AMERICAN GEAR MANUFACTURERS ASSOCIATION

A Rational Procedure for the Preliminary Design of Minimum Volume Gears

AGMA INFORMATION SHEET

(This Information Sheet is NOT an AGMA Standard)

AGMA 901–A92, A Rational Procedure for the Preliminary Design of Minimum Volume Gears

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ABSTRACT:

A simple, closed–form procedure is presented as a first step in the design of minimum volume spur and helical gearsets. The procedure includes methods for selecting geometry and dimensions, considering maximum pitting resistance, bending strength, and scuffing resistance. It also includes methods for selecting profile shift.

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FOREWORD

[The foreword, footnotes, and annexes are provided for informational purposes only and should not be construed as a part of AGMA 901-A92, *A Rational Procedure for the Preliminary Design of Minimum Volume Gears.*]

Gear design is a process of synthesis where gear geometry, materials, heat treatment, manufacturing methods, and lubrication are selected to meet the performance requirements of a given application. The designer must design the gearset with adequate pitting resistance, bending strength, and scuffing resistance to transmit the required power for the design life. With the algorithm presented here, one can select materials and heat treatment within the economic constraints and limitations of manufacturing facilities, and select the gear geometry to satisfy constraints on weight, size and configuration. The gear designer can minimize noise level and operating temperature by minimizing the pitchline velocity and sliding velocity. This is done by specifying high gear accuracy and selecting material strengths consistent with maximum material hardness, to obtain minimum volume gearsets with teeth no larger than necessary to balance the pitting resistance and bending strength.

Gear design is not the same as gear analysis. Existing gearsets can only be analyzed, not designed. While design is more challenging than analysis, current textbooks do not provide procedures for designing minimum volume gears. They usually recommend that the number of teeth in the pinion be chosen based solely on avoiding undercut. This information sheet, based on the work of R. Errichello [1]*, will show why this practice, or any procedure which arbitrarily selects the number of pinion teeth, will not necessarily result in minimum volume gearsets. Although there have been many technical papers on gear design (for example [2] and [3]), most advocate using computer-based search algorithms which are unnecessary. Tucker [4] came the closest to an efficient algorithm, but he did not show how to find the preferred number of teeth for the pinion.

This information sheet includes the design of spur and helical gears. Other gear types could be designed by a similar algorithm with some slight modifications to the one presented here.

AGMA 901-A92 was approved by the Helical Gear Rating Committee in March, 1992 and approved by the AGMA Board of Directors as of May, 1992.

Suggestions for the improvement of this information sheet will be welcome. They should be sent to the American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, Virginia, 22314.

* Numbers in brackets [ ] refer to the list of references in annex E.

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A Rational Procedure for the Preliminary Design of Minimum Volume Gears

1 Scope

This information sheet is intended for the student or beginning gear designer, to provide an outline of a preliminary design procedure which will lead to a rational design for spur and helical gear pairs within constraints such as:

– required gear ratio;

– required torque capacity;

– specified center distance;

– material selection.

This method could be extended to other gear types given the appropriate constants and factors.

1.1 Procedure

The simple closed form of the procedure allows the designer to explore options with a minimum of computation so that the important design decisions regarding loads, overloads, material, and tooling selections are not obscured by the need to spend a long time calculating each possibility.

This information sheet will demonstrate to the user that the traditional beginning point for gear design, selecting the minimum number of pinion teeth to avoid undercut, will rarely lead to the best design.

As this procedure is approximate, it is necessary to audit the design (see clause 6).

1.2 Exceptions

The procedure described herein incorporates major design considerations and leads toward minimum volume gear designs based upon the criteria chosen. For the final gear design, additional influencing factors beyond those in this information sheet include shaft deflection limits, sound level, cost, etc. Any of these could influence the design of the gear envelope and final volume.

It is not the intent of this information sheet to include the calculation of the profile shift coefficient (addendum modification coefficient). It is, however, necessary to inform the reader that profile shift exists, how it can affect gear design, and where it comes into play in designing a gearset. Some of the important factors relating to profile shift are discussed in 7.4.

Overhung pinions or gears are not covered by this information sheet because of the difficulty in determining an accurate value for the load distribution factor.

2 Definitions and symbols

2.1 Definitions

The terms used, wherever applicable, conform to the following standards:

ANSI Y10.3–1968, *Letter Symbols for Quantities Used in Mechanics of Solids*

ANSI/AGMA 1012–F90, *Gear Nomenclature, Definitions of Terms with Symbols*

AGMA 904–B89, *Metric Usage*

2.2 Symbols

The symbols used in this information sheet are shown in table 1.

NOTE – The symbols, definitions and terminology used in this information sheet may differ from other AGMA publications. The user should not assume that familiar symbols can be used without a careful study of these definitions.

3 Input variables

This clause discusses the significant parameters relating to a preferred gear design. It is not intended to be all inclusive, but to be limited by the scope of this information sheet.

3.1 Materials and heat treatment

Many materials have been used in gearing, but the most common today is steel. This information sheet only applies to steel gearing. There are two commonly used types of heat treatment for steel gear materials, surface hardening and through hardening. The choice of steel alloy must be compatible with the chosen heat treatment process.

Table 1 – Symbols used in equations

Symbols AGMA	Symbols ISO	Terms	Units	Equation where first used
b	N_b	number of power paths	—	8
C_a	K_A	application factor – pitting	—	9
C_d	Z_d	combined derating factor – pitting	—	30
C_L	Z_{NT}	pitting resistance life factor	—	26
C_m	K_H	load distribution factor – pitting	—	9
C_p	Z_E	elastic coefficient	$[lb/in^2]^{0.5}([N/mm^2]^{0.5})$	32
C_r	a_w	operating center distance	in (mm)	21
C_v	K_v	dynamic factor – pitting	—	30
d	d_1	operating pitch diameter of pinion	in (mm)	10
F	b	net face width (without gap for double helical)	in (mm)	25
H_B	H_B	Brinell hardness	—	1
I	Z_I	pitting resistance geometry factor	—	11
J	Y_J	bending strength geometry factor	—	12
K_a	K_A	application factor – bending	—	31
K_B	Y_B	rim thickness factor	—	31
K_c	Z_c	pitting resistance constant	in^3 (mm^3)	32
K_d	Y_d	combined derating factor – bending	—	31
K_L	Y_{NT}	bending strength life factor	—	27
K_m	K_H	load distribution factor – bending	—	9
K_t	Y_t	bending strength constant	in^3 (mm^3)	33
K_v	*	dynamic factor – bending	—	31
L	L	gear life	hours	3
m_a	u_b	aspect (F/d) ratio	—	4
m_F	ε_β	face contact ratio	—	25
m_G	u	gear ratio ($m_G \geq 1$)	—	4
m_{G1}	u_1	gear ratio of high speed gearset ($m_{G1} \geq 1$)	—	15
—	m_n	normal module	mm	25M
M_o	u_o	overall gear ratio of double stage gear drive ($M_o \geq 1$)	—	15
N	N_L	total number of load cycles in gear life	—	3
N_G	z_2	number of teeth in gear	—	6
N_P	z_1	number of teeth in pinion	—	6
$N_{P\ pre}$	$z_{1\ pre}$	preferred number of pinion teeth	—	34
n	ω_1	speed	rpm	3
n_c	S_H	pitting resistance factor of safety	—	32
n_t	S_F	bending strength factor of safety	—	33
P	P	input power	hp (kW)	7
P_{nd}	—	normal diametral pitch	in^{-1}	25

* The symbol K_v is also used for the dynamic factor in ISO standards. However, its value is the inverse (equal to $1/K_v$) of the value used in ANSI/AGMA 2001–B88.

continued

Table 1 (concluded)

Symbols AGMA	Symbols ISO	Terms	Units	Equation where first used
q	N_q	number of contacts per revolution	—	3
s_{ac}	σ_{HP}	allowable contact stress number	lb/in² (N/mm²)	1
s_{at}	σ_{FP}	allowable bending stress number	lb/in² (N/mm²)	2
s_{nc}	σ_{HN}	contact strength	lb/in² (N/mm²)	28
s_{nt}	σ_{FN}	bending strength	lb/in² (N/mm²)	29
T_1	T_1	torque on high speed shaft	in lbs (Nm)	7
T_P	T_P	transmitted pinion torque, per mesh	in lbs (Nm)	8
ϕ_n	α_{no}	normal profile angle of cutter	degrees	11
ψ	β_m	standard helix angle	degrees	25
	Subscripts/Sign convention			
1	high speed (pinion)			
2	low speed (wheel)			
n	normal (no subscript indicates transverse)			
G	gear (wheel)			
P	pinion			
r	operating or running			
(±)	upper sign external gearsets, lower sign internal gearsets			

3.1.1 Surface hardening

Surface hardening takes place after tooth cutting, usually on gears made from hot rolled bar or forged steel.

3.1.1.1 Carburized

Carburized steel is most commonly used for highly loaded, compact designs such as aircraft gears, vehicle transmissions of all types, machine tools, industrial gear drives, mining machines and similar uses. This material has the highest strength and greatest overload capacity, but carburized gears must be carefully manufactured. Carburized gears often result in the least expensive overall transmission design, if their advantage in small size for a given capacity can be utilized. Few manufacturers can produce carburized gears larger than 40 inches in diameter, though some can make them over 100 inches in diameter. Secondary finishing operations after carburizing, such as tooth grinding, are often required to achieve the desired tooth form. This is often required to eliminate the distortions caused by heating and cooling employed in the carburizing process.

