A Nash Power-Aware MAC Game for Ad Hoc Wireless Networks

Abdorasoul Ghasemi, Karim Faez
Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran
Email: {arghasemi,kfaez}@aut.ac.ir

***Abstract*—In this paper, a distributed power-aware Medium Access Control (MAC) algorithm for ad hoc wireless networks is presented. The algorithm is developed based on proposing a power-aware MAC game which is analyzed in the game theory framework. The aim is to adjust each active link persistence probability and power by maximizing a defined local link payoff function. The payoff function is such that its selfish maximization at the links leads to an efficient use of medium resources and has two terms. The first term is the link utility while the second one reflects the cost of using the medium resources. The existence and uniqueness of the game Nash equilibrium are investigated analytically. Also, it is shown that this equilibrium is Pareto optimal indicating its efficiency. Simulation results are provided to evaluate the algorithm and are compared to a scenario in which only powers are tuned. This results emphasize that the link persistence and power should be adjusted simultaneously according to the link location in ad hoc networks.**

I. Introduction

Due to the lack of any infrastructure, random multiple access schemes like slotted Aloha are appropriate candidates for the Medium Access Control (MAC) layer in wireless ad hoc and sensor networks [1]. The aim of the random access MAC layer design is to adapt the persistence probability or back-off window size of each active transmitter according to the network topology and load. On the other hand, the performance of the MAC layer is closely related to the power control at the physical layer [2]. The improvement in the performance of the random multiple access schemes by joint considering the power control and with different capture models are discussed in [3], [4]. By appropriate simultaneous control of these parameters, we can use the multi packet reception phenomenon at the network. This problem which is referred to as Power Aware MAC (PAM), is investigated as a cross layer network utility maximization in [5]. We use the results of [5] in defining the link payoff function.

In [6], [7] the persistence probability control in random access schemes is analyzed using the game theory by defining appropriate utility and cost functions. The Aloha protocol from the perspective of a selfish user is investigated in [6] by introducing the Aloha game. The Aloha game is a stochastic game in which the number of links who wish to transmit is the state of the game, and each time slot is a stage of game. At each stage, each player decides for transmitting or waiting and depends on whether its transmission is successful, receives a specified payoff. In Aloha game, delay and energy parameters are considered in the cost function. In [7] the back off-based random access MAC protocol is reversed engineered and it is shown that the contention resolution algorithm can be considered as a non-cooperative game. The extracted utility function has two terms. The first term is the expected reward that the link obtained by transmitting with a specified persistence probability and the second one is the cost for failure transmission.

In this paper we study the PAM problem in the game theory framework. We use this framework to develop a totally asynchronous distributed algorithm for jointly power and persistence probability control. In a totally asynchronous distributed algorithm, large communication delay between nodes can be tolerated [8]. These delays are unavoidable in a random multiple access environment between a link transmitter and receiver where the packets are exposed to collisions. On the other hand, game theory provides a nice mathematical framework where the network agents should work in a distributed fashion. Therefore, we use this framework for formulating and analyzing the PAM problem. As a game, the link payoff is defined as a function of the link persistence probability and power that consists of two terms. The first term is the link utility from the medium when transmitting with a specified persistence probability and power. The second one is the cost of using the medium resources. Under some assumptions, the existence, uniqueness and efficiency of the game equilibrium is analyzed and based on the game the PAM algorithm is presented. The rest of this paper is organized as follows. The system model and notations are presented in section II. In section III, the problem statement in the game theory framework is discussed. The game analysis and the PAM algorithm as well as its efficiency are presented in sections IV and V respectively. Section VI, contains the results of simulations, evaluation of the PAM, and comparison to the scenario in which only powers are adjusted.

