Pool Resource Management Based on Early Collision Detection in Random Access of Massive MTC over LTE

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Abstract

Collision in Random Access (RA) procedure of Long-Term Evolution (LTE) networks is the main problem in supporting massive Machine-Type Communications (MTC). Current solutions for resolving MTC Devices (MTCDs) collisions in the LTE incur additional overhead to existing MTCDs which may not be efficient for cost-effective MTCDs. Also, the lack of a proper Physical Resource Blocks (PRBs) management scheme may waste a large number of PRBs and leads to poor RA performance upon massive access of MTCDs . To this end, a simple PRBs management scheme based on the early preamble collision detection is proposed in this paper to improve the RA throughput of massive MTC. By exploiting the pool of RA resources and upon collision detection, a proper number of PRBs is granted to each collided preamble by estimating the number of collided MTCDs. To sustain the number of consumed PRBs around a certain threshold, the number of contending MTCDs is regulated using an appropriate Access Class Barring (ACB) factor. Simulation results show that the RA throughput and average RA delay can be improved using the proposed solution compared to the previous schemes; while the expected number of consumed PRBs in each RA procedure is sustained below a certain threshold.

Keywords: Machine-Type Communications (MTC), Long-Term Evolution (LTE), random access procedure.

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1. Introduction

Machine-to-Machine (M2M) communications is the main component of the Internet of Things (IoT) ecosystem which enables the connectivity between smart devices and remote servers via underlying communication networks [1]. One promising solution for connectivity among large distributed Machine-Type Communications Devices (MTCDs) is Long-Term Evolution (LTE) networks [2, 3]. However, the initial connection setup through Physical Random Access Channel (PRACH) of the LTE is designed to support the small number of Human-Type Communications (HTC) devices. Unlike the HTC, communicating a large number of MTCDs over the LTE will drastically reduce the average number of successfully transmitted access attempts during the connection setup in the LTE which is called Random Access (RA) throughput. That is, the shared RA resources in the PRACH and ever increasing number of coordinated MTCDs over the time will reduce the RA throughput because of the collisions. Moreover, it is crucial to keep the complexity of MTCDs ultra-low to ensure the cost of IoT networking as low as possible. Therefore, for the LTE to be able to accommodate the massive MTCDs (a list of used acronyms is provided in Table 1), a significant change in the ongoing evolution of current cellular network was considered crucial [2, 3, 4].

To enhance the RA throughput and reduce the access delay of the MTCDs in the LTE, various academic and industrial efforts have been made in the literature. In this regard, deploying different MTCDs' power levels [5], using Raptor codes [6], exploiting successive interference cancelation [7], and two-stage access control [8] have been proposed to resolve the collision during the RA procedure. Although these methods increase the complexity and power consumption of MTCDs which may not be efficient for power- and cost-efficient MTCDs. Without any additional overhead on existing MTCDs and by using the pool of available PRBs and early preamble collision, we show that how RA throughput of the LTE could be improve to support the massive MTC. To this end, we use the statistics of idle preambles, preambles chosen by no MTCDs, and successful preambles, preambles chosen by just one MTCD to assign a proper number of PRBs for each collided preamble. Then, each collided MTCD selects one PRB randomly from PRBs granted for Message 3 (Msg3) to transmit its schedul-

Table 1: Commonly used acronyms in the paper

Acronym	**Meaning**
3GPP	3rd Generation Partnership Project
ACB	Access Class Barring
eNB	evolved Node B
HTC	Human-Type Communications
IoT	Internet of Things
LTE	Long-Term Evolution
M2M	Machine-to-Machine
Msg	Message
MTC	Machine-Type Communications
MTCD	MTC Device
PRACH	Physical Random Access Channel
PRB	Physical Resource Block
PUSCH	Physical Uplink Shared Channel
RA	Random Access
RAR	Random Access Response
RA-RNTI	Random Access Radio Network Temporary Identifier

ing request message at the third step of the RA procedure. Using proposed scheme, the RA throughput is improved significantly compared to previous related works.

In addition, the average number of consumed PRBs of the proposed scheme is computed to find out how RA throughput could be improved by compromising on the total number of PRBs for the RA. Using a proper ACB factor, we show that the PRBs utilization during RA procedure of the proposed scheme will be always sustained around a threshold defined for the PRBs utilization in the MTC, i.e., R_T. In fact, the main idea behind the proposed scheme is to avoid over-and under-utilization of the total number of available PRBs assigned for the MTC to maximize the RA throughput. Unlike the traditional RA procedure which only considers the number of preambles for computing the optimal ACB factor, we show that by considering the number of PRBs, the RA capacity can be fully exploited to reduce the RA delay as much as possible. We should emphasize that the complexity of the receiver at the eNB may be increased using the proposed scheme because the eNB should be equipped with a proper collision detection scheme. To the best of our knowledge, this work is the first one which studies the effects of early preamble collision detection on the pool RA resource management. This pool resource management scheme can be deployed in scenarios with high traffic of emergency events as it shows good performance in dense networks.

In the rest of this paper, related works and background on RA procedure have been respectively reviewed in Section 2 and 3. The considered system model is presented in Section 4. In Section 5, the proposed resource allocation scheme is introduced. Section 6 is dedicated to the analysis of the achievable RA throughput and average consumed PRBs of the proposed scheme. Performance evaluation results are demonstrated in Section 7 before concluding in Section 8.

2. Related Works

To alleviate the collision in the shared access scenario of the RA procedure of the LTE, Access Class Barring (ACB) has been introduced by 3GPP to sustain the RA throughput near its optimum value upon massive access provided that the ACB factor is properly adjusted by the eNB. In the ACB, each active MTCD, i.e., MTCDs which request for connection establishment with the eNB, initiates the RA procedure if its randomly drawn number between 0 and 1 be less than the ACB factor. The ACB factor is computed by dividing the number of allocated preambles for the MTC by the estimated number of active MTCDs. The eNB needs a load estimation technique to estimate the number of active MTCDs. In [9], by considering the collision status, the authors developed a Markov chain to dynamically estimate the traffic load of the MTCDs and thus regulate the ACB factor. Wu et. al. in [10] used the statistics of sequential collision and idle slots to estimate the number of active event-driven MTCDs in a short time interval. Using the drift analysis, they also investigated the stability of their scheme under heavy and light load traffic conditions. In [11], a two-stage preamble transmission scheme has been investigated in which at the first stage, the MTCDs are balanced among several periods by exploiting an auction model. Then, at the second stage, a method for estimating the number of MTCDs in each period is introduced. The authors in [12] investigated two methods based on the maximum likelihood and Kalman filter to estimate the number of MTCDs during each RA opportunity. In these works, it is assumed that a specific number of preambles is assigned for the MTC. In [13] and [14], the preamble assignment problem with the objectives of reducing access delay and energy consumption for different groups of MTCDs have been investigated.

Mostly focused on preamble assignment and load estimation methods, a few number of works consider improving the achievable throughput of traditional RA procedure. Notice that the upper bound for the RA throughput can be achieved in an ideal scenario in which all available PRBs are successfully exploited by massive MTCDs during each RA procedure without any additional overhead on existing MTCDs. In order to improve the throughput of traditional RA, authors in [5] used the capture effect at the third step of the RA procedure to successfully decode the scheduling request message transmitted by the MTCD with a high enough transmission power. In this work, one message may be decoded if the interference caused by the transmissions from other MTCDs are sufficiently low compared to the received signal power. Shirvanimoghaddam et. al. in [6] incorporated the superposition modulation based on Raptor codes to resolve the collision of the MTCDs used the same PRBs. In [7], the authors employed the successive interference cancelation to resolve the collided data packets and thus ensure the quality of service requirements of different MTC groups. The authors in [8], exploit the early preamble collision detection to bar the access attempts of the collided MTCDs with an appropriate probability on granted PRBs at the third step of RA procedure. A special preamble has been used in [15] to grant unused PRBs to active MTCDs which did not pass the ACB check. In order to reduce the collision at the eNB, the authors in [16] used distributed queue approach to organize the scheduling requests of the MTCDs into a virtual queue in an autonomous and distributed manner.

Although by these approaches the throughput and delay of RA procedure could be somewhat improved, the high RA delay of the MTCDs upon massive access attempts still severely limits the RA performance of the LTE for time critical message dissemination in industrial applications. For example, real-time monitoring and control of substations in the power grid often contains the transmission of mission-critical messages with the guaranteed delays of a few milliseconds, in some cases below 10*ms*. Current LTE and subsequent developments barely able to meet such access delay. In addition, these approaches increase the complexity and power consumption of the MTCDs which may not be suitable for power- and cost-efficient MTCDs. In this paper, we show that how RA throughput could be improved using proposed resource allocation scheme compared to the traditional ACB-based RA procedure and what has been introduced in [8], named CARA in this paper, without increasing the complexity and energy consumption of the MTCDs. It is noteworthy that in the traditional ACB-based RA procedure there is not any access control or resource allocation for collided MTCDs at the third step of RA procedure upon early preamble collision detection. The main steps of the traditional RA procedure is introduced in Section 3.1

3. Background on RA Procedure

3.1. Connection Setup in the LTE

In the LTE, each MTCD is required to establish an air interface connection with the evolved Node B (eNB) to request data transmission. Due to the sporadic MTCDs traffic and for power efficiency, 3rd Generation Partnership Project (3GPP) specifies idle and connected modes for the MTC, in which the MTCDs will be in the idle mode with a low power consumption until they trigger for the data transfer. Upon triggering, MTCD starts the so-called RA procedure to be connected. Before each RA procedure, the eNB broadcasts the configuration of the PRACH along with the allocated preambles for the MTC in the cell [2, 17]. The RA procedure consists of four main messages as follows.