3.1.1.2 Nitrided

Nitrided steel is most commonly used for small gears, finer than 10 diametral pitch (2.5 module) because the maximum case depth is limited. Some large gears are nitrided to avoid the distortion inherent in the carburizing process. Typical applications are industrial gear drives and small machine tools. Nitrided gears have limited shock resistance. This information sheet does not address nitrided gears, as reference [5] does not provide life factor curves for nitrided gears.

3.1.1.3 Induction and flame hardened

Induction and flame hardened steels are used to achieve intermediate capacities between carburized and through hardened gears. These processes are difficult to control, but give good results when carefully controlled. This information sheet does not address induction or flame hardened gears, as they are not recommended for inexperienced designers.

3.1.2 Through hardening

Through hardened gears typically have teeth cut in pre–heat–treated gear blanks, with no further heat treatment after cutting. The raw material can be hot rolled, cast, or forged. Hardness is chosen on the basis of machinability, using the lowest hardness which will carry the load on the required center distance.

The allowable stress numbers shall be based on the lowest hardness in the heat treatment specifications. Typical heat treatment specifications have a 40 BHN tolerance between the minimum and maximum hardnesses. The hardest heat treatment range that can be machined without special techniques is 320–360 BHN. The normal lowest range of hardness is 180–220 BHN, because lower values are difficult to machine.

Through hardened gear sizes commonly range from less than one inch to over 20 feet in diameter. Typical applications vary from instrument gears to girth gears on large mills and kilns. When gears cannot be of minimum size because of required center distance, rigidity requirements or thermal limits, or when loads are low, through hardened gears are commonly used. Internal gears are often through hardened.

The selection of a proper alloy for hardenability and reliability as well as the quality control of the steel manufacturing and heat treatment process are beyond the scope of this information sheet. Guidance can be found in section 14 of [5], as well as [6] and [7].

3.1.3 Elastic coefficient, C_p

The rating of gears also depends on the elastic coefficient, C_p . Further information can be found in section 10 of [5]. The elastic coefficient for a steel pinion and gear is 2300[lbs/in^2]$^{0.5}$ (191[N/mm^2]$^{0.5}$).

3.1.4 Allowable stress numbers

The allowable stress numbers for some heat treatments, surface hardness and steel quality grades are shown in tables 2 and 3. There are two grades of allowables shown in tables 2 and 3. The allowable stress numbers are valid only when the requirements of ANSI/AGMA 2001–B88, section 14 are met.

Table 2 – Allowable contact stress numbers for steel gears[1]

Material designation	Heat treatment	Minimum hardness at surface	Allowable contact stress number, s_{ac} lb/in^2 (N/mm^2)	
			Grade 1	Grade 2
Steel	Through hardened	180 BHN and less	85 000 (590)	95 000 (660)
		240 BHN	105 000 (720)	115 000 (790)
		300 BHN	120 000 (830)	135 000 (930)
		360 BHN	145 000 (1000)	160 000 (1100)
		400 BHN	155 000 (1050)	170 000 (1150)
	Carburized and case hardened		180 000 (1250)	225 000 (1550)

1) The data in this table has been extracted from ANSI/AGMA 2001–B88. The metric values have been revised per AGMA 904–B89. The allowable stress numbers are valid only when the requirements of ANSI/AGMA 2001–B88, section 14 are met.

Table 3 – Allowable bending stress numbers for steel gears[1]

Material designation	Heat treatment	Minimum hardness at surface	Allowable bending stress number, s_{at} lb/in2 (N/mm2)	
			Grade 1	Grade 2
Steel	Through hardened	180 BHN and less	25 000 (170)	33 000 (230)
		240 BHN	31 000 (215)	41 000 (285)
		300 BHN	36 000 (250)	47 000 (325)
		360 BHN	40 000 (275)	52 000 (360)
		400 BHN	42 000 (290)	56 000 (385)
	Carburized and case hardened		55 000 (380)	65 000 (450)

1) The data in this table has been extracted from ANSI/AGMA 2001–B88. The metric values have been revised per AGMA 904–B89. The allowable stress numbers are valid only when the requirements of ANSI/AGMA 2001–B88, section 14 are met.

Allowable stress numbers for grade 1 through hardened steel for unity life factor are:

$$s_{ac} = 26\,000 + 327\,H_B \quad \text{...(1)}$$

$$\sigma_{HP} = 179 + 2.25\,H_B \quad \text{...(1M)}$$

$$s_{at} = -274 + 167\,H_B - 0.152\,H_B^2 \quad \text{...(2)}$$

$$\sigma_{FP} = -1.89 + 1.15\,H_B - 0.00105\,H_B^2 \quad \text{...(2M)}$$

where

s_{ac} (σ_{HP}) is allowable contact stress number in pounds per square inch (newtons per square millimeter);

s_{at} (σ_{FP}) is allowable bending stress number in pounds per square inch (newtons per square millimeter);

H_B is Brinell hardness.

3.2 Design life

When evaluating gearing, it is important to know how many stress cycles the individual gears will experience during the intended life of the equipment. Some machines will run twenty–four hours per day and operate for twenty or more years. Other machines have gears that have total life requirements of a few hours. The gear designer should obtain the design life that is appropriate for the application. The required life in load cycles, N, will be used to determine the life factors.

$$N = 60\,L\,n\,q \quad \text{...(3)}$$

where

N is the total number of load cycles in the gear life;

L is the life, in hours;

n is the speed, in rpm;

q is the number of contacts per revolution.

3.3 Aspect ratio

The aspect ratio, m_a, also known as the pinion face width to diameter ratio, F/d, is an indicator of how sensitive a gear set is to misalignment. In this algorithm the aspect ratio is input, rather than the face width. The ratio used will affect the value for C_m and K_m. This is because changes in the aspect ratio change the face width of a gearset. This will, in turn, increase or decrease the effect of any mounting errors or deflections under load. For this reason, as one changes the aspect ratio, one must appropriately alter C_m and K_m.

Opinions vary regarding what is good design practice for an aspect ratio. Some factors that influence one's selection are ratio, materials, shaft deflection, housing deflection, housing accuracy, bearing clearances, centrifugal and thermal deflections, and tooth crowning. In the absence of experience, equations 4 and 5 may be used. These will result in aspect ratios which will be conservative for most applications.

Aspect ratio, m_a (F/d):

$$m_a = \frac{m_G}{m_G + 1} \text{ (for spur and single helical)} \quad \text{...(4)}$$

$$m_a = \frac{2\,m_G}{m_G + 1} \text{ (for double helical without including the gap)} \quad \text{...(5)}$$

where

$$m_G = \frac{N_G}{N_P} \geq 1.0 \quad \text{...(6)}$$

3.4 Input power, P

Input power, P, is the total power input. If there are multiple power paths (where $b > 1$), each path takes a portion of the input power. This document assumes that each path receives an equal share of the input power, but in practice extra steps must be taken to approach equal load sharing between power paths. See figure 1 for an example where $b = 2$.

3.5 Combined derating factors, C_d and K_d

For the purposes of this information sheet, the additional factors (application factor, load distribution factor, dynamic factor and the rim thickness factor) that affect gearset rating are combined into one derating factor for pitting resistance, C_d, and a second for bending strength, K_d. These are defined in equations 30 and 31.

3.5.1 Application factor, C_a and K_a

The application factors make allowance for any externally applied loads in excess of the nominal tangential load. This factor must be based upon the past experience of gear drive users and manufacturers. Examples may be found in appendix A of [8]. Typical application factors are shown in table 4.

Table 4 – Typical application factors*, C_a and K_a

Application factor C_a, K_a	Driven Equipment
1.25	Uniformly loaded conveyors Pure liquid mixers Centrifugal compressors Rotary or centrifugal pumps
1.50	Non–uniformly fed conveyors Variable density mixers Lobe compressors Reciprocating pumps
1.75	Multi–cylinder reciprocating compressors Rubber extruders
2.0	Reciprocating conveyors Single–cylinder reciprocating compressors Laundry washers

*Taken from appendix A of [8], for gearing driven by electric or hydraulic motors.

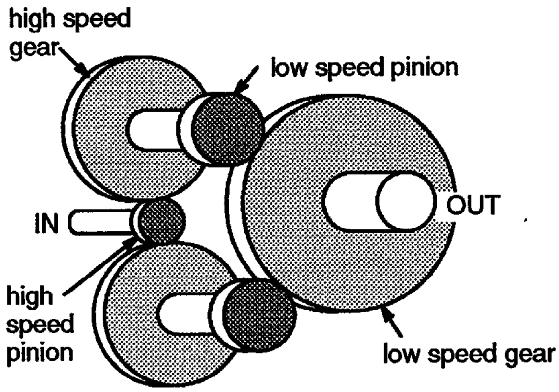

Figure 1 – Two branch double stage gearing

Most electric or hydraulic motors and steam or gas turbines are considered smoothly operating drivers, while multi–cylinder engines produce medium level shock loading and single–cylinder engines produce heavy shock loads. Add a value of 0.25 to the application factor given in table 4 if the driver is a multi–cylinder engine, or 0.50 if the driver is a single–cylinder engine.