II. System Model and Notation

From the MAC layer point of view, a wireless ad hoc network can be modeled with $2N$ mobile users that makes N pairs of transmitters and receivers, i.e, N active links and single hop transmissions from the transmitter to the corresponding receiver of each link. The transmitter nodes distributed randomly and uniformly in a square shape area of size $L \times L$. We assume that the corresponding receiver of each transmitter is located randomly in a circle around it of radius $R, R \leq L$. The parameter R is used to model the operation of the routing layer to find the next neighbor node

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in an end to end scenario for a real multihop wireless network. The link gain and the distance between the transmitter of link j to the receiver of link l are shown by G_{lj}, d_{lj} respectively; where $G_{lj} = \left(\frac{\ _0}{\ _{lj}}\right)^{\alpha}$, d_0 is the reference distance, and   is the path loss exponent [9]. We assume a minimum value for each distance, i.e., $d_{lj}^{min} = 1$.

Dedicated to each link l, there is a persistence probability q_l, $0 \leq q_l \leq 1$, representing the rate at which the link persists to access the channel, and transmission power p_l, $p_{min} \leq p_l \leq p_{max}$. $\mathbf{q} = (q_1, q_2, \ldots, q_N)$ and $\mathbf{p} = (p_1, p_2, \ldots, p_N)$ are the vectors of links persistence probabilities and powers. It is assumed that the system is time slotted and uses Code Division Multiple Access (CDMA) at the physical layer. The spreading gain effect is reflected in the link gains. The Signal to Interference plus Noise Ratio (SINR) of link l and its average are denoted by $\ _l$ and $\bar{\ }_l$ respectively. Let $\mathbf{1}_l(t)$ be an indicator function taking the value of one, if link l attempts to access the channel in time slot t. The conditional expectation of link l SINR at its receiver is given by [5]:

$$\bar{\ }_l = \mathbb{E}(\ _l|\mathbf{1}_l(t) = 1) \quad \frac{G_{ll}p_l}{\sum_{k \neq l} G_{lk}p_k q_k + \eta} \tag{1}$$

where η is the background noise. We use the right hand side of the inequality in (1) as the achievable average link SINR in each access to the channel. The attainable data rate of link l in a high SINR regime is $c_l = \log(\ _l)$ where for its average, denoted by $\bar{c}_l$, we have: $\bar{c}_l \leq \log(\bar{\ }_l)$ [9]. We use $\bar{\ }_l$ as a measure of the achievable average link data rate.

III. Problem Statement and Formulation

The objective of the PAM is to find the link persistence probability and power in each time slot. In a non-cooperative PAM Game (PAMG), this is attained by selfish maximization of a defined local utility function by each user in the 2D strategy space of these parameters.

A. PAM Game

Let $\mathcal{G} = [\mathcal{N}, \{Q_j, \ _j\}, \{u_j(.)\}]$ denotes the PAMG; where $\mathcal{N} = \{1, 2, \ldots N\}$ is the index of active links in the network and $Q_j = [0, 1]$, $\ _j = [p_{min}, p_{max}]$ are the strategy space of the link j. $u_j(q_j, p_j, \mathbf{q}\ _j, \mathbf{p}\ _j)$ is the MAC layer payoff function for link j. The payoff function indicates the link satisfaction when transmitting with persistence probability q_j and power p_j; where $\mathbf{q}\ _j, \mathbf{p}\ _j$ denote the vectors of persistence probabilities and powers of network links excluding link j. The PAMG is expressed as:

$$AMG \quad \max_{q_j \in Q_j, p_j \in P_j} \quad u_j(q_j, p_j, \mathbf{q}\ _j, \mathbf{p}\ _j), \ \forall j \in \mathcal{N} \tag{2}$$

Different games can be designed by defining different payoff functions. From the game theory viewpoint, the existence and uniqueness of the designed game is crucial. On the other hand, as a MAC layer protocol, its some other aspects like efficiency and fairness among active links should be considered. Also, the MAC layer link payoff depends on the physical layer characteristics like capture model and possibility of adaptive rate transmission.