In the first message, Msg1, the MTCD selects one preamble signature randomly from the dedicated preambles for the MTC and sends it toward the eNB via the PRACH. It is probable that multiple MTCDs transmit the same preamble at this stage. The primary goal of Msg1 is to enable the eNB to estimate the MTCD's transmission timing. In the second message, Msg2, then, the eNB replies by the RAR message to inform the MTCDs about the subsequent identifier, Random Access Radio Network Temporary

Identifier (RA-RNTI), uplink Physical Resource Blocks (PRBs) granted for transmitting third RA message, and the time-frequency slot in which the preamble has been detected. After receiving Msg2 from the eNB, the MTCDs transmit the third message, Msg3, through the granted PRBs on the Physical Uplink Shared Channel (PUSCH) [2, 18].

In the case that multiple MTCDs transmit the same signature in the same time-frequency slot, they will receive the same PRBs at Msg2. The collided MTCDs, MTCDs with the same transmitted preambles, will send their scheduling requests on the same uplink time-frequency PRBs that lead to the collision. Finally, in the fourth message, Msg4, the contention resolution message will be transmitted for those messages that have been successfully decoded by the eNB. The MTCDs which did not receive Msg2 or Msg4 considered as failed MTCDs and are configured for the upcoming PRACH opportunities [2, 17, 18].

3.2. Early Preamble Collision Detection

In the LTE, the eNB may be able to detect the preamble collision at the first step of RA procedure and thus does not assign the PRBs for such preambles [19].

The possibility of early preamble collision detection, i.e., preamble collision detection at Msg1, and its benefits are studied in recent literature of MTC over LTE. In [20], Zhang and et. al. exploited narrow guard bands on the PRACH for preamble collision detection. In [21], the authors used the tagged preambles which consist of both preamble and tag Zadoff-Chu sequences with different root numbers to detect the collision at Msg1. In [22], a preamble collision resolution scheme based on the capturing multiple timing advance values of a tagged preamble has been introduced for stationary MTCDs. Zhang in [23] proposed an extended guard band in the PRACH to avoid false collision detection. Also, a collision detection scheme based on the delay spread of received signals at PRACH receivers has been designed in [24]. The receiver assumes that a collision has happened if two correlation peaks of received signals at PRACH are separated in time by more than the expected maximum delay spread of the corresponding cell. See [24] for more details. The advantages of early preamble collision detection has been introduced in [8], where the authors proposed an access control scheme at the third step of RA procedure to bar each collided MTCD by applying an early collision detection scheme at the first step.

To better understand how the eNB may detect the preamble collision based on the delay spread of the received signals, we provide an example in which four MTCDs select a preamble with index 0 and transmit it toward the eNB at the same PRACH opportunity. The MTCDs' distances to the eNB are considered to be $2,7,4$ and $5km$. More details about the simulation parameters are provided in Section 7.2. In Fig. 1, the maximum discrete correlation between the received signals at the eNB and 64 predefined preambles codes are shown. The propagation delay of each received signal could be determined by the location of its corresponding shifted impulse from the origin, i.e., t_1, t_2, t_3, and t_4. Using these values, the eNB is able to find the difference between the least and the most delays, $t_2 - t_1$, and hence the preamble collision detection is possible by comparing this difference against an appropriate threshold value. In our example we have $t_2 - t_1 \simeq 17\mu s$. If the bandwidth of PRACH is assumed $b = 1.08MHz$, our expected threshold would be $\frac{1}{2b} = 0.46\mu s$ [25]. Since $t_2 - t_1$ is greater than this threshold, eNB is able to detect the collision of the preamble with index 0.

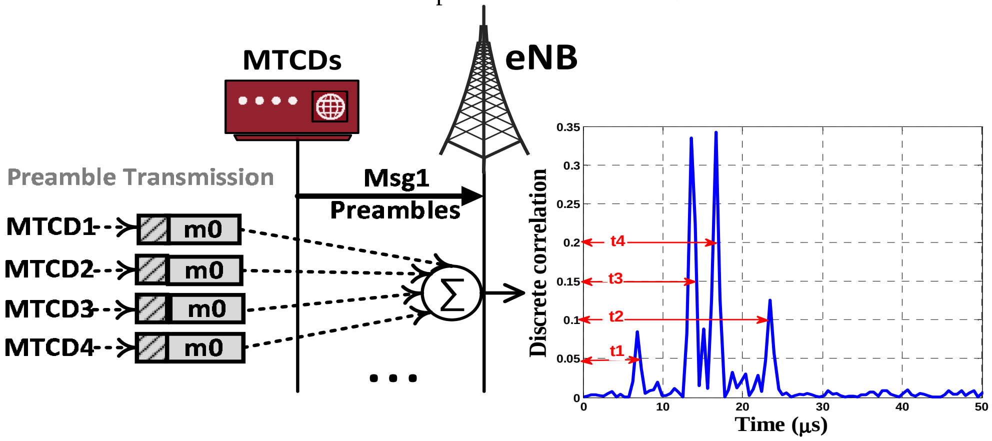


Figure 1: Preamble propagation delay evaluation in RA of LTE networks (SNR= -6dB)

In this paper, deploying early preamble collision detection we propose a pool RA resource management scheme to boost the performance of the LTE upon huge arrivals of MTCDs. In this scheme, the number of PRBs allocated for the MTC, denoted by R_T, has been fully exploited in an efficient manner to serve a greater number of MTCDs in each RA procedure. It is noteworthy that the radio resource manager unit at the eNB may allocate a dedicated number of the PRBs for the MTC to separate the RA resources between MTC and HTC [3, 17, 26, 27]. Partitioning orthogonal RA resources could separate the impact of the MTC over the HTC. However, the question is given R_T, how to optimally grant these PRBs to the MTCDs to maximize the RA throughput. To this end, we may use the advantages of early preamble collision detection to grant a proper number of PRBs to each collided preamble.

4. System Model

We consider a single cell in which N_T MTCDs communicate with an eNB located at the center of the cell. The MTCDs are activated during the activation time T_a according to Beta Probability Density Function (PDF) with parameters $\alpha = 3$ and $\beta = 4$, and $T_a = 10s$ as $g(t) = \frac{t^{\alpha-1}(T_a-t)^{\beta-1}}{T_a^{\alpha+\beta-1}Beta(\alpha,\beta)}$, where $Beta(\alpha,\beta)$ denotes the Beta function. This traffic model is introduced by 3GPP in [27] to characterize the highly coordinated arrivals of the massive MTCDs over a bounded time interval. The active MTCDs initiate the RA procedure after passing the ACB scheme with probability factor q. Fig. 2 shows different stages of the considered system model.

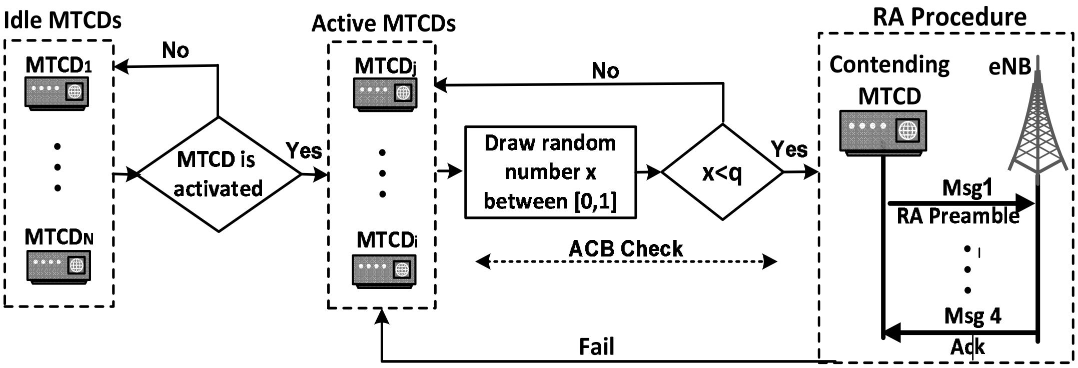


Figure 2: The procedure of connection establishment in the system model.

In the ACB mechanism [2, 27], each active MTCD initiates the RA procedure with probability q, see Fig. 2. In this paper, the active MTCDs which successfully passed the ACB mechanism named contending MTCDs and start the proposed RA procedure.

Let μ denote the average number of contending MTCDs which are allowed to initiate the RA procedure, called accepted number of MTCDs in this paper; $N_{act}(k)$ and $N_c(k)$ be the random variables of the numbers of, respectively, active and contending MTCDs at the k^{th} PRACH opportunity. Throughout the paper we use upper case letter X to show random variable and a lower case x to specify a sample of X. We first assume that the eNB knows the exact number of active MTCDs in each PRACH opportunity, i.e., $n_{act}(k)$. Then we evaluate the performance of the proposed scheme in the presence of the load estimation error. The eNB updates q at the k^{th} opportunity using $q(k) = min\left\{1, \frac{\mu}{n_{act}(k)}\right\}$, and then broadcasts $q(k)$ in the cell [9, 12, 27]. Having $n_{act}(k)$ and $q(k)$, the accepted number of MTCDs at the k^{th} opportunity can be obtained as $n_{act}(k)q(k)$. The active MTCDs which cannot pass the ACB check, named as failed MTCDs and retry for the next PRACH opportunity without any back-off time.

Our analysis deals with a perfect channel model where all transmitted preambles are successfully decoded by the eNB. In Section 7.2, the effect of wireless channel impairments on the successful preamble detection and thus on the performance of proposed scheme are investigated. Let M_0, M_1, and M_{cl} be the random variables that respectively represent the numbers of idle preambles, preambles chosen by no MTCDs, successful preambles, preambles chosen by just one MTCD, and collided preambles, preambles chosen by multiple MTCDs where $M_0 + M_1 + M_{cl} = M$.

Also, assume that the eNB fails to detect the preamble collision at the first stage of RA procedure with probability p_e; and M_e denote the fraction of M_{cl} for them collisions are not detected by the eNB. So we can simply conclude that the eNB can observe $M_{cl} - M_e$ collided preambles in each RA opportunity with probability $1 - p_e$, on average. As it is discussed in Section 3.2, in a real scenario of M2M, the eNB may be equipped with a proper collision detection technique to distinguish the preamble collision at the first step of RA procedure.