3.5.2 Load distribution factor, C_m and K_m

The load distribution factor modifies the rating equations to reflect the non–uniform distribution of load along the lines of contact. Reference [5] provides two methods for determining this factor, analytical and empirical. The analytical method requires knowledge of the design, manufacturing, and mounting to evaluate the load distribution factor. For this reason, the analytical method is typically used only by experienced engineers and should not be used for preliminary design. The empirical method requires a minimum amount of information. This method is recommended for relatively stiff gear designs which meet the following requirements:

- Aspect ratio, $F/d \leq 2.0$. (For double helical gears the gap is not included in the face width.);
- The gear elements are mounted between bearings. Designs having overhung pinions, overhung gears, or both, require extensive analysis to determine the load distribution factor and are not covered by this information sheet;
- Face width up to 40 inches;
- Contact across full face width of narrowest member when loaded.

When using the empirical method, the calculated value of C_m and K_m depends on many items but is basically a function of net face width and alignment. Unfortunately, at this point in the design, the net face width is not known. To get an approximation for C_m and K_m based on pinion torque, application factor, and aspect ratio, use equations 9 or 9M, or table 5.

$$T_1 = \frac{P\,(63\,000)}{n_{P1}} \qquad \text{...(7)}$$

$$T_1 = \frac{P\,(9550)}{\omega_1} \qquad \text{...(7M)}$$

$$T_{P1} = \frac{T_1}{b} \qquad \text{...(8)}$$

where

b is the number of power paths;

n_{P1} is the speed of high speed pinion, in rpm;

T_1 is the torque on high speed shaft, in in–lbs (Nm);

T_{P1} is the transmitted pinion torque, per high speed mesh, in in–lbs (Nm);

P is the input power, in hp (kW).

$$C_m = K_m = 1 + m_a\left[0.2 + 0.0054\left(\frac{T_P\,C_a}{m_a}\right)^{0.33}\right] \qquad \text{...(9)}$$

$$K_H = 1 + u_b\left[0.2 + 0.0112\left(\frac{T_P\,K_A}{u_b}\right)^{0.33}\right] \qquad \text{...(9M)}$$

Once the pinion operating pitch diameter, d, is known, a more accurate approximation of load distribution factor can be found with equations 10 or 10M.

$$C_m = K_m = 1 + m_a\,(0.2 + 0.03\,d) \qquad \text{...(10)}$$

$$K_H = 1 + u_b\,(0.2 + 0.0012\,d_1) \qquad \text{...(10M)}$$

Do not use equations 9, 9M, 10, 10M, or table 5 for a final design audit, but rather follow the procedure given in clause 6. See [5] for a complete discussion.

Table 5 – Typical load distribution factors

$T_P\,C_a$ (Pinion torque) x (Application factor)		C_m and K_m			
in lbs	(Nm)	$m_a = 0.25$	$m_a = 0.50$	$m_a = 0.75$	$m_a = 1.00$
5000	(550)	1.10	1.15	1.25	1.30
50 000	(5600)	1.15	1.25	1.30	1.40
500 000	(56 500)	1.20	1.35	1.50	1.65
5 000 000	(565 000)	1.40	1.70	1.90	2.10

3.5.3 Dynamic factor, C_v and K_v

Dynamic factors, C_v and K_v, account for internally generated gear tooth loads which are induced by non–conjugate meshing action of the gear teeth. Externally applied dynamic loading or resonance operation is not applicable to the dynamic factor and is usually accommodated in the application factor. Dynamic factors are essentially dependent on gear pitch line velocity and gear quality. For simplicity, this information sheet uses $C_v = K_v = 0.7$ as a first approximation, which is conservative for most applications. See [5] for a more detailed explanation of dynamic factor.

3.5.4 Rim thickness factor, K_B

Where the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim, rather than at the root fillet. The rim thickness factor, K_B, adjusts the calculated bending stress number for thin rimmed gears.

In general, this factor can be taken as unity if the rim section below the tooth is at least 20% greater than the tooth height, unless keyways, splines, or notches are present in the bore. Otherwise, see appendix C of [5] for a more detailed analysis.

3.6 Geometry factors, I and J

Geometry factors for pitting resistance, I, and bending strength, J, are approximated by the following equations.

Geometry factors for spur gears:

$$I \approx \frac{\sin \phi_n \cos \phi_n}{2} \frac{m_G}{m_G \pm 1} \quad ..(11)$$

$$J \approx 0.45 \quad ...(12)$$

Geometry factors for helical gears:

$$I \approx \frac{1 + 0.00682\,\phi_n}{4.0584} \frac{m_G}{m_G \pm 1} \quad ...(13)$$

$$J \approx 0.50 \quad ...(14)$$

where

- $(\pm)$ upper sign is for external gearsets, lower sign is for internal gearsets;
- ϕ_n is the normal profile angle in degrees. See 3.8.

Equations 13 and 14 above assume $\psi = 15°$ and will change for other values of ψ.

Values of $J = 0.45$ for spur gears and $J = 0.50$ for helical gears are starting points that can normally be achieved through good design procedures when selecting profile shift and tooling. J factors as low as 0.20 and as high as 0.75 are not uncommon, but usually are not found in optimum designs.

3.7 Gear ratio

The gear ratio, m_G, is an input that must be known. In many situations the ratio is so large that there should be more than one stage. When the gear ratio approaches or exceeds 6:1, it may be more economical to add a second stage. To optimize the design, one must first optimize the overall ratio split between the various meshes. This subclause provides two methods for optimizing the ratio split; one based on minimum volume and a second based on preexisting housing designs.

In either case, internal clearances between rotating and stationary parts must be maintained. Also, adequate room must be available for bearings with enough capacity for the application. Both methods balance the rating of each mesh by equalizing the pitting rating of each pinion.

3.7.1 Minimum volume gearsets

One method is to split the overall ratio to minimize the sum of the solid rotor volumes. This method is discussed in [9] and annex B. The "minimum volume" gear ratio of the high speed gearset of a two stage gearbox with b power branches is given by:

$$\left(\frac{M_o^2}{b\, m_{G1}^2}\right) - 1 = \left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \times \left[\left(\frac{0.112}{b^{0.888}\, m_{G1}^{0.888}}\right) + 2.112\; b^{0.112}\, m_{G1}^{1.112}\right] \quad ...(15)$$

where

- C_{m1} is the load distribution factor for pitting resistance – high speed mesh;
- C_{m2} is the load distribution factor for pitting resistance – low speed mesh;
- I_1 is the geometry factor for pitting resistance – high speed mesh;

I_2 is the geometry factor for pitting resistance – low speed mesh;

m_{G1} is the gear ratio ($m_{G1} \geq 1$) – high speed mesh

M_o is the overall gear ratio ($M_o \geq 1$);

s_{ac1} is the allowable contact stress number – high speed mesh;

s_{ac2} is the allowable contact stress number – low speed mesh.

Iteration is necessary to solve for m_{G1} since it appears three places in equation 15. Equation 15 may be solved by defining:

$$X = m_{G1} \qquad ...(16)$$

$$A = \left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \qquad ...(17)$$

$$B = 0.112\, b^{0.126} \qquad ...(18)$$

$$C = 2.112\, b^{0.112} \qquad ...(19)$$

And iterating this equation:

$$X1 = M_o \left\{ A \left[\left(\frac{B}{X^{0.888}} \right) + C\, X^{1.112} \right] + b \right\}^{-0.5} \qquad ...(20)$$

Assuming an initial approximation for $X = M_o^{0.5}$, it is successively improved upon by iterating equation 20 each time setting $X = X1$ (from the previous iteration). Iterate until $|X - X1| \leq 0.001$.

3.7.2 Fixed housing designs

The method in 3.7.1 will not normally work when designing gears for an existing gearcase where the center distances are fixed and face widths may be constrained. In such situations, the best way to split the overall gear ratio may be to balance the pitting ratings of each mesh.

By reworking the same equations from reference [9], a balanced gear train can be achieved by proper splitting of the overall ratio of a multiple stage gearcase. This method is discussed for a two stage existing gearcase in annex C. The "balanced rating" gear ratio of the high speed gearset of an existing two stage gearbox, with b power branches, is given by:

$$\left(\frac{M_o + m_{G1}}{m_{G1} + 1}\right)^3 m_{G1}^{-2.112} = b^{0.112}\left(\frac{m_{a2}}{m_{a1}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{C_{r2}}{C_{r1}}\right)^3\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \qquad ...(21)$$

where

C_{r1} is the center distance of high speed mesh;

C_{r2} is the center distance of low speed mesh;

m_{a1} is the aspect ratio of high speed mesh;

m_{a2} is the aspect ratio of low speed mesh.

Iteration is necessary to solve for m_G since it appears three places in equation 21. Equation 21 may be solved by defining:

$$X = m_{G1} \qquad ...(22)$$

$$B = \left\{ b^{0.112}\left(\frac{m_{a2}}{m_{a1}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{C_{r2}}{C_{r1}}\right)^3\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \right\}^{1/3} \qquad ....(23)$$

and iterating this equation:

$$X1 = \left(\frac{M_o + X}{B\; X^{0.704}}\right) - 1 \qquad ...(24)$$

Assuming an initial approximation for $X = M_o^{0.5}$, it is successively improved upon by iterating equation 24 each time setting $X = X1$ (from the previous iteration). Iterate until $|X - X1| \leq 0.001$.

3.8 Cutter profile angle

The cutter normal profile angle, ϕ_n, is generally chosen from the range between 14.5° and 25°. Standard values are 14.5°, 17.5°, 20°, 22.5°, and 25°. The starting value should be 20°, since the majority of cutting tools use that angle, are universally available, and will usually provide satisfactory gear tooth geometry.

