in that time slot. We should note that in average link

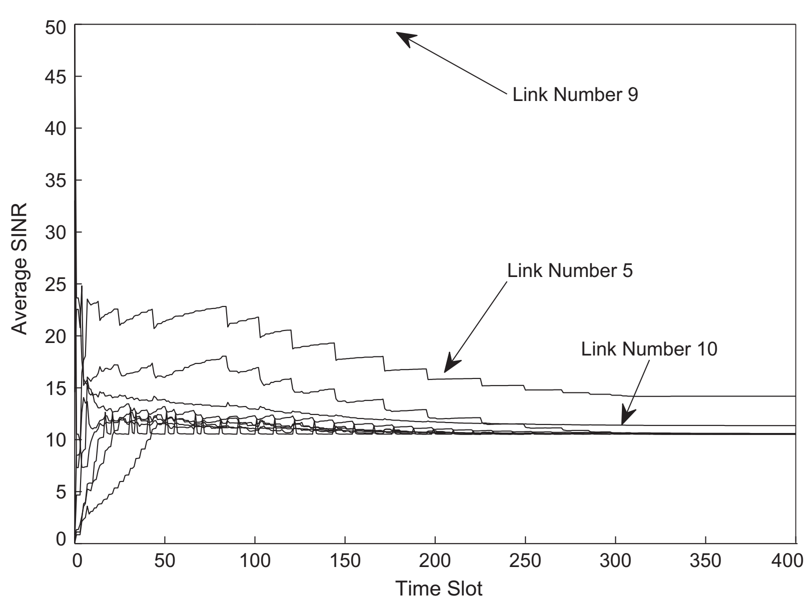

Fig. 5. Convergence of the links' SINR using $\delta_j = 0.05$ in PAM Algorithm 1.

j is in active or transmit mode with probability q_j, and in inactive or listening mode with probability $1 - q_j$.

Applying the PAM requires that link j estimates $\bar{\gamma}_j$ and M_j based on the transmitted packets when it is active and the received messages when it is inactive, respectively. The following structures are provided at link j to track these parameters based on the samples.

W_j is a counter that counts the number of transmissions or instantaneous SINR samples from the previous decision update. Upon receiving a new sample, this counter is used to update the current estimation of $\bar{\gamma}_j$. A message list of length $N - 1$ stores the neighboring links messages information and is named $\mathscr{L}_j$. Each element of $\mathscr{L}_j$ has three fields: the path loss to the receiver of link l, $\mathscr{L}_j \cdot G_{lj}$, the number of received messages from link l, $\mathscr{L}_j \cdot c_l$, and the estimated message from l, $\mathscr{L}_j \cdot m_l$. Since link j is in listening mode with probability $1 - q_j$, the received message from link l is divided by $1 - q_j$ to speed up the estimation. The broadcasted message of link j is constructed based on the current estimate of $\bar{\gamma}_j$ while it updates its power and persistence after W_0 transmissions. Algorithm 2 presents the PAM based on the measurements; where α_j is the smoothing factor in averaging the SINR at each W_0 transmissions.

In this algorithm, the links' parameters are initialized in lines 1 to 6. Then, at each time slot the subsets of active and inactive links are determined, i.e., lines 9 and 10. For each link belongs to the subset of active links, the link's transmission, message broadcasting and average SINR updating are done in lines 12 and 13. Then the link's power and persistence probability are updated in lines 14–17. For each link belongs to the subset of inactive links, the message list is updated in line 21.

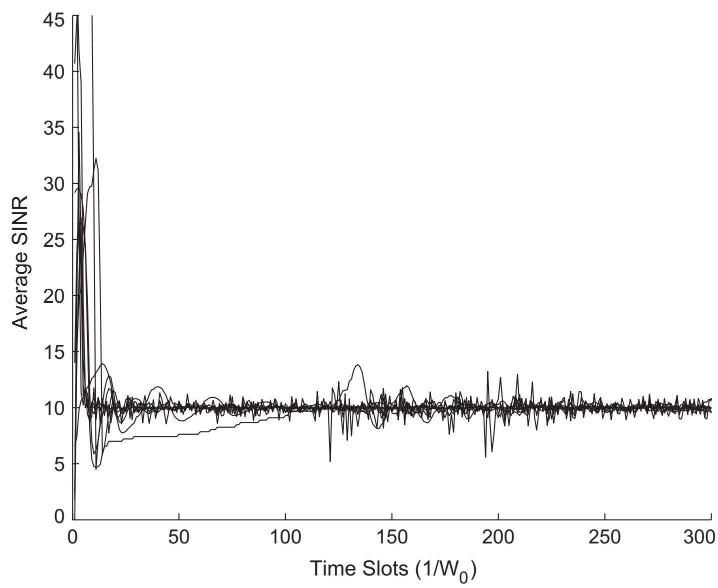

Fig. 6. Convergence of the links' SINR using PAM Algorithm 2.