B. MAC Layer Payoff Function

Assume that the link transmission is successful in a time slot if the average attained SINR at the corresponding receiver is greater than some threshold named . Therefore, the link should adjust its power to hit this minimum threshold in the presence of the other links interference. The PAM problem under this assumption is analyzed in the cross layer framework in [5] when considering its interaction with the transport layer. According to the results of [5] and using them in the reverse direction, the MAC layer link payoff function is defined as:

$$u_j(q_j, p_j, \mathbf{q}\ _j, \mathbf{p}\ _j) = \log(q_j) + \ _j \log(\bar{\ }_j \quad) \quad \lambda_j q_j p_j \tag{3}$$

where $\ _j, \lambda_j$ are appropriate constants. The payoff function of link j has three terms. The first two terms show the link utility and the third one is the cost of transmission with persistence probability q_j and power p_j. In general, a link can increase its long term average data rate, by increasing its persistence probability or its power. The former leads to more frequent access to the channel, increases the first term in (3), while the latter provides higher SINR in each access, increases the second term in (3). The cost function, $\lambda_j q_j p_j$, increases monotonically with the link persistence probability and power. In fact, it shows the link tradeoff for transmitting with higher persistence probability or higher power.

If $\ _j \ll 1$, $\forall j \in \mathcal{N}$, then the second term in (3) acts as a pure barrier function. With this assumption, the link objective is to maximize its persistence probability while adjusting its power to hit the minimum required average threshold . In section IV we use this assumption to analyze some aspects of the game. The link pricing factor, λ_j, should be chosen appropriate according to the number of active links at the MAC layer which is discussed in section IV. This factor can be selected adaptively if there exists knowledge about the network topology.

C. Interpretation Of Payoff Function in PAMG

The medium is utilized by a link in time and space when it adjusts its persistence probability and power respectively. In PAMG each link selfishly decides on its persistence probability and power to maximize its payoff from the medium. The first term in (3), shows that the higher is the link persistence probability, the more the link utility from the medium will be. The log function is used to provide the fairness among links. The second term reflects the effect of the link power on the utility function. It acts as a barrier function to ensure the minimum required average SINR threshold, .

Satisfying the minimum average SINR, the link utility from the medium is determined by the link persistence probability, i.e., the frequency at which the link attempts to access the channel. Therefore, the best decision of each link is transmitting in all time slots. This, however, causes to severe mutual interference and degrades the network performance. To relief this problem, a cost function should be provided in the payoff function to coordinate the selfish decisions of links to have better network utilization of medium. This cost is applied by the last term in (3).

IV. PAMG Analysis

The Nash equilibrium study of a game can be used to predict the outcome of the game where each player decisions are based on self-optimization. Briefly, at the Nash equilibrium no player can improve his payoff by making individual changes in his decisions [9]. Therefore, the decision of link j by self-optimizing of (3), which is denoted by $\tilde{q_j}, \tilde{p_j}$ is computed by solving: $\frac{\partial u_j(.)}{\partial q_j} = 0, \frac{\partial u_j(.)}{\partial p_j} = 0$. Taking the differentiation and doing some simplification, we have:

$$\tilde{q_j} = \frac{1}{\lambda_j \tilde{p_j}} \tag{4}$$

$$\tilde{p_j} = I_j + \frac{\ _j}{\lambda_j \tilde{q_j}} \tag{5}$$

Where $I_j = \frac{p_j}{\ _j} = \frac{\sum_{k \neq j} G_{jk} p_k q_k + \eta}{G_{jj}}$ is the effective interference at the receiver of link j.

We note that adjusting the power to satisfy (5) is equivalent to hit $\ _j^{tar}$ given in (6).

$$\bar{\ }_j^{tar} = \ + \frac{\ _j}{\lambda_j q_j I_j} \tag{6}$$

Lemma 1: Provided that $\lambda_j, \forall j \in \mathcal{N}$ is sufficiently large and $\ _j, \forall j \in \mathcal{N}$ is sufficiently small, using the self-optimizing scheme by each link we have $\bar{\ }_j \rightarrow \ , \forall j \in \mathcal{N}$.