Tools with smaller profile angles can be used to obtain higher transverse contact ratios when lower noise levels or less sensitivity to center distance change are desirable. These gears usually have high numbers of teeth and are relatively lightly loaded, as in telescopes, antenna drives, and precision instruments. It is generally easier to obtain gears which operate quietly when pressure angles are low.

Tools with higher profile angles are sometimes used when bending strength is the most important requirement. These gears usually have lower numbers of teeth and are heavily loaded, as in mining machines, rotary actuators, and rock crushers. These gears often operate at low speeds in noise tolerant environments.

The effect of higher or lower cutter profile angles can also be achieved with 20° cutters by increasing or decreasing the operating center distance to change the operating pressure angle of the gearset. Many companies use nonstandard cutter profile angles to optimize tooth root geometry or to accomplish specific design goals. The selection of those special cutters is beyond the scope of this information sheet.

3.9 Tool selection

The number of gears being produced can influence the geometry on the basis of gear cutter selection. For high production situations, an ideal tool can be developed. But when only a few gear components are required, the design should be based on standard or readily available cutters. This can usually be accomplished by varying the helix angle or profile shift to match the required center distance and ratio (see 7.4).

3.10 Selecting a helix angle

Helix angle selection is not an arbitrary procedure, but one that requires a knowledge of what the proper choice can provide. The main function of a helix angle is to provide a high enough face contact ratio to ensure smooth transmission of power from one tooth to the next during meshing. See table 6 for other considerations. For single helical gears, a good starting point is 15° and for double helical gears, a helix angle of either 23° for hobbed gears or 30° for shaped gears, will be adequate in the beginning. Before selecting the final helix angle, check tooling, bearing rating, shaft deflection, and equipment availability.

The face contact ratio, m_F, must be greater than 1.0 for a gear to be considered a conventional helical gear. If the face is too narrow, the pitch too coarse, or the helix angle too low, the gear will have m_F equal to or less than 1.0 and have no effective overlaps. The following equation shows the effect of gear geometry on m_F:

$$m_F = \frac{F\, P_{nd} \sin \psi}{\pi} \qquad \text{...(25)}$$

$$\varepsilon_\beta = \frac{b \sin \beta_m}{\pi\, m_n} \qquad \text{...(25M)}$$

where

F is the net face width, in inches (millimeters);

P_{nd} is the normal diametral pitch, in inches^{-1};

ψ is the standard helix angle;

m_n is the normal module, in millimeters.

From this equation it can be seen that the normal diametral pitch (or normal module), net face width, and helix angle all influence the face contact ratio.

3.11 Factor of safety

The factors of safety are n_c and n_t. The term "factor of safety" has historically been used in mechanical design to describe a general derating factor to limit the design stress in proportion to the material strength. A factor of safety accounts for uncertainties in:

- design analysis;
- manufacturing;
- applied load;
- quality consistent with design requirements.

Factor of safety also takes into consideration:

- human safety risk;
- economic risk.

The greater the uncertainties or consequences of the above parameters, the higher the factor of safety should be. As the values of all the variables within the algorithm are known with more certainty, the value of the factor of safety can be reduced.

The factor of safety for bending strength is often selected to be greater than the factor of safety for pitting resistance to reduce the likelihood of catastrophic failure (see clause 4).

For inexperienced gear designers, conservative selection of variables within the algorithm is recommended.

Table 6 – Effects of helix angle in parallel shaft gearing

Spur gearing: – Spur gears impose mainly radial loads on bearings. In practice, thrust loads may exist; – Usually noisier than helical gears because there are fewer teeth in contact, and the dynamic loading occurring during a mesh cycle is greater; – Pitchline velocities are usually limited to 3000 feet per minute without special design considerations; – Less load capacity than helical gears of the same proportions.
Single helical gearing: – Usually quieter than spur gears because there are more teeth in contact, and because less severe dynamic loading occurs during a mesh cycle; – Very high pitchline velocities are obtainable with proper design considerations; – Higher load capacity than spur gears of the same proportions; – Thrust loads and overturning moments are generated due to the helix angle, so bearings and housings must be designed accordingly.
Double helical gearing: – Gear tooth generated thrust loads oppose each other; – Usually quieter than spur gears because there are more teeth in contact, and less severe dynamic loading occurs during a mesh cycle; – Very high pitchline velocities are obtainable with proper design considerations; – More difficult to manufacture; – Gap between opposing helices must be wide enough for tool clearance. This gap adds weight and length to the gear design; – At least one member of the gearset must be allowed to float axially; – The two helices cannot be matched perfectly, so dynamic loading of a member occurs; – External thrust loads (i.e., couplings, bearings, inertial forces, and the like) on the floating member can create an overload condition on one of the helices.

4 Preferred number of pinion teeth

The preferred number of pinion teeth attempts to maximize the load capacity of a gearset. Figure 2 shows that load capacity is limited by pitting fatigue, bending fatigue, or scuffing failure depending on the number of teeth in the pinion. Also, there is a lower limit to the number of teeth, below which undercut occurs. The hatched zone is bounded by all three failure mode curves and the undercut limit. It applies to a homologous class of gears with a specific combination of gear geometry, material properties, and application requirements.

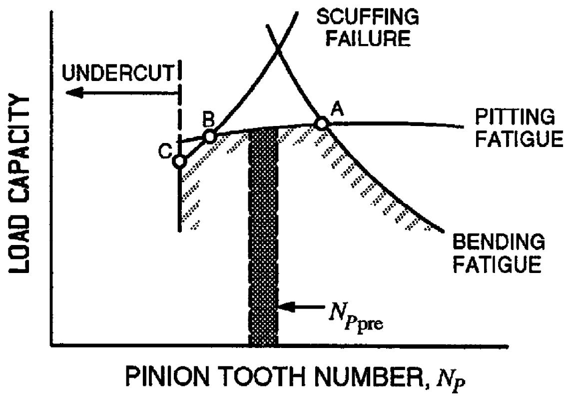

Figure 2 – Preferred number of pinion teeth

The relative positions of the curves change as these parameters change, as demonstrated in figures 3 through 5. The algorithm presented here directly solves for the preferred number of pinion teeth making it unnecessary to draw these figures, which are shown strictly for demonstrating the concept of the preferred number of pinion teeth. The curve marked "Pitting Fatigue", representing the pitting resistance of the gearset, is relatively flat, being only weakly influenced by the number of pinion teeth. In contrast, the curve marked "Bending Fatigue", representing the bending strength, depends strongly on the number of pinion teeth and it drops rapidly as the number of teeth increases. Maximum load capacity occurs at point "A" where the pitting resistance and bending strength are balanced. For more pinion teeth (to the right of point "A") load capacity is controlled by bending fatigue, while for fewer teeth (to the left of point "A") load capacity is controlled by pitting fatigue.

The two failure modes are quite different. Pitting fatigue usually progresses relatively slowly, starting with a few pits which may increase in number and coalesce into larger spalls. As the tooth profiles deteriorate with pitting, the gears typically generate noise and vibration which warns of the pitting fatigue failure. In contrast, bending fatigue may progress rapidly as a fatigue crack propagates across the base of a tooth, breaking the tooth with little or no warning. Hence, pitting fatigue is often less serious than bending fatigue, which is frequently catastrophic.

Considering the differences between pitting fatigue and bending fatigue, it is prudent to select the number of pinion teeth somewhat to the left of point "A" (shown by the vertical column marked N_{Ppre}) where pitting fatigue controls rather than bending fatigue. With this design approach, not much load capacity is lost because the pitting fatigue curve is relatively horizontal, while a margin of safety against bending fatigue is gained. This practice should not be carried to extremes, because pinions with few, large teeth (with high specific sliding ratios) are prone to scuffing (see point "B" on curve marked "Scuffing Failure").

Some textbooks recommend using a number of teeth for the pinion equal to the minimum required to avoid undercut. This gives gearsets with less than optimum load capacity which are prone to scuffing (see point "C"). For information on scuffing, see appendix A of reference [5]. A pinion tooth number near N_{Ppre} provides a good balance between pitting resistance and bending strength, and since the teeth are no larger than necessary, the risk of scuffing is reduced.

Figures 3 through 5 demonstrate how the three limiting elements—pitting fatigue, bending fatigue, and scuffing failure—interact to identify a preferred number of pinion teeth based on gear geometry. Figure 3 illustrates an unmodified spur gearset. Here, the three elements converge, providing a limited tooth number selection at maximum load capacity. Figure 4 shows how the range of preferred teeth selection broadens by modifying the addendum of the same gearset. Figure 5 illustrates how some gearset designs may not make full use of the calculated load capacity. This is an example where redesign should be considered.

Some designers may require a hunting tooth combination for the gear set. This will restrict the acceptable combinations of teeth. A hunting combination is one in which any pinion tooth contacts every gear tooth. The tooth combination of 108/33 would not be hunting since there is a common factor of 3, meaning that any pinion tooth would eventually contact every gear tooth, but no other gear teeth. The combination of 109/34 is a hunting tooth combination as it has no common factors (other than unity) and therefore each pinion tooth will eventually contact every gear tooth.