In the proposed scheme it is probable that in addition to those MTCDs with successfully transmitted preambles, some other MTCDs which have sent the same preamble and hence collided at the first stage are also able to pass the RA procedure. Let N_{s1} and N_{s2} refer, respectively, to the random variables of the number of successful access attempts from the first and second group of MTCDs. The total number of successful access attempts or RA throughput is denoted by $N_s = N_{s1} + N_{s2}$. Without loss of generality, we assume that each scheduling request message can be transmitted via one PRB as in [7]. The number of consumed PRBs by the contending MTCDs in each RA procedure is shown by random variable R.

The objective is to increase the number of successful connection establishments during the RA procedure of massive MTCDs to meet the desired RA delay of emergency situations. That is, by using the information of probable preamble collision detection at the first stage of RA procedure, we can determine the proper number of granted PRBs to each collided preamble to boost the performance of RA procedure. Using proposed scheme, the RA throughput of the traditional RA procedure can be increased from the expected number of successful transmitted preambles, i.e., $N_{s1} = \mathbb{E}[M_1]$, to $N_{s1} + N_{s2}$. The main system parameters and notations are summarized in Table 2.

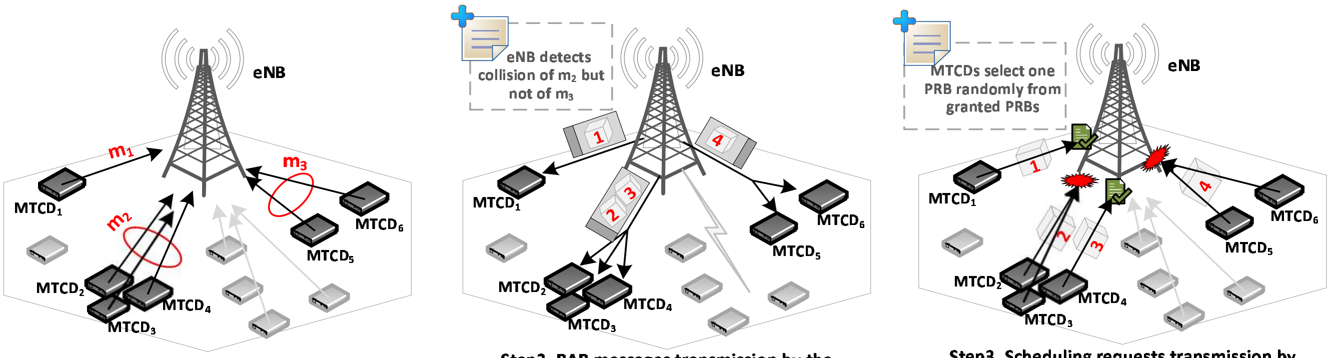
5. Proposed RA procedure

The proposed scheme consists of four consecutive steps as follows. In the first step, each active MTCD which passes the ACB check chooses a preamble randomly from M available preambles, and sends it toward the eNB. For each transmitted preamble, the eNB proceeds to the collision detection stage if it can decode the transmitted preamble successfully at the second step of RA procedure. In the case that the eNB cannot detect a preamble collision, it will grant one PRB for that preamble similar to the traditional RA procedure. However, when the collision is detected, the eNB should compute the proper number of granted PRBs for each detected collided preamble which is shown by N_r. Despite the traditional RA procedure, in the proposed scheme, the eNB can assign multiple PRBs for each received preamble in the case of collision detection. These PRBs are embedded in the RAR messages which are transmitted by the eNB at the third step of RA procedure. After communicating the RAR messages by the eNB, each contending MTCD which finds the identifier of the transmitted preamble in the RAR messages sends its scheduling request in the third step to the eNB through one PRB randomly selected from the granted PRBs for that preamble. It is probable that

Table 2: System Notation

Notation	**Description**
N_T	Total number of MTCDs
T	Time interval between two PRACH opportunities
M	Number of preambles
M_0	Number of idle preambles
M_1	Number of successful preambles
M_{cl}	Number of collided preambles
M_e	Number of collided preambles which cannot be detected by the eNB
N_{act}	Number of active MTCDs
N_c	Number of contending MTCDs
N_r	Number of granted PRBs to each collided MTCD
μ	Accepted number of contending MTCDs
N_{s1}	Number of successful access attempts resulted from successful preambles
N_{s2}	Number of successful access attempts resulted from collided preambles
N_s	Total number of successful access attempts
q	Probability of access class barring
p_e	Probability of detecting no collision

more than one collided MTCD select the same PRB and thus collision will happen at this stage. Summing up the collision in the proposed RA procedure occurs either if the preamble collision cannot be successfully detected by the eNB or multiple collided MTCDs transmit their Msg3 using the same PRB to the eNB. Eventually, the eNB sends acknowledge message in the fourth step.



Step1. Preambles transmission by MTCDs

Step2. RAR messages transmission by the eNB

Step3. Scheduling requests transmission by MTCDs via granted RBs for m1,m2, and m3

Figure 3: Steps of proposed scheme.

An example of proposed scheme is shown in Fig. 3 where $MTCD_1$ sends preamble m_1, $MTCDs_{2,3,4}$ send preamble m_2, and $MTCDs_{5,6}$ send preamble m_3 to the eNB among other MTCDs participating in the RA procedure. In step 2 of this figure, we assume that the eNB could not detect the collision of m_3 and thus grants only one PRB in Msg2 to preamble m_3 similar to that for m_1. That is, it assigns PRB_1 and PRB_4 for m_1 and m_3, respectively. For m_2, the eNB computes N_r as 2 via the information of all idle and collided preambles in current RA procedure and then grants PRB_2 and PRB_3 to m_2. In step 3 of Fig. 3, $MTCDs_{2,3}$ transmit their scheduling requests through PRB_2 which lead to the collision in Msg3, while the transmission of $MTCD_4$ used PRB_3 is successful. The other collision arises when the eNB cannot detect preamble collision and thus assigns only one PRB to $MTCDs_{5,6}$. Notice that in the traditional RA procedure, all scheduling request messages transmitted by $MTCD_{2,3,4,5,6}$ will be failed because of the collision at the first stage.

In order to compute proper N_r, we recall that the objective is to enhance the RA throughput by efficiently using PRBs available for the MTC. Define R_T as the desired number of PRBs which are expected to be utilized by the contending MTCDs in each RA opportunity. In order to maximize $\mathbb{E}[N_s]$, the optimum of μ, i.e., μ^*, should be equal to R_T to ensure on average one PRB is available for each contending MTCD. That is $\mu^* = R_T$. It is emphasized that in the traditional ACB-enabled RA procedure, the accepted number of contending MTCDs is regulated around M to ensure one preamble for each contending MTCD on average, i.e., $\mu^* = M$ [8, 9, 12, 28]. This leads to the over-and under-utilization of PRBs when $m_1 + m_e > R_T$ and $m_1 + m_e < R_T$, respectively.

Since in the case of collision detection, the eNB cannot be able to find how many MTCDs have selected the collided preamble, it should estimate N_r as follows. Assume that the eNB observed $m_0(k)$ and $m_1(k) + m_e(k)$ idle and successful preambles at the k^{th} RA opportunity, respectively. Here, $N_r(k)$ can be computed by dividing the estimated number of collided MTCDs, i.e., $\mu^* - m_1 - m_e$, by the number of observed collided preambles, i.e., $M - m_0(k) - m_1(k) - m_e(k)$, as:

$$N_r(k) = \left\lceil \frac{\mu^* - m_1(k) - m_e(k)}{M - m_0(k) - m_1(k) - m_e(k)} \right\rceil \tag{1}$$

which indicates that each collided preamble has been chosen by how many MTCDs, on average. Now, by substituting μ^* in (1) by its value, i.e., R_T, we have:

$$N_r(k) = \left\lceil \frac{R_T - m_1(k) - m_e(k)}{M - m_0(k) - m_1(k) - m_e(k)} \right\rceil$$

where $R_T - m_1(k) - m_e(k)$ denotes the total number of PRBs which will be granted to the collided preambles assuming that one PRB are granted to each observed successful preamble, i.e., one PRB to each of $m_1(k) - m_e(k)$ observed successful preamble. These PRBs will be equally divided among observed collided preambles, i.e., $M - m_0(k) - m_1(k) - m_e(k)$. Notice that the eNB will grant $N_r(k)$ PRBs to each of $M - m_0(k) - m_1(k) - m_e(k)$ collided preambles provided that at least one preamble collision has been observed by it. Hence, in this case, the expected number of granted PRBs, $\mathbb{E}[R]$, would be equal to the sum of the observed successful preambles, $m_1(k) + m_e(k)$, and $N_r\Big(M - m_0(k) - m_1(k) - m_e(k)\Big)$; which results in R_T as it is expected.

The performance of proposed scheme is compared with the CARA and traditional RA procedure. To this end, in the next section, the RA throughput and average PRBs utilization of these schemes are computed. It is noted that in the CARA, the access attempts of the collided MTCDs at the third step of RA procedure are barred upon early preamble collision detection. However, in the proposed scheme, instead of barring collided MTCDs which incurs complexity at the MTCDs, we have used the pool of available PRBs to grant multiple PRBs to each collided preamble and thus increase the probability of successful transmission. As it is shown in Section 7, using proposed scheme the performance of RA procedure is improved compared to the CARA.