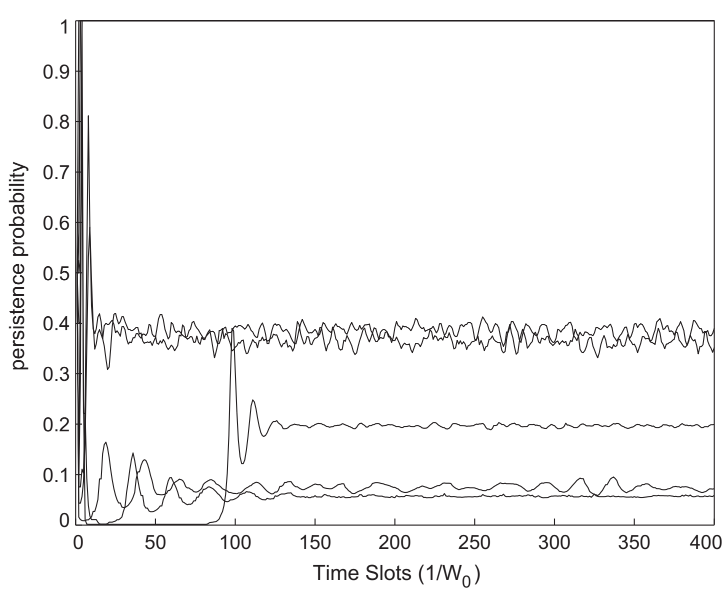

Fig. 7. Convergence of the links' persistence probabilities using PAM Algorithm 2.

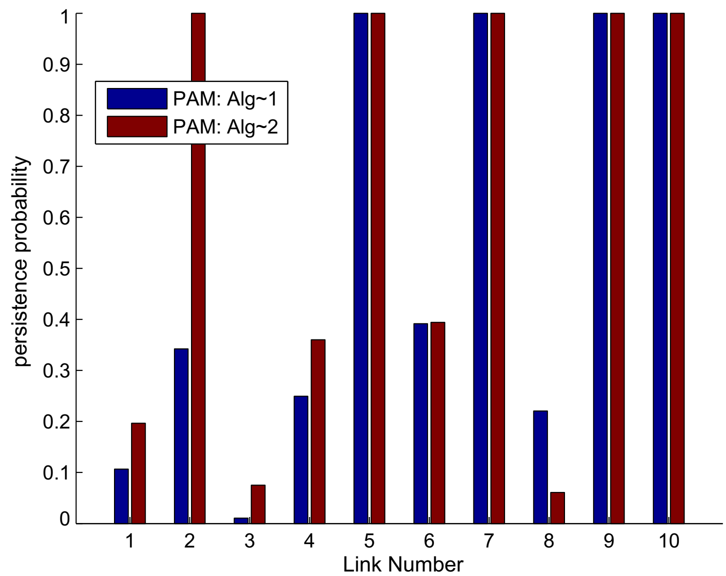

Fig. 8. Final links' persistence probabilities using PAM Algorithm 1 and PAM Algorithm 2.

Algorithm 2. The measurement based PAM algorithm

```
Initialization Step:
for all j ∈ N do
  p_j^0 ← p_j, q_j^0 ← q_j, where p_j, q_j are randomly selected from
  P_j, Q_j, respectively.
  γ̄_j ← 0, W_j ← 0
  L_j · m_l ← 0, L_j · c_l ← 0, ∀l≠j
end for
Recursion Step: At time slot t = 1, 2, ...
loop
  A^t ⊆ N: The subset of active links in time slot t
  Ā^t ⊆ N: The subset of inactive links in time slot t
  for all j ∈ A^t do
    Transmit and broadcast its message
    W_j ← W_j + 1, Update γ̄_j
    if W_j == W_0 then
      W_j ← α_j W0
      Estimate cost factor, M_j ← Σ_l G_lj m_l
      Update p_j, q_j using the current γ̄_j, M_j.
    end if
  end for
  for all j ∈ Ā^t do
    Update L_j
  end for
end loop
```

Algorithm 2 is applied to the network topology of Fig. 1. We set $W_0 = 200$. This window should be selected wide enough to ensure correct estimation of the cost factor at each link. The variations of the average links' SINR estimation until the convergence in a long term study is depicted in Fig. 6. As it is expected, there exist fluctuations around the target average SINR. The variations in the links' persistence probabilities until convergence are shown in Fig. 7. These figures show that links can adjust their persistence probabilities based on the received messages and the estimated cost factor.

The final values of links' persistence probabilities using Algorithms 1 and 2 are also depicted in Fig. 8 for comparison. Again, we see the same trend on the relation of the appropriate decision for link persistence and its location in the network. On the other hand, since $\bar{\gamma}_j$ is a lower bound for the average attainable link SINR, we achieve more utility in a practical scenario. The network utility when we apply Algorithm 1 is $U_1 = -11.7$ and when we apply Algorithm 2 is $U_2 = -9.1$. This indicates an approximate 28% increase in network utilization.

8. Conclusion

We investigate the power-aware MAC problem in ad hoc wireless networks and propose a new algorithm to adjust the links' persistence probabilities and powers. This algorithm is based on a non-cooperative static power-aware MAC game that can be implemented totally asynchronous. In this game, each link decides on its persistence probability and power to maximize a defined utility function that is derived from the optimization framework insights. The included cost factor in the utility function is computed by message passing in the network. The practical implementation issues are discussed and the results are compared with those of the optimal scheme. The results emphasize on the location dependency of the appropriate link decision for a better utilization of network resources.

Acknowledgement

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