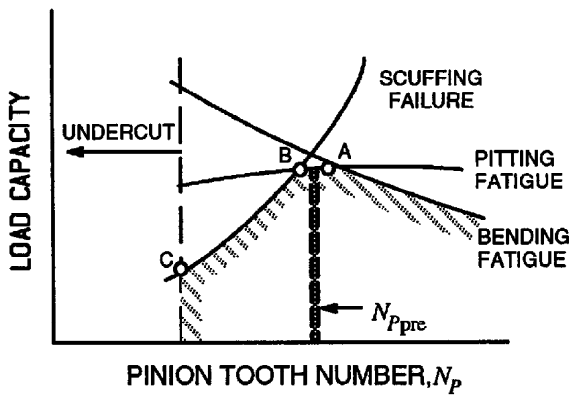

Figure 3 – Preferred number of pinion teeth for spur gear (unmodified)

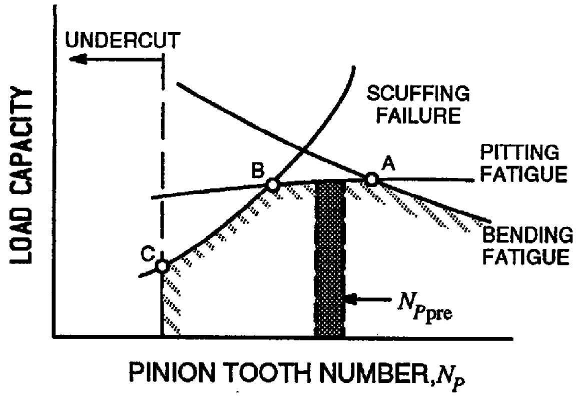

Figure 4 – Preferred number of pinion teeth for spur gear (modified)

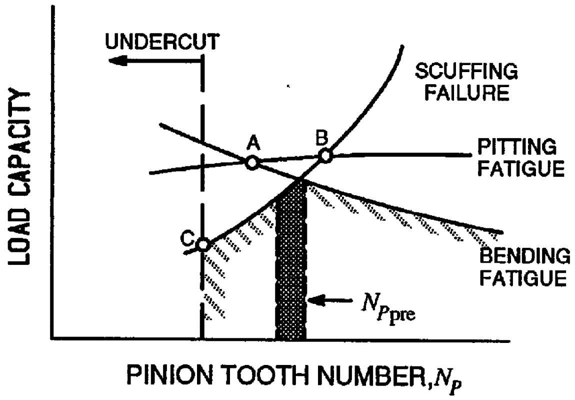

Figure 5 – Preferred number of pinion teeth for spur gear where redesign should be considered

5 Design algorithm

There is no need for cut–and–try procedures for gear design if one exploits the near independence of pitting resistance and the number of pinion teeth. The following algorithm solves for the diameter and face width of the pinion based on surface fatigue and solves for the preferred number of pinion teeth by simultaneously satisfying the surface fatigue and the bending fatigue constraints. It is derived from equations given in [5] and [10], and is limited to steel. Because it is necessary to approximate the geometry factors I and J, the final design must be verified using [5] and [10].

Life factors are:

$$1.0 \geq C_L = 2.4660\,(N)^{-0.0560} \quad \text{...(26)}$$

$$1.0 \geq K_L = 1.6831\,(N)^{-0.0323} \quad \text{...(27)}$$

C_L and K_L must be less than or equal to 1.0 for this algorithm.

Contact strength is:

$$s_{nc} = C_L\, s_{ac} \quad \text{...(28)}$$

Bending strength is:

$s_{nt} = K_L\, s_{at}$ (for constant reverse bending, such as idler gears, multiply s_{nt} by 0.7) ...(29)

The contact strengths and bending strengths are calculated for both the pinion and gear and the lesser of the pinion and gear values for s_{nc} and s_{nt} are used in the following equations.

Combined derating factor:

$$C_d = \frac{C_a\, C_m}{C_v} \quad \text{...(30)}$$

$$K_d = \frac{K_a\, K_m K_B}{K_v} \quad \text{...(31)}$$

where:

C_a , K_a is the application factor;
C_m, K_m is the load distribution factor;
C_v , K_v is the dynamic factor;
K_B is the rim thickness factor.

Pitting resistance constant in inches cubed (millimeters cubed):

$$K_c = \frac{126\,000\,P\,C_d}{b\,I\,n_p}\left(\frac{C_P\;n_c}{s_{nc}}\right)^2 \quad \text{...(32)}$$

$$Z_c = \frac{1.91 \times 10^7\,P\,Z_d}{N_b\,Z_I\,\omega_1}\left(\frac{Z_E\;S_S}{\sigma_{HN}}\right)^2 \quad \text{...(32M)}$$

where

C_p = 2300 (lbs/in2)0.5 (191[N/mm2]0.5).

Bending strength constant in inches cubed (millimeters cubed):

$$K_t = \frac{126\,000\,P\,K_d}{b\,J\,n_P}\,\frac{n_t}{s_{nt}} \qquad \text{...(33)}$$

$$K_t = \frac{1.91 \times 10^7\,P\,Y_d}{N_b\,Y_I\,\omega_1}\,\frac{S_F}{\sigma_{FN}} \qquad \text{...(33M)}$$

Preferred number of pinion teeth:

$$N_{P_{pre}} = \frac{K_c}{K_t} = \frac{1}{K_B}\,\frac{J}{I}\,\frac{s_{nt}}{s_{nc}^2}\,\frac{\left(C_P\,n_c\right)^2}{n_t} \qquad \text{...(34)}$$

(round to an integer)

Pinion operating pitch diameter:

$$d = \left(\frac{K_c}{m_a}\right)^{1/3} \qquad \text{...(35)}$$

Face width:

$$F = d\,m_a \qquad \text{...(36)}$$

The equations in clause 5 calculate a preliminary value for the pinion operating pitch diameter, d. With this, one can calculate an approximate operating center distance. This value is rarely the actual center distance to be used. It only gives the designer a starting point.

If the operating center distance, C_r, is known, d and F may be calculated with equations 37 and 38.

$$d = \frac{2\,C_r}{m_G \pm 1} \qquad \text{...(37)}$$

$$F = \frac{K_c}{d^2} \qquad \text{...(38)}$$

If d and F are obtained from equations 37 and 38, the aspect ratio should be calculated with equation 39.

$$m_a = \frac{F}{d} \qquad \text{...(39)}$$

If m_a is greater than recommended by equation 4 for spur or single helical or equation 5 for double helical, then the operating center distance, C_r, should be increased or the pitting resistance constant, K_c, should be decreased.

6 Design audit

When the profile shift (7.4 and annex A) is selected, the gear design is complete. It is necessary to audit the design by analyzing the stresses and life cycles (using [5] and [10]) because approximate values were used for I and J. The only change that is usually required to meet the design life is a small adjustment of the face width. Before proceeding, it is suggested that the design be reviewed by an experienced gear designer for practicality and economic feasibility.

Although it is beyond the scope of this information sheet, the selection of the lubricant type and viscosity should be verified by calculating the film thickness and flash temperature to ensure that they are within allowable limits as they relate to scuffing resistance. See appendix A of [5].

7 Considerations for improved rating

If the gearset being designed does not meet the rating requirements of bending fatigue, pitting fatigue, or scuffing resistance, the design should be altered to improve the power or life rating of the weak area(s). Changing a gear design parameter may help one area and hurt another, and may also affect other non–gear items such as bearings. Some things that help improve the bending fatigue, pitting fatigue, and scuffing resistance are listed below.

7.1 Improve bending fatigue resistance with:

- lower load;
- increased center distance;
- coarser pitch (fewer teeth on the same diameter gear);
- higher operating pressure angle;
- helical (vs. spur) tooth design;
- carburized material;
- higher surface hardness with appropriate core hardness of material;
- improved gear accuracy;
- higher quality material;
- wider effective face width (up to a specified F/d limit);
- profile shift for balanced bending fatigue life;
- large, smooth root fillets in teeth;
- shot peening roots of teeth*.

7.2 Improve pitting fatigue resistance with:

- lower load;
- increased center distance;
- finer pitch (more teeth on the same diameter gear);
- higher operating pressure angle;
- helical (vs. spur) tooth design;
- carburized material;
- higher surface hardness with appropriate core hardness of material;
- improved gear accuracy;
- higher quality material;
- wider effective face width (up to a specified F/d limit);
- profile shift for balanced specific sliding;
- proper tip and/or root relief*;
- higher EHD film thickness of the lubricating oil*;
- smoother tooth surfaces by careful manufacture and run-in;
- improved quality of lubricant*.

7.3 Improve scuffing resistance with:

- lower load;
- reduced gear bulk temperature;
- higher operating pressure angle;
- higher EHD film thickness of the lubricating oil;
- anti-scuff EP additives in the oil;
- smoother tooth surfaces by careful manufacture and run-in;
- proper tip and/or root relief;
- profile shift for balanced flash temperature;
- improved gear accuracy;
- finer pitch (more gear teeth);
- reduced speed;
- silver or copper plating of gear teeth*;
- improved quality of lubricant*;
- nitriding steel.

7.4 Profile shift (addendum modification)

Once the pitch diameter, face width, and preferred number of teeth for the pinion are determined with the design algorithm, routine methods are used to select most of the other necessary design parameters. Included in this list are the number of teeth on the gear, normal diametral pitch, and operating center distance. However, a final gear design is not complete until the profile shift has been selected.