6. Analysis of Proposed Scheme

In this section, we calculate the achievable RA throughput and the average number of consumed PRBs of the proposed scheme in each RA procedure, analytically. We consider a specific PRACH opportunity where there are n_c contending MTCDs. It is noted that $n_c(k)$ can be replaced by the expected number of contending MTCDs at the k^{th} RA opportunity, i.e., $\mathbb{E}[N_c(k)]$. $\mathbb{E}[N_c(k)]$ can be obtained using the ACB factor as $q(k)\mathbb{E}[N_{act}(k)]$; where $\mathbb{E}[N_{act}(k)]$ includes the expected number of new arriving requests at the k^{th} opportunity, $\mathbb{E}[A(k)]$ and unsuccessful access attempts from previous RA opportunity. Hence, $\mathbb{E}[N_{act}(k)]$ can be computed as $\mathbb{E}[N_{act}(k)] = \mathbb{E}[A(k)] + \mathbb{E}[N_{act}(k-1)] - \mathbb{E}[N_s(k-1)]$, where $\mathbb{E}[A(k)]$ is obtained from Beta distribution as $\mathbb{E}[A(k)] = N_T \int_{t_{k-1}}^{t_k} g(t)dt$, and $\mathbb{E}[N_{act}(k-1)] - \mathbb{E}[N_s(k-1)]$ denotes the number of failed MTCDs from $(k-1)th$ opportunity. In Section 6.1 we explain how to compute $\mathbb{E}[N_s(k-1)]$.

In the following, the RA throughput and average PRBs utilization are computed as sum of two possible cases that there is at least one collided preamble and the case that there is no collision. Notice that these two cases include all possible events that may occur when n_c MTCDs randomly select one preamble from M preambles. Index k is ignored in the following for simplicity of presentation

6.1. RA Throughput

In order to obtain the RA throughput, we need to obtain the joint probability mass function (pmf) of $M_1 = m_1$ and $M_0 = m_0$. Let E_i be the event that i specific preambles are chosen by at most one MTCD. From [12], the occurrence probability of E_i is obtained as

$$\Pr(E_i; n_c, M) = \sum_{j=0}^{i} \binom{i}{j} \binom{n_c}{j} \frac{j!}{M^j} (1 - \frac{i}{M})^{n_c - j}, \quad i = 1, ..., M \tag{2}$$

Using Inclusion-Exclusion principal [29, 30], the probability that exactly k preambles are chosen by at most one MTCDs can be computed as given in (3).

$$\Pr(M_1+M_0=\upsilon;n_c,M)=\binom{M}{\upsilon}\sum_{i=\upsilon}^{M}(-1)^{i-\upsilon}\binom{M-\upsilon}{i-\upsilon}\Pr(E_i;n_c,M) \tag{3}$$

When there is at least one collision, i.e., $m_1+m_0<M$, the probability that $M_1=m_1$ and $M_0=m_0$ can be obtained, by the following pmf [12]:

$$\begin{aligned}\Pr(M_1=m_1,M_0=m_0;n_c,M)=&\frac{\binom{M}{m_1+m_0}\binom{m_0+m_1}{m_1}\binom{n_c}{m_1}m_1!(m_{cl})^{n_c-m_1}}{M^{n_c}}\\&\times\Pr(M_1+M_0=0;n_c-m_1,m_{cl}),\quad m_{cl}>0\end{aligned} \tag{4}$$

where m_{cl} is equal to $M-m_1-m_0$. The first term of the right-hand side (RHS) of (4) indicates the joint probability of $M_1=m_1$ and $M_0=m_0$ provided that all remaining preambles, i.e., m_{cl}, are chosen by more than one MTCDs; the second term is the probability of m_{cl} collided preambles when n_c-m_1 MTCDs select m_{cl} preambles. This is equivalent to $\Pr(M_1+M_0=0;n_c-m_1,m_{cl})$ which is computed using (3).

Given that $M_1=m_1$ and $M_0=m_0$, we can now compute the RA throughput of the proposed scheme for $m_{cl}>0$. Recall that the eNB will grant N_r PRBs to each collided preamble if it can detect the preamble collision. Assume that the eNB fails to detect the collision of m_e collided preambles among all m_{cl}. By considering (1), N_r can be written for m_e collided preambles and an arbitrary value of μ as:

$$N_r^{m_e}=\frac{\mu-m_1-m_e}{m_{cl}-m_e} \tag{5}$$

Let ζ_{cl}^{i} be the event that a specific collided preamble named preamble m, has been chosen by i contending MTCDs. Given that m is chosen by more than one MTCD, the occurrence probability of ζ_{cl}^{i} can be computed using (6)

$$\Pr(\zeta_{cl}^{i}|i\geqslant 2)=\frac{B(i;n_c,\frac{1}{M})}{\sum_{l=2}^{n_c}B(l;n_c,\frac{1}{M})} \tag{6}$$

where $B(i;n_c,\frac{1}{M})$ denotes the binomial pmf[1]. Here, the expected number of successful access attempts among all collided MTCDs using granted PRBs for preamble m is

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[1] In this paper, the binomial pmf, $\binom{N}{n}p^n(1-p)^{n-1}$, is shown by $B(n;N,p)$.

equal to the multiplication of $N_r^{m_e}$ by the probability that only one out of i collided MTCDs transmits its scheduling request with probability $\frac{1}{N_r^{m_e}}$ and others do not, i.e., $N_r^{m_e} B(1; i, \frac{1}{N_r^{m_e}})$, where $i \geqslant 2$. Since $m_{cl} - m_e$ collided preambles have been observed by the eNB, the total number of successful transmissions resulted from observed collided preambles, N_{s2}, can be obtained for all i as given in (7).

$$\begin{aligned} \mathbb{E}[N_{s2}|M_1 = m_1, M_0 = m_0, M_e = m_e] &= \sum_{i=2}^{n_c} (m_{cl} - m_e) N_r^{m_e} B(1; i, \frac{1}{N_r^{m_e}}) \Pr(\zeta_{cl}^{i} | i > 2) \\ &= (\mu - m_1 - m_e) \frac{n_c}{N_r^{m_e}} \Big(\frac{(M - \frac{1}{N_r^{m_e}})^{n_c - 1} - (M-1)^{n_c - 1}}{M^{n_c} - (M-1)^{n_c-1}(M - 1 + n_c)} \Big) \end{aligned} \tag{7}$$

Also, each collided preamble is independently recognized as successful preamble with collision detection error p_e, thus, the probability that $M_e = m_e$ is computed using $B(m_e; m_{cl}, p_e)$. For all m_e we have:

$$\mathbb{E}[N_{s2}|M_1 = m_1, M_0 = m_0] = \sum_{m_e=0}^{m_{cl}-1} \mathbb{E}[N_{s2}|M_1 = m_1, M_0 = m_0, M_e = m_e] B(m_e; m_{cl}, p_e) \tag{8}$$

Therefore, for $m_{cl} > 0$, the RA throughput of the proposed scheme can be computed as sum of successfully transmitted preambles, i.e., $N_{s1} = m_1$, and $\mathbb{E}[N_{s2}|M_1 = m_1, M_0 = m_0]$ as:

$$\mathbb{E}[N_s|M_1 = m_1, M_0 = m_0, m_{cl} > 0] = m_1 + \mathbb{E}[N_{s2}|M_1 = m_1, M_0 = m_0] \tag{9}$$

For the case that m_{cl} is zero, the RA throughput would be equal to n_c because all contending MTCDs will successfully pass the RA procedure. Here, we have $m_1 = n_c$ and $m_0 = M - n_c$, and thus $\Pr(M_1 = m_1, M_0 = M - m_1)$ for all m_1 that $m_1 \neq n_c$ is zero. Hence, the probability that $M_1 = n_c$ and $M_0 = M - n_c$ can be computed as:

$$\begin{aligned} \Pr(M_1 = n_c, M_0 = M - n_c; n_c, M) &= \Pr(M_1 + M_0 = M; n_c, M) \\ &= \frac{M!}{(M - n_c)! M^{n_c}}, \quad n_c \leqslant M \end{aligned} \tag{10}$$

It is clear that for $n_c > M$, $\Pr(M_1 = n_c, M_0 = M - n_c)$ would be equal to zero because there is at least one collision.

In sum, the RA throughput of the proposed scheme for all possible m_1 and m_0 can be written as given in (11).

$$\mathbb{E}[N_s] = \sum_{\substack{m_1=0 \\ m_1 \neq n_c}}^{M} \sum_{m_0=0}^{M-m_1} \mathbb{E}[N_s | M_1 = m_1, M_0 = m_0, m_{cl} > 0] \Pr(M_1 = m_1, M_0 = m_0; n_c, M)$$
$$+ n_c \Pr(M_1 = n_c, M_0 = M - n_c; n_c, M) \tag{11}$$

According to Lemma 6.1, at the worst case if the early collision detection is not successful, i.e., $p_e = 1$, the throughput of the proposed scheme reaches to the throughput of traditional ACB-enabled RA procedure.

Lemma 6.1. *For the proposed scheme, if $p_e = 1$, the RA throughput would be reduced to:*

$$\mathbb{E}[N_s] = n_c(1 - \frac{1}{M})^{n_c - 1} \tag{12}$$

Proof. Please see Appendix A. ☐

It is noted that in an excessive load condition, n_c can be approximated by μ using the ACB factor. In this condition and for $\mu \gg 1$, we have $m_{cl} = M$ and $m_0, m_1 = 0$. Hence $\mathbb{E}[N_s]$ in (11) would be approximated by $(1 - p_e)e^{-1}\mu$ for $\mu \gg 1$; where μ can be replaced by its optimal value, i.e., R_T.

6.2. Average Utilization of PRBs

In a similar way to Section 6.1, the average number of consumed PRBs in each RA opportunity, $\mathbb{E}[R]$, is computed as sum of two possible cases that $m_{cl} > 0$ and $m_{cl} = 0$. When there is at least one collision, $\mathbb{E}[R]$ is the sum of the granted PRBs for both successful and collided preambles observed by the eNB. Assuming $M_e = m_e$, then, the eNB grants one PRB to each of $m_1 + m_e$ observed successful preambles and $N_r^{m_e}$ PRBs to each of $m_{cl} - m_e$ observed collided preambles. Hence, by considering the pmf of $M_e = m_e$, $\mathbb{E}[R]$ for $m_{cl} > 0$ is equal to:

$$\mathbb{E}[R | M_1 = m_1, M_0 = m_0, m_{cl} > 0] = m_1 + m_{cl} B(m_{cl}; m_{cl}, p_e) + \sum_{m_e=0}^{m_{cl}-1} \Big((m_{cl} - m_e) N_r^{m_e} + m_e \Big)$$
$$\times B(m_e; m_{cl}, p_e) = \mu - (\mu + m_0 - M) B(m_{cl}; m_{cl}, p_e) \tag{13}$$

For the case that $m_{cl} = 0$, $\mathbb{E}[R]$ as RA throughput would be equal to n_c.