Proof: According to (3) if $\ _j$ is sufficiently small, link j should attempt to adjust its power to the minimum required that hit because using a higher power there is no increase in the link utility while its cost will increase. Now if $\lambda_j, \forall j \in \mathcal{N}$ is sufficiently large, all link can achieve the SINR threshold. The reason is that substituting (4) in (1) we have:

$$\bar{\ }_j = \frac{G_{jj} p_j}{\sum_{k \neq j} G_{jk} p_k q_k + \eta} = \frac{G_{jj} p_j}{\sum_{k \neq j} \frac{G_{jk}}{\lambda_k} + \eta} \tag{7}$$

which shows that $\bar{\ }_j > \ , \forall j \in \mathcal{N}$ can be achieved with the mentioned condition. ■

A. Existence and Uniqueness of Nash Equilibrium

We first investigate the existence of the Nash equilibrium for PAMG.

Proposition 1: There exists a Nash equilibrium for PAMG under the assumptions in lemma 1.

Proof: The existence of the Nash equilibrium is proved by showing that the strategy space of each user is a nonempty compact and convex set of $\mathbb{R}^2$ and $u_j(\mathbf{q}, \mathbf{p})$ is continuous in $(\mathbf{q}, \mathbf{p})$ and quasi-concave in (q_j, p_j) [10]. In PAMG, the link strategy space is the box $[0\ 1] \times [p_{min}\ p_{max}]$ which is a convex and compact subset of $\mathbb{R}^2$. Also, according to (3), the payoff function is continuous in $(\mathbf{q}, \mathbf{p})$. We show that the Hessian of u_j is negative definite and hence u_j is strictly and therefore quasi concave in (q_j, p_j). We have,

$$\nabla^2_{(q_j, p_j)} u_j(.) = \begin{matrix} \frac{1}{q_j^2} & \lambda_j \\ \lambda_j & \frac{\delta_j\ _j^2}{(\ _j\ \beta)^2 p_j^2} \end{matrix}$$

Since $\frac{1}{q_j^2} < 0$, the sufficient condition for the Hessian to be negative definite is [11]:

$$\frac{\ _j\ _j^2}{(\ _j\quad)^2 p_j^2 q_j^2} \quad \lambda_j^2 > 0 \tag{8}$$

Using lemma 1, if $\ _j \rightarrow 0$ then $\ _j \rightarrow\ $. Therefore, the first term in the left hand side of (8) tends to infinity and the inequality is satisfied. ■

Proposition 2: The best response of link j in PAMG is given by: $p_j^* = \max\{p_{min}, \min\{p_{max}, \tilde{p_j}\}\}, q_j^* = \min\{1, \frac{1}{\lambda_j p_j^*}\}$ where $\tilde{q_j}, \tilde{p_j}$ are computed using (4) and (5).

Proof: The proof is clear by noting that (4) and (5) are obtained by unconstrained optimization of a strictly concave function. Also, $p_j = p_{max}$ and $q_j = 1$ are the largest power and persistent probability that can be assigned to a link. ■

Therefore, according to (6) in PAMG the power control scheme is reduced to a simple SINR based target hitting algorithm and adjusting the power, the persistence probability is adjusted by a simple formula in (4). The following proposition shows that the Nash equilibrium of PAMG is also unique.

Proposition 3: The Nash equilibrium for PAMG is unique.