The profile shift is the amount that is added to, or subtracted from, the gear teeth addendum to enhance the operational performance of the gearset or meet fixed design criteria. The determination of the amount of the shift is based on the following criteria:

- avoiding undercut teeth;
- balanced specific sliding;
- balanced flash temperature;
- balanced bending fatigue life;
- avoiding narrow top lands.

It is not the intent of this information sheet to include the calculation of the profile shift coefficient. It is, however, necessary to inform the reader that profile shift exists, how it can affect gear design, and where it comes into play in designing a gearset. A discussion on the determination and effects of profile shift is presented in annex A.

7.5 Summary

Be sure to re-analyze the design, if it has been altered, to determine the amount of benefit gained from the alteration(s).

* Denotes an item that should help, but no benefit is shown in the analytical rating per [5].

Annex A
(informative)
Profile shift (addendum modification)

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 901–A92, *A Rational Procedure for Designing Minimum Volume Gears.*]

A.1 Profile shift

Once the diameter, face width, and preferred number of teeth for the pinion are determined with the design algorithm, routine methods are used to select the number of teeth in the gear, normal diametral pitch, and operating center distance. However, the gear design is not complete until the profile shift has been selected after considering the following criteria:

– avoiding undercut;
– balanced specific sliding;
– balanced flash temperature;
– balanced bending fatigue life;
– avoiding narrow top lands.

The profile shift should be large enough to avoid undercut and small enough to avoid narrow top lands. The profile shifts required for balanced specific sliding, balanced flash temperature and balanced bending fatigue life are usually different. Therefore, the value used should be based on the criterion that is judged to be the most important for the particular application.

Table A.1 – Symbols used in equations

Symbols	Terms	Units	Equation where first used
B_n	normal operating circular backlash	in	A.19
C	standard center distance	in	A.4
C_1	distance to SAP (see figure A.1)	—	A.12
C_5	distance to EAP (see figure A.1)	—	A.12
C_6	distance between interference points (see figure 2)	—	A.12
C_r	operating center distance	in	A.6
h_{a1}, h_{a2}	addendum, pinion and gear	in	A.22
J_1, J_2	bending strength geometry factor, pinion and gear	—	A.18
k_s	tip–shortening coefficient	—	A.20
m_G	gear ratio $\geq$ 1.0	—	A.1
n_1, n_2	number of teeth, pinion and gear	—	A.1
P_{nd}	normal diametral pitch	in^{-1}	A.2
R_1, R_2	standard pitch radius, pinion and gear	in	A.2
R_{b1}, R_{b2}	base circle radius, pinion and gear	in	A.14
R_{o1}, R_{o2}	outside radius, pinion and gear	in	A.14
s_{n1}, s_{n2}	reference normal circular tooth thickness, pinion and gear	in	A.32
s_{nt1}, s_{nt2}	bending strength, pinion and gear	lb/in^2	A.18
x_1, x_2	profile shift coefficient, pinion and gear	—	A.10
$x_{1\min}$	minimum profile shift coefficient to avoid undercut	—	A.11
x_{g1}, x_{g2}	generating rack shift coefficient, pinion and gear	—	A.30
ΔC	center distance modification	in	A.20
$\Delta s_{n1}, \Delta s_{n2}$	tooth thinning for backlash, pinion and gear	in	A.19
Σx	sum of profile shift coefficients	—	A.9
Σx_g	sum of generating rack shift coefficients	—	A.36

continued

Table A.1(concluded)

Symbols	Terms	Units	Equation where first used
ϕ	standard transverse pressure angle	—	A.5
ϕ_n	standard normal pressure angle	—	A.5
ϕ_r	operating transverse pressure angle	—	A.6
ψ	standard helix angle	—	A.5
	Subscripts/ sign convention		
1	pinion		
2	gear		
n	normal (no subscript indicates transverse)		
r	operating or running		
(±)	upper sign external gearsets, lower sign internal gearsets		

A.2 Basic gear geometry

$$m_G = \frac{n_2}{n_1} \quad \text{...(A.1)}$$

$$R_1 = \frac{n_1}{2\, P_{nd} \cos\psi} \quad \text{...(A.2)}$$

$$R_2 = R_1\, m_G \quad \text{...(A.3)}$$

$$C = R_2 \pm R_1 \quad \text{...(A.4)}$$

$$\phi = \arctan\left(\frac{\tan\phi_n}{\cos\psi}\right) \quad \text{...(A.5)}$$

$$\phi_r = \arccos\left(\frac{C}{C_r}\cos\phi\right) \quad \text{...(A.6)}$$

$$\text{inv}\,\phi = \tan\phi - \phi \quad \text{...(A.7)}$$

$$\text{inv}\,\phi_r = \tan\phi_r - \phi_r \quad \text{...(A.8)}$$

A.3 Sum of profile shift coefficients for zero backlash

$$\Sigma x = \frac{C\, P_{nd}\, (\text{inv}\,\phi_r - \text{inv}\,\phi)}{\tan\phi} \quad \text{...(A.9)}$$

$$\Sigma x = x_2 \pm x_1 \quad \text{...(A.10)}$$

A.4 Avoiding undercut teeth

There are a number of design options to compensate for undercut teeth, including profile shift. The design algorithm presented here will usually provide a number of pinion teeth considerably larger than the number needed to avoid undercut pinion teeth. It is important, however, to understand what undercut is and how it is produced.

Undercut is a condition in generated gear teeth where any part of the fillet curves lies inside a line drawn tangent to the working profile at its point of juncture with the fillet. For such gears, the end of the cutting tool has extended inside of the point of tangency of the base circle and the pressure line, and removed an excessive amount of material. This removal of material can weaken the tooth and also may reduce the length of contact, since gear action can only take place on the involute portion of the flank. Should a gear be made by another method that would not undercut the flanks, there may be interference of material and generally the gear would not mesh or roll with another gear. See [1]*.

Conditions which lead to the design of gears with small numbers of possibly undercut teeth are: high material hardness, short design life, large gear ratios, and high bending fatigue safety factors. With reasonable selections of these parameters, the algorithm gives $N_{P\ pre}$ a value usually greater than 20. In any case, the minimum profile shift coefficient (to avoid undercut) for the pinion is given by:

$$x_{1min} = 1.1 - n_1\left(\frac{\sin^2\phi}{2\cos\psi}\right) \quad \text{...(A.11)}$$

A.5 Balanced specific sliding

Maximum pitting and wear resistance is obtained by balancing the extreme specific sliding ratio at each end of the path of contact. This is done by iteratively varying the profile shift coefficients of the pinion and gear until the following equation is satisfied:

$$\left(\frac{C_6}{C_1} \mp 1\right)\left(\frac{C_6}{C_5} \mp 1\right) = m_G^2 \quad \text{...(A.12)}$$

where

C_1 is the distance to SAP (see figure A.1);

C_5 is the distance to EAP (see figure A.1);

C_6 is the distance between interference points (see figure A.1).

$$C_6 = C_r \sin\phi_r \quad \text{...(A.13)}$$

$$C_1 = \pm[C_6 - (R_{o2}^2 - R_{b2}^2)^{0.5}] \quad \text{...(A.14)}$$

$$C_5 = (R_{o1}^2 - R_{b1}^2)^{0.5} \quad \text{...(A.15)}$$

$$R_{b1} = R_1 \cos\phi \quad \text{...(A.16)}$$

$$R_{b2} = R_{b1}\, m_G \quad \text{...(A.17)}$$

A.6 Balanced flash temperature

Maximum scuffing resistance is obtained by minimizing the contact temperature. This is done by iteratively varying the profile shift coefficients of the pinion and gear, while calculating the flash temperature by Blok's equation (see appendix A of [2]), until the flash temperature peaks in the approach, and recess portions of the line of action are equal. The flash temperature should be calculated at the points SAP, LPSTC, HPSTC, EAP and at several points in the two pair zones (between points SAP and LPSTC and between points HPSTC and EAP, see figure A.1).

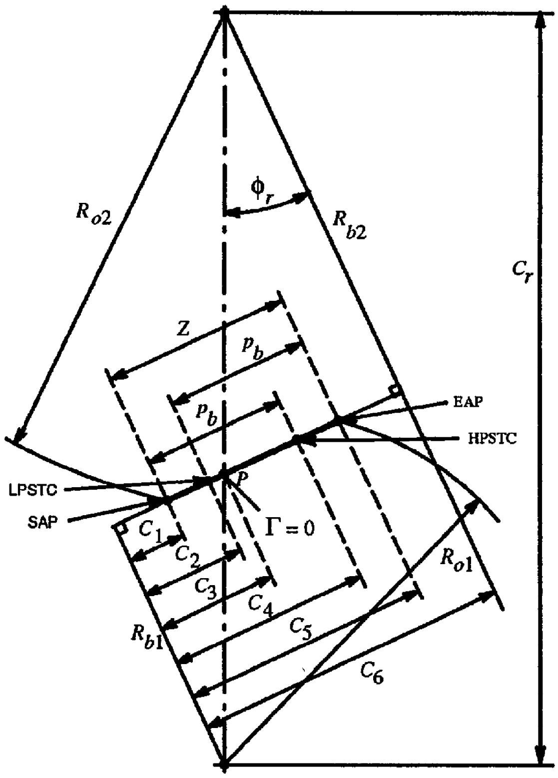

Figure A.1 – Distances along the line of action

A.7 Balanced bending fatigue life

Maximum bending fatigue resistance is obtained by iteratively varying the profile shift coefficients of the pinion and gear until the ratio of the bending strength geometry factors equals the ratio of bending strengths, i.e.,

$$\frac{J_1}{J_2} = \frac{s_{nt2}}{s_{nt1}} \quad \text{...(A18)}$$

A.8 Avoiding narrow top lands

The maximum permissible profile shift coefficients are obtained by iteratively varying the profile shift coefficients of the pinion and gear until their top land

* Numbers in brackets [ ] refer to the reference list at the end of this annex.

thicknesses are equal to the minimum allowable (usually $0.3/P_{nd}$).