Therefore, the average number of consumed PRBs of the proposed scheme for all m_1 and m_0 can be written as:

$$\mathbb{E}[R] = \sum_{\substack{m_1=0 \\ m_1 \neq n_c}}^{M} \sum_{m_0=0}^{M-m_1} \mathbb{E}[R|M_1 = m_1, M_0 = m_0, m_{cl} > 0] \Pr(M_1 = m_1, M_0 = m_0; n_c, M) \\ + n_c \Pr(M_1 = n_c, M_0 = M - n_c; n_c, M) \tag{14}$$

which with substituting $\mathbb{E}[R|M_1 = m_1, M_0 = m_0, m_{cl} > 0]$ by its value from (13) and approximating n_c by μ for a high load condition, after simplify, yields:

$$\mathbb{E}[R] = \mu - \sum_{\substack{m_1=0 \\ m_1 \neq n_c}}^{M} \sum_{m_0=0}^{M-m_1} (\mu + m_0 - M) B(m_{cl}; m_{cl}, p_e) \Pr(M_1 = m_1, M_0 = m_0; n_c, M) \tag{15}$$

From (15) it can be found that the maximum of $\mathbb{E}[R]$ is bounded by μ, also, when $B(m_{cl}; m_{cl}, p_e) \ll 1$, it would be equal to μ. This emphasizes that we can sustain the average number of consumed PRBs below a predefined threshold, μ, using the proposed scheme while the RA throughput can be improved as it is computed using (11).

According to Lemma 6.2, for the case that $p_e = 1$, $\mathbb{E}[R]$ of the proposed scheme reduced to the one of traditional RA procedure.

Lemma 6.2. *For the case that $p_e = 1$, the average number of consumed PRBs of the proposed scheme is equal to:*

$$\mathbb{E}[R] = M\Big(1 - (1 - \frac{1}{M})^{n_c}\Big), \tag{16}$$

which is the same as both CARA and traditional ACB-enabled RA procedures.

Proof. Please see Appendix B. □

Notice that in the CARA the eNB will grant one PRB to each of preambles chosen by the MTCDs [8], and thus $\mathbb{E}[R]$ is equal to the expected number of selected preambles by the MTCDs, i.e., $\mathbb{E}[R] = M\Big(1 - (1 - \frac{1}{M})^{n_c}\Big)$. For $n_c \gg 1$, $\mathbb{E}[R]$ of the CARA would be equal to the number of available preambles, i.e., M. In the traditional ACB-enabled

RA procedure, the PRBs are granted to those preambles which are observed by the eNB as successful preambles and thus $\mathbb{E}[R]$ can be obtained using Lemma 6.3.

Lemma 6.3. *For traditional ACB-enabled RA procedure we have:*

$$\mathbb{E}[R] = p_e M\Big(1-(1-\frac{1}{M})^{n_c}\Big)+(1-p_e)n_c(1-\frac{1}{M})^{n_c-1} \tag{17}$$

Proof. Please see Appendix C. □

From Lemma 6.3 it can be found that for $n_c \gg 1$ and $p_e = 1$, the average PRBs utilization reaches, respectively, to Mp_e and expected number of selected preambles by the MTCDs. That is for $n_c \gg 1$ all preambles will be collided and thus the total number of granted PRBs depends on the collision detection error multiplied by M.

7. Performance Evaluation

In this section, the performance of the proposed scheme is evaluated in terms of RA throughput, average RA delay of the MTCDs, and average PRBs utilization. The RA delay is defined as the time duration required for an arbitrary active MTCD to successfully gain access to the eNB. We compare the performance of the proposed scheme with the CARA [8] and traditional ACB-enabled RA procedure.

For the simulations, a given number of preambles, $M = 54$ [27], is considered to be available for N_T MTCDs in each RA procedure. The total number of MTCDs is assumed to be $N_T = 40000$. We assume that there is no limit on the total number of granted PRBs for the MTC, however, as it is shown through simulations this number is always below R_T using an appropriate ACB factor. We use the PRACH configuration with index 6 in which RA opportunity happens every $5ms$ [27]. At first, we assume that the eNB can exactly estimate the number of active MTCDs before each PRACH opportunity and thus broadcasts the optimal ACB factor in the cell. Then in Section 7.2, the effect of load estimation error on the performance of the proposed scheme is examined. The power ramping step and index of backoff parameter are considered to be $0dB$ [31] and 0 [18], respectively. We assume that each scheduling request of the

MTCD can be transmitted using one of the granted PRBs. Upon failure in the RA, the contending MTCDs retry for the next PRACH opportunity with no limit on the number of retransmission attempts. In fact, this assumption can be easily relaxed by increasing the total number of MTCDs. The simulation results are the average of 20 independent runs. Also, to show the achievable RA throughput and consumed PRBs in each RA procedure of the proposed scheme, $\mathbb{E}[N_s]$ in Figs. 5 and 7, and $\mathbb{E}[R]$ in Figs. 6 and 8 are drawn for an excessive load condition. In these figures, the results of average RA delay are shown for $N_T = 40000$.

In the following figures, the analysis of $\mathbb{E}[N_s]$ and $\mathbb{E}[R_T]$ are respectively computed according to (11) and (14) for the proposed scheme, (12) and (17) for the traditional RA procedure, and what have been discussed in [8] and in paragraph 4 of Section 6.2 for the CARA. The analysis results of average RA delay which is shown by $\mathbb{E}[D]$ is computed as in [8]. In addition, as it is discussed in Section 5, the values of μ^* in the proposed and traditional RA procedures are, respectively, equal to R_T and M; which is equal to 54 in figures 4, 5, and 6. For the CARA, we compute μ^* as in [8] for each value of p_e in these figures.

7.1. Comparisons with CARA and traditional RA Procedure

In Fig. 4, the maximum achievable RA throughput of the proposed scheme is compared with the maximum throughputs of the CARA and traditional RA procedure for collision detection errors $p_e = 0, 0.4$. From Fig. 4, the throughput of the proposed scheme for both scenarios of $p_e = 0$ and 0.4 are greater than that for the CARA and traditional RA procedure. This causes the total service time to the active MTCDs to be decreased as it is indicated in Fig. 4. Since achieving enhanced throughput in both proposed and CARA schemes depends on the success of the early collision detection at the eNB, the RA throughput of both schemes are reduced by increasing the detection errors as it is expected from Fig. 4.

In Fig. 5 the achievable RA throughput normalized by R_T along with the MTCDs' average RA delay, and in Fig. 6, the average number of consumed PRBs normalized by R_T of the proposed scheme are compared with the ones of CARA and traditional RA procedure for different p_e and $R_T = 54$. From Fig. 5 we find that $\mathbb{E}[D]$ and $\frac{\mathbb{E}[N_s]}{R_T}$

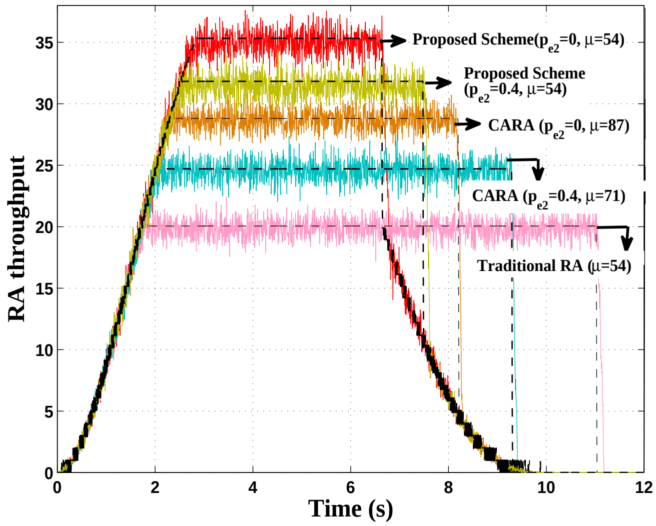

Figure 4: The RA throughput of the proposed scheme, CARA, and traditional RA scheme over the time for $R_T = 54$ (dashed lines:analysis; solid lines:simulation).

of the proposed scheme are respectively less and greater than that for the CARA and traditional RA. By increasing p_e, the RA delay and achievable throughput of both proposed and CARA schemes are respectively increased and reduced until they reach to the ones of traditional RA procedure at $p_e = 1$. Notice that for $p_e = 1$ both CARA and proposed scheme behave as traditional RA procedure. Also, since $\mathbb{E}[D]$ and $\frac{\mathbb{E}[N_s]}{R_T}$ of the traditional RA only depend on the number of successful preambles, they did not vary by changing p_e.