Proof:

To prove the uniqueness of the Nash equilibrium, it is sufficient to show that the best response function of each player for the power is a standard function [12]. The reason is that according to the utility function, each player adjusts his persistence probability according to (4). Hence, adjusting the power, the link persistence probability is determined uniquely. In other words, if all trajectories of the best response function of power control converge to a unique point, the same will happen for the powers. Let $\mathcal{T}(\mathbf{p}) = (\mathcal{T}_1(\mathbf{p}), \ldots, \mathcal{T}_N(\mathbf{p}))$ denotes the best response vector of links for power updates. According to (4-5), we have:

$$\mathcal{T}_j(\mathbf{p}) = \begin{cases} \frac{I_j}{1\quad _j}, & if\ q_j < 1 \quad (9) \\ I_j + \frac{\ _j}{\lambda_j}, & if\ q_j = 1 \quad (10) \end{cases}$$

It is easy to show that both (9) and (10) are standard functions of $\mathbf{p}$, i.e, they have positivity, monotonicity, and scalability properties. ■

Therefore under the assumptions in lemma 1, PAMG has a unique Nash equilibrium point. Since the link power control is a standard function, it converges to its fixed point asynchronously in a distributed implementation. The feasibility of distributed and asynchronous implementation is crucial for the PAM algorithm since the link gets feedback from its receivers with frequency not greater that its persistence probability. This feedback is necessary for the power control algorithm.

B. The PAM Algorithm

Based on the PAMG, Algorithm 1 is proposed as the PAM algorithm for the MAC layer of ad hoc networks.

Algorithm 1 The PAM Algorithm

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(1) Initialization: For each link $j \in \mathcal{N}$ choose $p_j(t_0) \in P_j, q_j(t_0) \in Q_j$ randomly.
(2) Power Update: Each link updates its power in time slots $t = t_1, t_2, \ldots$ when attempts to access the channel according to:

$$p_j(t_k) = \max\{p_{min}, \min\{p_{max}, \frac{\bar{\gamma}_j^{tar}}{\bar{\gamma}_j(t_{k-1})} p_j(t_{k-1})\}\} \quad (11)$$

where $\bar{\gamma}_j^{tar}$ comes from (6).
(3) Persistence Probability Update: Each link updates its persistence probability to:

$$q_j(t_k) = \min\{1, \frac{1}{\lambda_j p_j(t_k)}\} \quad (12)$$

V. Efficiency of PAMG

We already showed that under the lemma 1 assumptions, there exists unique Nash equilibrium point for PAMG and presented the PAM algorithm reached to that equilibrium. Another important question is about the efficiency of that equilibrium. The efficiency of a Nash equilibrium can be investigated by examining its Pareto efficiency [10]. Briefly, a Nash equilibrium is Pareto optimal if it is not possible for a subset of players to increase their utility without hurting any players.

Proposition 4: The Nash equilibrium solution of PAMG is Pareto optimal provided that $_j, \forall j \in \mathcal{N}$ is sufficiently small.

Proof: Assume that the Nash equilibrium point of PAMG is denoted by $(\mathbf{q}^*, \mathbf{p}^*)$. We must show that $\nexists\, (\mathbf{q}', \mathbf{p}')$: $u_j(\mathbf{q}', \mathbf{p}') \quad u_j(\mathbf{q}^*, \mathbf{p}^*), \forall j \in \mathcal{N}$ and $u_k(\mathbf{q}', \mathbf{p}') > u_k(\mathbf{q}^*, \mathbf{p}^*)$ for some $k \in \mathcal{N}$ and $(\mathbf{q}', \mathbf{p}')$ is feasible.
By contradiction, let there exists such $(\mathbf{q}', \mathbf{p}')$. Without loss of generality, let $q'_j = \phi_j q^*_j, \phi_j \quad 1\ \forall j \in \mathcal{N}$ and $\phi_k > 1$ for some $k \in \mathcal{N}$. First we note that if such a $\mathbf{q}'$ exists, then for the corresponding $\mathbf{p}'$, we have: $p'_j = \psi_j p^*_j, \psi_j > 1,\ \forall j \in \mathcal{N}$. The reason is that for a given link j, using the new persistence probability vector, the interference at the receiver of j is strictly increased and to hit the minimum required threshold, , the link must increase its power. Now we consider two cases.
If there exists a link like k that $q^*_k = 1$, its payoff will decrease and this is a contradiction. The reason is that while its utility doesn't change, because $q^*_k = q'_k = 1$ and $_j \to 0$, its cost $C'_k = \lambda_k \psi_k p_k{}^*$ will increase because $\psi_k > 1$.
If all links persistence probability are less that one, then according to (4), $q^*_j p^*_j \lambda_j = 1\ \forall j \in \mathcal{N}$. The necessary condition to have increasing in the utility function with the new persistence probability, is: $\frac{\partial u_j(q'_j, p'_j)}{\partial \phi_j} = \frac{1}{\phi_j}$ $\psi_j q^*_j p^*_j \lambda_j \quad 0\ \forall j \in \mathcal{N}$. Therefore, we must have:

$$1 \quad \phi_j \psi_j q^*_j p^*_j \lambda_j = 1 \quad \phi_j \psi_j \quad 0 \quad (13)$$

This leads to $\phi_j \psi_j \leq 1$, which is a contradiction since $\phi_j, \psi_j > 1$. ■

The pricing factor should be selected appropriately based on an estimate of the number of active links. Therefore, it is important to investigate the sensitivity of the PAMG equilibrium to this parameter. The following sensitivity analysis shows that the equilibrium of PAMG is not very sensitive to the absolute pricing factor. According to (5), the PAMG solution for link j power is independent of λ_j provided that $_j$ is small enough. Also, using (4) the persistence probability variation is related to the cost factor by:

$$\Delta q_j = \frac{\partial q_j}{\partial \lambda_j} \Delta \lambda_j = \frac{1}{p_j \lambda_j^2} \Delta \lambda_j \quad (14)$$

Equ. (14) shows that the persistence probability changes have inverse relation to the square of the cost factor and hence are not much sensitive to the absolute value of this parameter.

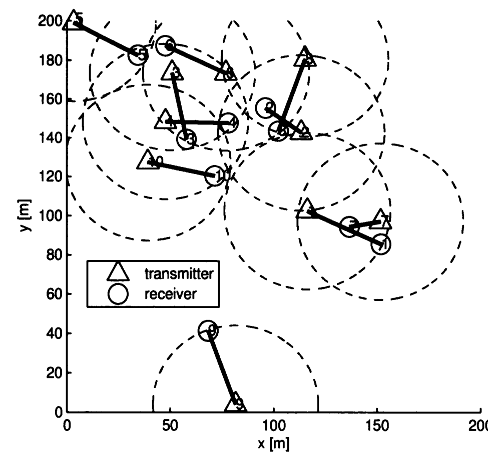

Fig. 1. Random Network Topology

VI. Numerical Evaluation

For numerical study of the PAM algorithm, a random network topology is generated with $N = 10$, $L = 200\ [m]$, and $R = 40\ [m]$ according to the system model in II and is shown in Fig.(1). The corresponding transmitters and receivers are connected together and included in a circle to show the high level interference regions of the medium. For simulation we set $_j = 0.05$, $\lambda_j = 100$, $\forall j \in \mathcal{N}$. Other simulation parameters are: $= 4$, $d_0 = 10\,[m]$, $\eta = 5 \times 10^{\ 15}\,[mW]$, $p_{max} = 500\,[mW]$, $p_{min} = 0.1\,[mW]$.

Applying the PAM algorithm, the variations in the links average SINR till the convergence are shown in Fig. 2. In simulation, link j parameters are updated with frequency $\frac{1}{q_j}$ since it can get feedback from the corresponding receiver with this frequency. The final links SINR vector is:
$\bar{} = [10.5, 11.2, 10.5, 10.5, 10.5, 10.5, 13.7, 10.5, 12.3, 10.7]$.

The final values of links persistence probabilities and powers, in [mW], after the algorithm convergence are:
$\mathbf{q} = (0.10, 1.00, 0.05, 0.60, 0.15, 0.08, 1.00, 0.03, 1.00, 1.00)$
$\mathbf{p} = (103.2, 4.7, 216.0, 16.5, 67.5, 125.5, 1.8, 322.3, 2.6, 8.0)$.