A.9 Tooth thinning for backlash

The small adjustments of the position of the cutting tool to thin the gear teeth for backlash are considered independently of the profile shift coefficients (x_1 and x_2) by specifying the amount the pinion and gear teeth are thinned for backlash, Δs_{n1} and Δs_{n2}. This way, the outside diameters are independent of tooth thinning for backlash, and are based solely on the profile shift coefficients x_1 and x_2. The tooth thinning coefficients are selected such that:

$$\Delta s_{n1} + \Delta s_{n2} = B_n \, P_{nd} \left(\frac{C}{C_r} \right) \qquad \text{...(A.19)}$$

A.10 Tip–shortening coefficient for external gearsets

For gears operating on extended centers ($C_r > C$), the outside radii of the gears are shortened to maintain adequate tip–to–root clearance. The amount of adjustment of outside radii is proportional to the tip–shortening coefficient:

$$k_s = \Sigma x - \Delta C \, P_{nd} \qquad \text{...(A.20)}$$

where

$$\Delta C = C_r - C \qquad \text{...(A.21)}$$

A.10.1 Tip–shortening options

Three of the tip shortening options are as follows:

A.10.1.1 Full length teeth – option 1

$$h_{a1} = \frac{1 + x_1}{P_{nd}} \qquad \text{...(A.22)}$$

$$h_{a2} = \frac{1 + x_2}{P_{nd}} \qquad \text{...(A.23)}$$

CAUTION – Option 1 (full length teeth) may give insufficient tip–to–root clearance if $C_r >> C$. Check clearances or use option 3 to be safe.

A.10.1.2 Full working depth – option 2

$$h_{a1} = \frac{1 + x_1 - k_s / 2}{P_{nd}} \qquad \text{...(A.24)}$$

$$h_{a2} = \frac{1 + x_2 - k_s / 2}{P_{nd}} \qquad \text{...(A.25)}$$

CAUTION – Option 2 (full working depth) may give insufficient tip–to–root clearance if $C_r >> C$. Check clearances or use option 3 (full tip–to–root clearance) to be safe.

A.10.1.3 Full tip–to–root clearance – option 3

$$h_{a1} = \frac{1 + x_1 - k_s}{P_{nd}} \qquad \text{...(A.26)}$$

$$h_{a2} = \frac{1 + x_2 - k_s}{P_{nd}} \qquad \text{...(A.27)}$$

A.11 Tip–shortening coefficient for internal gearsets

For internal gearsets, there are several requirements which must be met in addition to those that apply to external gearsets. There must be no tip interference between the pinion and gear or between the cutter and gear. Also, there must be no rubbing between the cutter and gear during the return stroke of the cutter. A likely place for interference is between the tooth root fillets of the pinion and the tips of the gear teeth, and it is common practice to shorten the teeth of the internal gear to prevent this. Likewise, the tip radius of the pinion must be selected to ensure that the pinion tips do not interfere with the root fillets of the gear. As with external gearsets, undercut should be avoided and adequate tip–to–root clearances must be maintained. Reference [3] describes a design procedure for a generalized form of profile shift which includes all of the above considerations.

A.12 Outside radii

$$R_{o1} = R_1 + h_{a1} \qquad \text{...(A.28)}$$

$$R_{o2} = R_2 \pm h_{a2} \qquad \text{...(A.29)}$$

A.13 Generating rack shift coefficients

$$x_{g1} = x_1 - \frac{\Delta s_{n1}}{2 \tan \phi_n} \quad \text{...(A.30)}$$

$$x_{g2} = x_2 \mp \frac{\Delta s_{n2}}{2 \tan \phi_n} \quad \text{...(A.31)}$$

A.14 Normal circular tooth thickness

$$s_{n1} = (\pi/2 + 2\, x_{g1} \tan \phi_n) / P_{nd} \quad \text{...(A.32)}$$

$$s_{n2} = (\pi/2 \pm 2\, x_{g2} \tan \phi_n) / P_{nd} \quad \text{...(A.33)}$$

A.15 Determining profile shift coefficients of existing gears

If the normal circular tooth thicknesses are known, the generating rack shift coefficients are found from equations A34 and A35.

$$x_{g1} = \frac{s_{n1} P_{nd} - \pi/2}{2 \tan \phi_n} \quad \text{...(A.34)}$$

$$x_{g2} = \pm \left(\frac{s_{n2} P_{nd} - \pi/2}{2 \tan \phi_n} \right) \quad \text{...(A.35)}$$

A.15.1 Sum of generating rack shift coefficients

$$\Sigma x_g = x_{g2} \pm x_{g1} \quad \text{...(A.36)}$$

A15.2 Normal operating circular backlash

$$B_n = \pm \left(\frac{2\, C_r \tan \phi_n}{C\, P_{nd}} \right) (\Sigma x - \Sigma x_g) \quad \text{...(A.37)}$$

A.15.3 Tooth thinning for backlash

The tooth thinning coefficients must satisfy equation A19. However, it is usually impossible to determine the ratio $\Delta s_{n1} / \Delta s_{n2}$ that was used for existing gears. The following analysis is based on common practice where $\Delta s_{n1} = \Delta s_{n2}$, in which case:

$$\Delta s_{n1} = \Delta s_{n2} = \frac{B_n P_{nd}}{2} \left(\frac{C}{C_r} \right) \quad \text{...(A.38)}$$

A.15.4 Profile shift coefficients

From equations A.30 and A.31:

$$x_1 = x_{g1} + \frac{\Delta s_{n1}}{2 \tan \phi_n} \quad \text{...(A.39)}$$

$$x_2 = x_{g2} \pm \frac{\Delta s_{n2}}{2 \tan \phi_n} \quad \text{...(A.40)}$$

Annex A References

1. AGMA 908-B89, *Information Sheet – Geometry Factors for Determining the Pitting Resistance and Bending Strength of Spur, Helical and Herringbone Gear Teeth*, 1989.

2. ANSI/AGMA 2001-B88, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, 1988.

3. Colbourne, J.R., *The Geometric Design of Internal Gear Pairs*, AGMA Paper No. 87 FTM 2.

Annex B
(informative)
Ratio split for minimum volume

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 901–A92, *A Rational Procedure for the Preliminary Design of Minimum Volume Gears.*]

B.1 Minimum volume considerations in new gear box designs

Often the situation arises where more than one stage is needed in a gear box. If the gear box has more than one stage, the designer must decide how to split the overall gear ratio between the stages. With the right splitting, the total volume of the gears can be minimized. This minimizes the total weight, and usually the cost, of the gearing. Previously, choosing the ratio splitting was usually done by trial and error. Equation 15 can be used to determine how to split the overall ratio in a two stage gear box. This annex shows how that equation was derived.

B.2 Assumptions

In order to simplify equation 15, the following assumptions have been made:

- Balanced pinion pitting ratings;
- C_v, C_H, C_a, C_s, C_f, C_p, C_T, and C_R are constant for both meshes;
- Both meshes consist of steel external gears.

B.3 Symbols used

Table B.1 contains the definitions of the symbols used in this annex. The table also lists the number of the first equation to use each symbol.

Table B.1 – Symbols used in equations

Symbols	Terms	Units	Equation where first used
b	number of power paths	—	B.9
C_a	application factor for pitting resistance	—	B.2
C_f	surface condition factor for pitting resistance	—	B.2
C_G	gear ratio factor	—	B.2
C_H	hardness ratio factor for pitting resistance	—	B.2
C_L	life factor for pitting resistance	—	B.2
C_m	load distribution factor for pitting resistance	—	B.2
C_p	elastic coefficient	$[lb/in^2]^{0.5}$($[N/mm^2]^{0.5}$)	B.2
C_R	reliability factor for pitting resistance	—	B.2
C_s	size factor for pitting resistance	—	B.2
C_T	temperature factor for pitting resistance	—	B.2
C_v	dynamic factor for pitting resistance	—	B.2
d	operating pitch diameter of pinion	in (mm)	B.1
F	face width of narrowest member	in (mm)	B.1
I	geometry factor for pitting resistance	—	B.2
K	contact load factor for pitting resistance	lb/in^2 (N/mm)	B.1
K_{ac}	allowable contact load factor	lb/in^2 (N/mm)	B.2
K_k	constant	—	B.4
m_G	gear ratio ($m_G \geq 1$)	—	B.1
m_{G1}	gear ratio of high speed mesh ($m_{G1} \geq 1$)	—	B.8

continued

Table B.1 (concluded)

Symbols	Terms	Units	Equation where first used
M_o	overall gear ratio ($M_o \geq 1$)	—	B.8
N	total number of pinion load cycles	—	B.10
s_{ac}	allowable contact stress number	lb/in²(N/mm²)	B.2
T_P	transmitted pinion torque, per mesh	in lbs (Nm)	B.1
T_1	torque on high speed shaft	in lbs (Nm)	B.9
Subscripts/ sign convention			
1	high speed mesh		
2	low speed mesh		
±	upper sign external gearsets, lower sign internal gearsets		