Fig. 6 shows that the average number of consumed PRBs of the proposed scheme is equal to R_T as it is expected, however, this figure implies significant under-utilization of a given R_T for two other schemes. By increasing p_e in the traditional RA procedure, it is more probable that the eNB recognizes some collided preambles as successful and grants one PRB to each of them which causes $\frac{\mathbb{E}[R]}{R_T}$ to be increased. At $p_e = 1$, $\mathbb{E}[R]$ equals to the expected number of selected preambles because the eNB cannot differentiate which preamble has been chosen by more than one MTCD. As it is discussed in Section 6.2, for the CARA $\mathbb{E}[R]$ is always equal to the expected number of selected

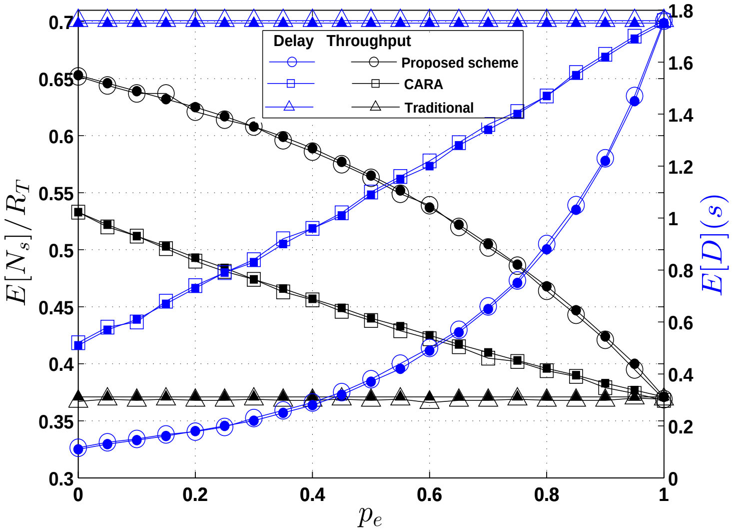

Figure 5: Normalized RA throughput and average RA delay of the proposed scheme, CARA, and traditional RA scheme versus different p_e for $R_T = 54$ (filled markers:analysis).

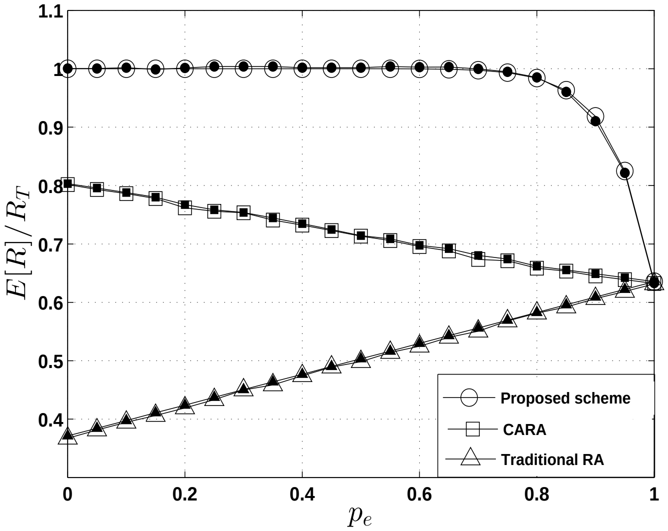

Figure 6: Normalized average number of consumed PRBs of the proposed scheme, CARA, and traditional RA scheme versus different p_e for $R_T = 54$ (filled markers:analysis).

preambles which itself depends on μ^*. It is noted that in the CARA, μ^* is reduced as p_e increases which leads to the reduction in $\frac{\mathbb{E}[R]}{R_T}$ by the increase in collision detection error. We note that for the lower p_e, $\frac{\mathbb{E}[R]}{R_T}$ of the proposed scheme is equal to 1, while it is reduced for the higher p_e until it reaches to the average PRBs utilization of the traditional RA procedure. The reason is that for higher p_e, it is likely that the eNB cannot detect any preamble collision.

In Figs 7 and 8, the effects of the accepted number of contending MTCDs on $\mathbb{E}[N_s]$, $\mathbb{E}[D]$, and $\mathbb{E}[R]$ have been shown for $p_e = 0.3$. From Fig. 7, $\mathbb{E}[N_s]$ of the proposed scheme outperforms two other schemes for all considered μ including both light and heavy load conditions. Also, the RA delay of the proposed scheme is much less than the CARA and traditional RA procedure. The inset of Fig. 7 shows that $\mathbb{E}[D]$ of the proposed scheme reaches to its minimum value for $\mu = 90$ and almost remain constant for $\mu > 90$. The reason is that for these μ the number of active MTCDs is much less than the achievable throughput by the proposed scheme and thus all active MTCDs can be served upon one or two RA opportunity. In addition, we see that $\mathbb{E}[N_s]$ of the proposed scheme is increased by increasing μ, while it is diminished for both CARA and traditional RA procedures after it reaches to its maximum value for each scheme. It is noted that for both CARA and traditional RA procedure, μ^* depends on M, while in the proposed scheme it is determined in accordance with R_T. This behavior of proposed scheme emphasizes that in an excessive load condition, the RA throughput can be improved almost linearly by increasing μ. The factor that limits this improvement is the average number of consumed PRBs, as it is shown in Fig. 8.

According to Fig. 8, $\mathbb{E}[R]$ of the proposed scheme is linearly increased by increasing μ with slope one; i.e., $\mathbb{E}[R]$ is always equal to the accepted number of contending MTCDs. Hence, it can be concluded that with the proposed scheme, $\mathbb{E}[R]$ can be sustained around a given number of available PRBs, R_T, by adjusting the ACB factor as $\frac{\mu^*}{n_{act}}$ that $\mu^* = R_T$. In this situation, $\mathbb{E}[N_s]$ can be enhanced as much as possible for a given number of PRBs, as it is shown in Fig. 7. Also, from Fig. 8, we see that $\mathbb{E}[R]$ of the CARA is increased by increasing μ up to $M = 54$. In other words, for higher μ, it is more probable that all preambles are selected by the MTCDs and thus $\mathbb{E}[R]$ of the CARA equals to M. For traditional RA procedure, $\mathbb{E}[R]$ reaches to p_eM because for higher μ, there are no successful preambles and thus the eNB assigns the PRBs to only undetected collided preambles.

7.2. Practical Issues

Up to now, we have assumed a perfect channel condition in our simulations which means there was no error in preamble detection at the eNB. In this section, the performance of the proposed scheme under preamble detection errors in a simulated wireless channel has been shown. Also, we implement the maximum likelihood estimator technique in [12] to evaluate the RA throughput of the proposed scheme in each RA procedure upon possible load estimation errors.

7.2.1. Preamble detection errors

In order to investigate the effects of wireless channel as well as unsuccessful preamble detection on the RA throughput, we perform a simulation study and provide the results in Fig. 9. In the considered scenario, the selected preambles by the MTCDs are configured and passed through a multipath fading channel model. Also, the additive

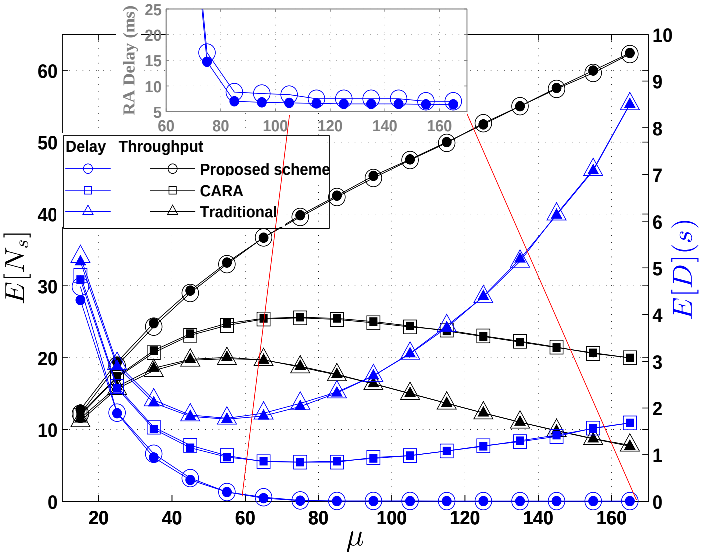

Figure 7: The RA throughput and delay of the proposed scheme, CARA, and traditional RA scheme versus different μ for $p_e = 0.3$. The inset shows that $\mathbb{E}[D]$ of the proposed scheme reaches to its minimum value and remains constant for $\mu > 90$ (filled markers:analysis).

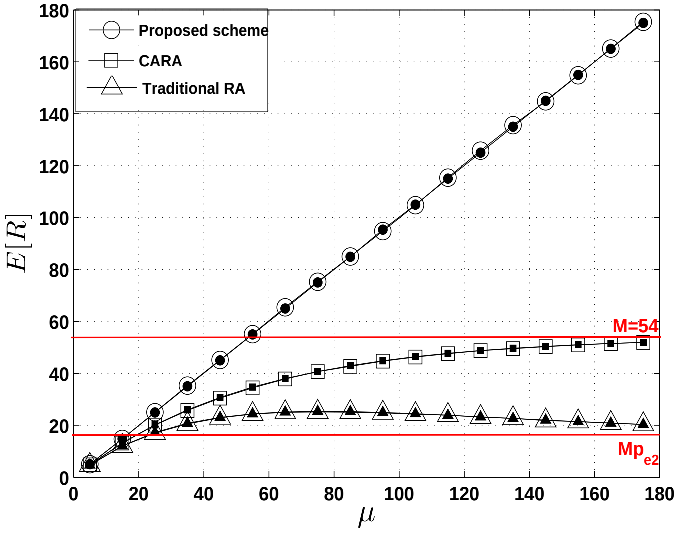

Figure 8: The average number of consumed PRBs of the proposed scheme, CARA, and traditional RA scheme versus different μ for $p_e = 0.3$ (filled markers:analysis).

white Gaussian noise is added to the considered PRACH before SC-FDMA demodulation. The eNB performs the preamble detection using the given channel configuration and user equipment specific setting. At the eNB's preamble detection module, the set of root sequences which are required for all predefined preamble indices is prepared. The correlation of the received signal and each root sequence is then calculated in the frequency domain. The preamble index is then extracted through searching the position of the detected peak at the correlator's output. The threshold of the correlator for preamble detection is specified according to the received signal power estimated on each antenna within PRACH bandwidth. Other important simulation parameters are as follows: initial phases of the line-of-sight component of all paths are randomly initialized according to the seed value, delay profile is set according to the Extended Typical Urban (ETU) model, and the delay profile power has been normalized. It is noted that successful preamble detection depends on the SNR of the received signals because as the received SNR is increased, it is more likely that the peaks appeared at the output of the correlator exceed the threshold for preamble detection. In Fig. 9, the throughputs of the proposed scheme and traditional RA procedure for $SNR = -6dB$ and $SNR = -12dB$ are shown. From this figure, we find that throughput of the proposed scheme is greater than that for the traditional RA scheme for both $SNR = -6dB$ and $SNR = -12dB$. In addition, as it is expected, by decreasing the SNR, the RA throughput of both RA schemes are decreased which is rooted to the unsuccessful preamble detection at the first stage of RA procedure.