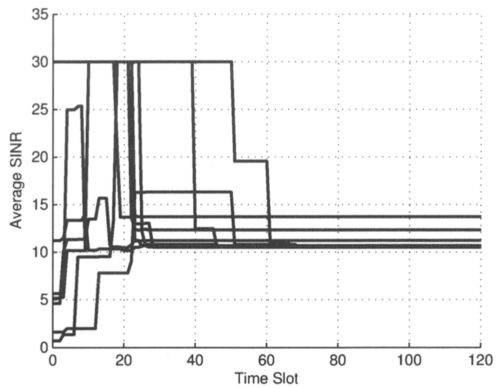

Fig. 2. Variations on Links SINR Till the Algorithm Convergence

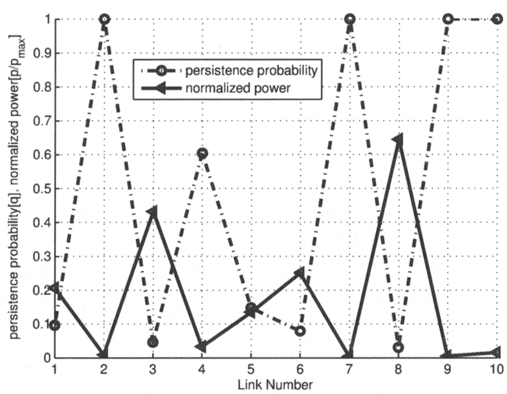

Fig. 3. Links Persistence Probability and Normalized Power at Equilibrium

These parameters are depicted in Fig. 3 which shows the the relation of the final link parameter to its location in the network. We can see from this figure that some links, e.g., link numbers 2,4,7,9,10, converge to higher persistence probability and lower power while some other, e.g., link numbers 1,3,5,6,8, converge to higher power and lower persistence probability. According to the network topology in Fig. 1, we see the latter links are those which their receivers are exposed to severe interference from the neighboring links. This result is reasonable because in a dense region with high level of interference, theses links should decrease their persistence to mitigate the mutual interference. On the other hand, using higher power these links have higher SINR in each access. This result emphasizes that the link persistence and power should be adjusted simultaneously according to the link location in the network.

Since all the links are to hit the average SINR threshold, we can use the sum of all links persistence probabilities as a measure of the algorithm efficiency in the network. In following we use $U = \sum_{j\in\mathcal{N}} \log(q_j)$ to consider the fairness criterion as well in our discussions. The efficiency of the PAM algorithm with this measure is $U^{PAM} =$ 13.82.

The results of PAM algorithm are compared with a scenario in which only powers are adjusted. In this scenario, the links persistence are equal and increased incrementally until the point at which all links could hit the minimum required SINR threshold, by power control. In other words, the links persistence are the maximum possible for which there exists a power vector that all links could hit . The powers are adjusted using the target hitting algorithm in [12]. Simulation results show that the maximum possible achievable persistence for all links is 0.15. Therefore, using this scheme the efficiency would be $U' =$ 18.97. Comparing the results, we conclude that by simultaneous persistence and power control using the PAM algorithm the efficiency of resource assignment is increased about 37%. This gain is achieved because the PAM algorithm considers the links location in the network in adjusting the links parameters.

VII. Conclusion

The paper presents a distributed algorithm for power-aware MAC in ad hoc wireless networks. The algorithm is based on a non-cooperative game that is used for persistence probability and power control by each active link in the network. An appropriate payoff function is defined for each link and it is proved that with mild conditions and by self optimizing, the game converges to its unique Nash equilibrium. Simulation results and discussions are provided to investigate the performance of the algorithm. Comparing the results with a scenario in which only powers are adjusted, it is shown that considering the link location by simultaneous control of persistence and power, the proposed algorithm yields better efficiency.

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