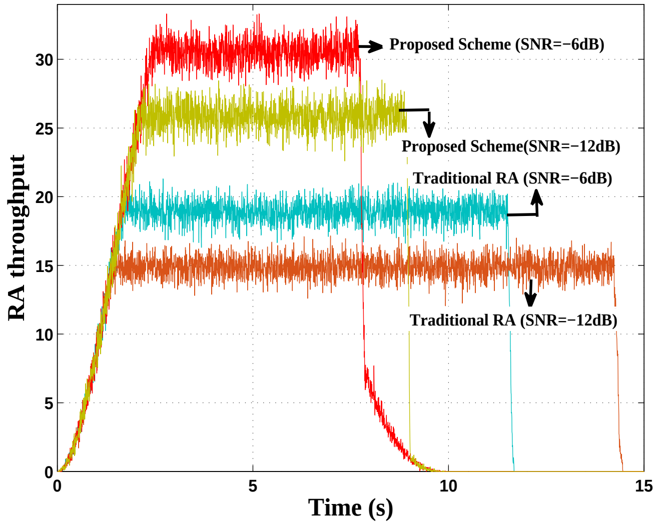

Figure 9: The effect of preamble detection error on RA throughput of proposed and traditional RA schemes for $p_e = 0.4$ and $\mu = 54$

7.2.2. Load estimation errors

We implement a maximum likelihood estimator as in [12] to determine the number of active MTCDs, n_{act}, before each PRACH opportunity. Recall that in a LTE-base MTC system, the eNB can observe m_0 idle preambles and $m_1 + m_e$ successful preambles after a RA procedure. At first, we assume that there is no collision detection error, i.e., $p_e = 0$, then, the effect of p_e on the accuracy of considered estimator and thus on achievable RA throughput is examined. By considering the joint pmf given by (4), the estimated number of contending MTCDs at the k^{th} opportunity, $\hat{n}_c(k)$, can be obtained by [12]:

$$\hat{n}_c(k) = \underset{n_c}{\arg\max}\ \Pr(M_1 = m_1, M_0 = m_0 | n_c) \tag{18}$$

where $\Pr(M_1 = m_1, M_0 = m_0|n_c)$ is substituted with (4) for a given n_c. We use the lookup table method introduced in [12] to solve (18). After obtaining $\hat{n}_c(k)$ form (18), the number of active MTCDs at the k^{th} PRACH opportunity can be estimated using the ACB factor from:

$$\hat{n}_{act}(k) = \frac{\hat{n}_c(k)}{q(k)}$$

Now the ACB factor at the $(k+1)th$ opportunity can be updated as:

$$q(k+1) = \begin{cases} \min\{1, \frac{\mu}{\hat{n}_{act}(k)+\mathbb{E}[A(k)]-m_1}\} & k \leq \gamma \\ \min\{1, \frac{\mu}{\hat{n}_{act}(k)-m_1}\} & k > \gamma \end{cases}$$

where γ denotes the number of PRACH opportunities within the activation time interval. For more information about this estimation method see [12]. By estimating the number of active MTCDs, the eNB broadcasts an updated value of ACB factor before each RA opportunity to control the number of contending MTCDs. In Fig. 10, the RA throughput of the proposed and traditional RA schemes is shown for $p_e = 0, 0.2$, and $\mu = 54$. As it is shown in Fig. 10, $\mathbb{E}[N_s]$ of the proposed and traditional RA schemes using this estimation method follow the corresponding ideal scenarios in Fig. 4 where the proposed scheme outperforms the traditional one. Also, by increasing the preamble detection error, the RA throughput of both schemes are reduced as it is expected.

8. Conclusion

In this paper, we deploy the early preamble collision detection at the eNB to design a new resource management scheme which improves the number of successful attempts upon massive access of the MTCDs. It is shown that the RA throughput can be enhanced for a given number of PRBs using a proper ACB factor. Comparing with similar works, it is found that the proposed scheme outperforms them in terms of RA throughput as well as RA delay. This improvement in RA throughput of the LTE may be beneficial for delay-sensitive M2M applications in the smart grid such as condition

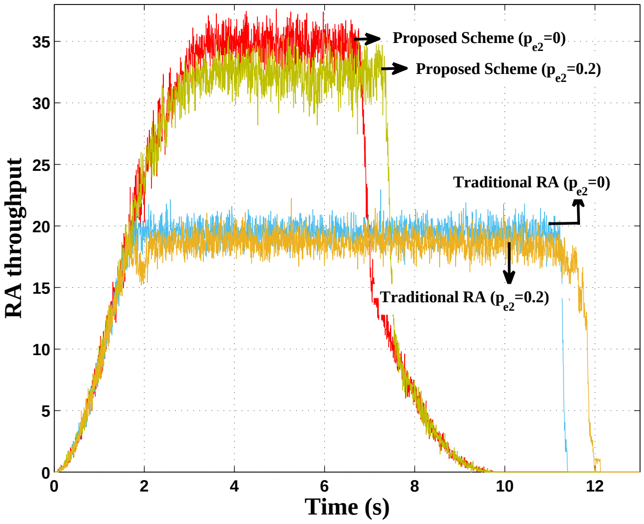

Figure 10: The effect of load estimator on RA throughput of the proposed and traditional RA schemes for $p_e = 0, 0.2$ and $\mu = 54$

monitoring. In future work, we will extend our resource allocation scheme to take the effects of some smart grid traffic classes.

Appendix A

Proof of Lemma III.1

In order to derive the throughput of proposed scheme which is given by (12), we compute $\mathbb{E}[N_s]$ using (11) for the case that $m_e = m_{cl}$, that is:

$$\mathbb{E}[N_s] = \sum_{\substack{m_1=0 \\ m_1 \neq n_c}}^{M} m_1 \sum_{m_0=0}^{M-m_1} \Pr(M_1 = m_1, M_0 = m_0; n_c, M) + n_c \Pr(M_1 = n_c, M_0 = M - n_c; n_c, M) \tag{A-1}$$

In order to compute (12), we continue to find the inner summation in (A-1) for a given m_1 and all possible m_0. By substituting (4) and (10) in (A-1) and after simplifying, we have

$$\sum_{m_0=0}^{M-m_1} \Pr(M_1 = m_1, M_0 = m_0; n_c, M) = \frac{n_c! M!}{M^{n_c} m_1!} \sum_{m_0=0}^{M-m_1} \sum_{i=0}^{m_{cl}} \sum_{j=0}^{i} \frac{(-1)^i (m_{cl}-i)^{n_c-m_1-j}}{m_0!(M_{cl}-i)!(i-j)!(n_c-m_1-j)!j!} \tag{A-2}$$

where m_{cl} is equal to $M - m_1 - m_0$. Now, by replacing i with $k - m_0$ and subsequently interchanging the order of summation in (A-2), we have

$$\begin{aligned} \sum_{m_0=0}^{M-m_1} \Pr(M_1 = m_1, M_0 = m_0; n_c, M) = \frac{n_c! M!}{M^{n_c} m_1!} \sum_{k=0}^{M-m_1} \sum_{j=0}^{k} \frac{(-1)^k (M-m_1-k)^{n_c-m_1-j}}{(k-j)!(M-m_1-k)!(n-j)!j!} \\ \times \sum_{m_0=0}^{k-j} (-1)^{m_0} \binom{k-j}{m_0} \end{aligned} \tag{A-3}$$

The value of the third summation in (A-3) is equal to zeros for all j that $j < k$. For $j = k$, (A-3) can be simplified as given in (A-4).

$$\Pr(M_1 = m_1; n_c, M) = \binom{M}{m_1} \sum_{i=m_1}^{M} (-1)^{i-m_1} \binom{M-m_1}{i-m_1} \binom{n_c}{i} \frac{i!}{M^i} (1 - \frac{i}{M})^{n_c-i} \tag{A-4}$$

which is the probability that m_1 preambles each are chosen by exact one MTCD. $\binom{n_c}{i} \frac{i!}{M^i} (1 - \frac{i}{M})^{n_c-i}$ in (A-4) denotes the probability that i specific preambles are chosen by only one MTCD as an special case of (3) with $j = i$. Also, notice that $\Pr(M_1 = n_c, M_0 = M - n_c; n_c, M) = \Pr(M_1 = n_c; n_c, M)$ as used in (A-2). Next, by substituting (A-4) in (A-1) and interchanging the order of summation, after simplifying, we have

$$\mathbb{E}[N_s] = \sum_{i=0}^{M} \frac{i(-1)^{i-1} M!}{M^i (M-i)!} (1 - \frac{i}{M})^{n_c-i} \binom{n_c}{i} \sum_{m_1=0}^{i-1} \binom{i-1}{m_1} (-1)^{m_1} \tag{A-5}$$

The inner summation is equal to zero for all i except for $i = 1$. As result, by selecting $i = 1$ in (A-5) the throughput of traditional RA procedure can be obtained as given in (A-6).

$$\mathbb{E}[N_s] = n_c (1 - \frac{1}{M})^{n_c-1} \tag{A-6}$$

Appendix B

Proof of Lemma III.2

In order to derive the expected number of consumed PRBs in the proposed scheme for $p_e = 1$, $\mathbb{E}[R]$ in (15) is simplified by replacing $B(m_{cl}; m_{cl}, p_e)$ by its value for $p_e = 1$, i.e., 1, as given in (B-1).

$$\mathbb{E}[R] = M - \sum_{m_1=0}^{M} \sum_{m_0=0}^{M-m_1} m_0 \Pr(M_1 = m_1, M_0 = m_0; n_c, M) \tag{B-1}$$

The inner summation of (B-1) can be simplified in a similar way to what is done in Appendix A, as:

$$\mathbb{E}[R] = M - \sum_{m_1=0}^{M} M \binom{M-1}{m_1} \sum_{i=m_1}^{M} (-1)^{i-m_1} \binom{M-m_1-1}{i-m_1-1} \binom{n_c}{i-1} \frac{(i-1)!}{M^{i-1}} (1-\frac{i}{M})^{n_c-i+1} \tag{B-2}$$

Now by interchanging the order of summation, after simplify, we have:

$$\mathbb{E}[R] = M - M \sum_{i=0}^{M} \frac{(-1)^{i-1}(M-1)!}{M^{i-1}(M-i)!} (1-\frac{i}{M})^{n_c-i+1} \binom{n_c}{i-1} \sum_{m_1=0}^{i-1} (-1)^{m_1} \binom{i-1}{m_1} \tag{B-3}$$

which is valid for $i = 1$ and is given by (B-4).

$$\mathbb{E}[R] = M - M(1-\frac{1}{M})^{n_c} \tag{B-4}$$

Appendix C

Proof of Lemma III.3

For traditional ACB-enabled RA procedure $\mathbb{E}[R]$ can be computed as sum of the successful preambles provided that the eNB can detect the preamble collision and all selected preamble if the eNB cannot detect the collision, that is;

$$\begin{aligned}\mathbb{E}[R] = \sum_{\substack{m_1=0 \\ m_1 \neq n_c}}^{M} \sum_{m_0=0}^{M-m_1} \Big(p_e(M-m_0) + (1-p_e)m_1\Big) \Pr(M_1 = m_1, M_0 = m_0; n_c, M) \\ + (M-m_0) \Pr(M_1 = n_c, M_0 = M - n_c; n_c, M)\end{aligned} \tag{C-1}$$

Using the Appendices A and B, (C-1) can be simplified as given in (C-2).

$$\mathbb{E}[R] = p_e M\Big(1-(1-\frac{1}{M})^{n_c}\Big) + (1-p_e)n_c(1-\frac{1}{M})^{n_c-1} \tag{C-2}$$

References

[1] K.C. Chen and S.Y. Lien, “Machine-to-Machine Communications: Technologies and Challenges,” *Ad Hoc Networks*, vol. 18, pp. 3-23, July 2014.

[2] C. A.Haro and M. Dohler, *Machine-to-Machine(M2M) Communications: Architecture, Performance and Applications*. Elsevier, 2015, 1st edn, pp. 158-164.

[3] K. Zheng, S. Ou, J. A.Zarate, M. Dohler, F. Liu, and H. Zhu, “Challenges of Massive Access in Highly Dense LTE-Advanced Networks with Machine-to-Machine Communications,” *IEEE Wireless Communications*, vol. 21, no. 3, pp. 12-18, June 2014.

[4] 3rd Generation Partnership Project (3GPP), “Standardization of Machine-type Communications (V0.2.2),” Dec. 2013. [Online] Available: http://www.3gpp.org/DynaReport/TDocExMtg–RP-63–30564.htm

[5] Z. Alavikia and A. Ghasemi, “A Multiple Power Level Random Access Method for M2M Communications in LTE-A Network,” *Transaction on emerging telecommunications technologies,* vol. 28, no. 6, June 2017.

[6] M. Shirvanimoghaddam, M. Dohler, and S. J. Johnson, “Massive Multiple Access Based on Superposition Raptor Codes for Cellular M2M Communications,” *IEEE Transaction on Wireless Communications,* vol. 16, no. 1, pp. 307-319, Jan. 2017.

[7] R. Abbas, M. Shirvanimoghaddam, Y. Li, and B. Vucetic, “Random Access for M2M Communications with QoS Guarantees,” *IEEE Transaction on Communications,* vol. 65, no. 7, pp. 2889-2903, July 2017.

[8] Z. Alavikia and A. Ghasemi, “Collision-Aware Resource Access scheme for LTE-based Machine-to-Machine communications,” *IEEE Transaction on Vehicular Technology,* vol. 67, no. 5, pp. 4683-4688, May 2018.

[9] H. He, Q. Mansouri, H. Song, W. Li, Y. Wang, and P. Ren, “Traffic-Aware ACB Scheme for Massive Access in Machine-to-Machine Networks,” *IEEE International Conference on Communications (ICC 2015), London, UK,* June 2015.

[10] H. Wu, C. Zhu, R. J.La, X. Liu, and Y. Zhang, “FASA: Accelerated S-ALOHA Using Access History for Event-Driven M2M Communications,” *IEEE/ACM Transactions on Networking*, vol. 21, no. 6, pp. 1904-1917, Dec. 2013.

[11] G-Y. Lin, and H-Y. Wei, “Auction-Based Random Access Load Control for Time Dependent Machine-to-Machine Communications,” *IEEE Internet of Things Journal*, vol. 3, no. 5, pp. 658-672, Oct. 2016.

[12] M. Tavana, V.S.Mansouri, and V. W.S.Wong, “Congestion Control for Bursty M2M Traffic in LTE Networks,” *IEEE International Conference on Communications (ICC 2015), London, UK*, June 2015.

[13] T. Kim, K. Seok-ko, I. Bang, and D. K. Sung, “A Random Access Scheme Based on a Special Preamble for Supporting Emergency Alarms,” *IEEE Wireless Communications and Networking Conference (WCNC 2014), Istanbul, Turkey*, pp. 1768-1773, Apr. 2014.

[14] O. Arouk, A. Ksentini, and T. Taleb, “Group Paging-Based Energy Saving for Massive MTC Accesses in LTE and Beyond Networks,” *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 5, pp. 1086-1102, May 2016.

[15] F. Morvari and A. Ghasemi, “Two-Stage Resource Allocation for Random Access M2M Communications in LTE Network,” *IEEE Communications Letters*, vol. 20, no. 5, pp. 982-985, May 2016.

[16] F.M. Awuor and C.Y. Wang, “Massive Machine Type Communication in Cellular System: A Distributed Queue Approach,” *IEEE International Conference on Communications (ICC 2016), Kuala Lumpur, Malaysia*, May 2016.

[17] D. T. Wiriaatmadja and K. W. Choi, “Hybrid Random Access and Data Transmission Protocol for Machine-to-Machine Communications in Cellular Networks,” *IEEE Transactions on Wireless Communications*, vol. 14, no. 1, pp. 33-46, Jan. 2015.

[18] 3rd Generation Partnership Project (3GPP), “LTE; Evolved Universal Terrestrial Radio Access (E-UTRAN);Medium Access Control (MAC) protocol specifica-

tion; Release 13, v.13.2.0," Sophia-Antipolis Cedex, France, TS 36.321, Aug. 2016.

[19] S. Sesia, I. Toufik, and M. Baker, *LTE-The UMTS Long Term Evolution: From Theory to Practice.* New York: Wiley, 2009.

[20] N. Zhang, G. Kang, J. Wang, Y. Guo, and F. Labeau, "Resource Allocation in a New Random Access for M2M Communications," *IEEE Communications Letters*, vol. 19, no. 5, pp. 843-846, May 2015.

[21] H. Seung-Jang, S. Min-Kim, H-S. Park, and D. Keun-Sung, "An early preamble collision detection scheme based on tagged preambles for cellular M2M random access," *IEEE Transaction on Vehicular Technology*, vol. 66, no. 7, pp. 5974-5984, July 2017.

[22] H. Seung-Jang, S. Min-Kim, H-S. Park, and D. Keun-Sung, "A Preamble Collision Resolution Scheme via Tagged Preambles for Cellular IoT/M2M Communications," *IEEE Transaction on Vehicular Technology*, vol. 67, no. 2, pp. 1825-1829, Feb. 2018.

[23] N. Zhang, "An improved random access procedure for M2M communications," *IEEE Wireless Communications and Networking Conference (WCNC 2016)*, Doha, Qatar, April 2016.

[24] J. Haartsen, Telefonaktiebolaget LM Ericsson (Publ), Random Access Collision Detection. United States patent application US 11/754,329. 2007 May 28.

[25] M. Polese, M. Centenaro, A. Zanella, and M. Zorzi, "M2M Massive Access in LTE: RACH Performance Evaluation in a Smart City Scenario," *IEEE International Conference on Communications (ICC 2016)*, Kuala Lumpur, Malaysia, May 2016.

[26] N. Xia, H.H. Chen, and C.S. Yang, "Radio Resource Management in Machine-to-Machine Communications-A Survey," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 1, pp. 791-828, Oct. 2017.

[27] 3rd Generation Partnership Project (3GPP), “Study on RAN improvements for machine type communications; Release 11, v.11.0.0,” Sophia-Antipolis Cedex, France, TR 37.868, Sep. 2011.

[28] C-Y. Oh, D. Hwang, and T-J. Lee, “Joint Access Control and Resource Allocation for Concurrent and Massive Access of M2M devices,” *IEEE Transaction on Wireless Communications*, vol. 14, no. 8, pp. 4182-4192, Aug. 2015.

[29] A. Hald, *A History of Probability and Statistics and Their Applications before 1750 (Vol. 501)*, John Wiley & Sons, 2003.

[30] W. Feller, *An Introduction to Probability Theory and its Applications*, Wiley Publisher, 1970, 3st edn.

[31] 3rd Generation Partnership Project (3GPP), “LTE; Evolved Universal Terrestrial Radio Access(E-UTRAN); Radio Resource Control (RRC); Protocol Specification Release 12, v.12.4.1,” Sophia-Antipolis Cedex, France, TS 36.331, Feb. 2015.