B.4 Derivation

This is how equation 15 was derived:

From [1]*:

$$F(d)^2 = \frac{2T_P}{K}\left(\frac{m_G \pm 1}{m_G}\right) \qquad \text{...(B.1)}$$

From [2]:

$$K_{ac} = \frac{I}{C_G}\left(\frac{C_v}{C_a\, C_s\, C_m\, C_f}\right)\left(\frac{s_{ac}\; C_L\; C_H}{C_p\; C_T\; C_R}\right)^2 \qquad \text{...(B.2)}$$

where

$$C_G = \frac{m_G}{m_G \pm 1} \qquad \text{...(B.3)}$$

and $C_s = C_f = C_H = C_T = C_R = 1.0$

Group variables which are approximately equal for both high and low speed meshes and call it K_k.

$$K_{ac} = \left(\frac{I(s_{ac}\, C_L)^2}{C_G\, C_m}\right)K_k \qquad \text{...(B.4)}$$

where

$$K_k = \left(\frac{C_v}{C_a C_s C_f}\right)\left(\frac{C_H}{C_p C_T C_R}\right)^2 \qquad \text{...(B.5)}$$

Setting $K = K_{ac}$:

$$F(d)^2 = \left(\frac{m_G \pm 1}{m_G}\right)\left(\frac{2\,T_P\, C_m}{I(s_{ac}\, C_L)^2 K_k}\right)\left(\frac{m_G}{m_G \pm 1}\right) \qquad \text{...(B.6)}$$

$$F(d)^2 = \frac{2\,T_P\, C_m}{I\,(s_{ac}\, C_L)^2\, K_k} \qquad \text{...(B.7)}$$

In the case of a two stage box with multiple power branches, the total volume of all the gears is equal to the volume of one high speed pinion, b high speed gears, b low speed pinions, and one low speed gear. The volume of a gear or pinion is proportional to $F(d)^2$. Since the gear operating pitch diameter equals the mating pinion operating pitch diameter multiplied by the gear ratio:

$$\sum[Fd^2] = F_1 d_1^2 + bF_1(d_1 m_{G1})^2 + bF_2 d_2^2 + F_2(d_2(M_o/m_{G1}))^2 \qquad \text{...(B.8)}$$

Combining with equation B7:
Since: $T_{P1} = T_1/b$ and $T_{P2} = (T_1 m_{G1})/b$

$$\sum[Fd^2] = \left(\frac{2T_1 C_{m1}}{bI_1(s_{ac1}C_{L1})^2 K_k}\right) + \left(\frac{2T_1 C_{m1} m_{G1}^2}{I_1(s_{ac1}C_{L1})^2 K_k}\right) + \left(\frac{2T_1 C_{m2} m_{G1}}{I_2(\,s_{ac2}C_{L2}\,)^2 K_k}\right) + \left(\frac{2T_1 C_{m2} M_o^2}{bI_2\,(\,s_{ac2}C_{L2}\,)^2 K_k m_{G1}}\right) \qquad \text{...(B9)}$$

* Numbers in brackets [ ] refer to the reference list at the end of this annex.

In order to eliminate C_L and realizing that:

$$N_1 = N_2\, b\, m_{G1} \quad \text{...(B.10)}$$

$$C_{L1} = 2.466(N_1)^{-0.056} = 2.466(N_2\, b m_{G1})^{-0.056} \quad \text{...(B.11)}$$

$$C_{L2} = 2.466(N_2)^{-0.056} \quad \text{...(B.12)}$$

$$\left(\frac{C_{L1}}{C_{L2}}\right) = \left(\frac{N_2\, b m_{G1}}{N_2}\right)^{-0.056}$$

Reduces to:

$$\left(\frac{C_{L1}}{C_{L2}}\right)^2 = (b m_{G1})^{-0.112} \quad \text{...(B.13)}$$

Substituting into equation B9:

$$\sum[Fd^2] = \left(\frac{2T_1 C_{m1}(b m_{G1})^{0.112}}{b I_1 (s_{ac1} C_{L2})^2 K_k}\right) + \left(\frac{2T_1 C_{m1} b^{0.112} m_{G1}^{2.112}}{I_1 (s_{ac1} C_{L2})^2 K_k}\right) + \left(\frac{2T_I C_{m2} m_{G1}}{I_2 (s_{ac2} C_{L2})^2 K_k}\right) + \left(\frac{2T_I C_{m2} M_o^2}{b I_2 (s_{ac2} C_{L2})^2 K_k m_{G1}}\right) \quad \text{...(B.14)}$$

$$\sum[Fd^2] = \left(\frac{2T_I}{C_{L2}^2 K_k}\right)\left[\left(\frac{C_{m1} m_{G1}^{0.112}}{b^{0.888} I_1 s_{ac1}^2}\right) + \left(\frac{C_{m1} b^{0.112} m_{G1}^{2.112}}{I_1 s_{ac1}^2}\right) + \left(\frac{C_{m2} m_{G1}}{I_2 s_{ac2}^2}\right) + \left(\frac{C_{m2} M_o^2}{b I_2 s_{ac2}^2 m_{G1}}\right)\right] \quad \text{...(B.15)}$$

The minimum volume occurs where the first derivative with respect to m_{G1} of equation B.15 is zero. This causes T_I, K_k and C_L to drop out. Taking the first derivative and setting it equal to zero:

$$0 = \left(\frac{0.112 C_{m1} m_{G1}^{-0.888}}{b^{0.888} I_1 s_{ac1}^2}\right) + \left(\frac{2.112 C_{m1} b^{0.112} m_{G1}^{1.112}}{I_1 s_{ac1}^2}\right) + \left(\frac{C_{m2}}{I_2 s_{ac2}^2}\right) + \left(\frac{(-1) C_{m2} M_o^2}{b I_2 s_{ac2}^2 m_{G1}^2}\right) \quad \text{...(B.16)}$$

Rearranging terms:

$$\left(\frac{C_{m2}}{I_2 s_{ac2}^2}\right)\left[\left(\frac{M_o^2}{b m_{G1}^2}\right) - 1\right] = \left(\frac{C_{m1}}{I_1 s_{ac1}^2}\right)\left[\left(\frac{0.112}{b^{0.888} m_{G1}^{0.888}}\right) + 2.112 b^{0.112} m_{G1}^{1.112}\right] \quad \text{...(B.17)}$$

$$\left(\frac{M_o^2}{b m_{G1}^2}\right) - 1 = \left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \times \left[\left(\frac{0.112}{b^{0.888} m_{G1}^{0.888}}\right) + 2.112 b^{0.112} m_{G1}^{1.112}\right] \quad \text{...(B.18)}$$

Equation B.18 is identical to equation 15. This completes the derivation.

Annex B References

1. Willis Jr., R. J., "Lightest–Weight Gears", *Product Engineering*, Jan. 21, 1963.

2. ANSI/AGMA 2001–B88, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, 1988.

Annex C
(informative)
Ratio split for an existing two stage box

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 901-A92, *A Rational Procedure for the Preliminary Design of Minimum Volume Gears.*]

C.1 Existing gear boxes

Sometimes a situation arises where an existing gear box is used with a new gear ratio. If the gear box has more than one stage, the designer must decide how to split the overall gear ratio between the stages. With the right splitting, each stage can have the same horsepower rating maximizing the capacity of the design. Choosing the right splitting has usually been done by trial and error. Equation 21 can be used to determine how to split the overall ratio in a two stage gear box. This annex shows how that equation was derived.

C.2 Assumptions

In order to simplify equation 21, the following assumptions have been made:

– Balanced pinion pitting ratings;

– C_v, C_H, C_a, C_s, C_f, C_p, C_T, and C_R are constant for both meshes;

– Both meshes consist of steel external gears.

C.3 Symbols used

Table C.1 contains the definitions of symbols used in this annex. The table also lists the number of the first equation to use each symbol.

Table C.1 – Symbols used in equations

Symbols	Terms	Units	Equation where first used
b	number of power paths	—	C.11
C_a	application factor for pitting resistance	—	C.2
C_f	surface condition factor for pitting resistance	—	C.2
C_G	gear ratio factor	—	C.2
C_H	hardness ratio factor for pitting resistance	—	C.2
C_L	life factor for pitting resistance	—	C.2
C_m	load distribution factor for pitting resistance	—	C.2
C_p	elastic coefficient	$[lb/in^2]^{0.5}$ $([N/mm^2]^{0.5})$	C.2
C_r	operating center distance	in (mm)	C.8
C_R	reliability factor pitting resistance	—	C.2
C_s	size factor for pitting resistance	—	C.2
C_T	temperature factor for pitting resistance	—	C.2
C_v	dynamic factor for pitting resistance	—	C.2
d	operating pitch diameter of pinion	in (mm)	C.1
F	net face width of narrowest member	in (mm)	C.1
I	geometry factor for pitting resistance	—	C.2
K	contact load factor for pitting resistance	lb/in^2 (N/mm^2)	C.1
K_{ac}	allowable contact load factor	lb/in^2 (N/mm^2)	C.2
K_k	constant	—	C.4
m_a	aspect (F/d) ratio	—	C.8
m_G	gear ratio ($m_G \geq 1$)	—	C.1
m_{G1}	gear ratio of high speed mesh ($m_{G1} \geq 1$)	—	C.11
M_o	overall gear ratio ($M_o \geq 1$)	—	C.14

continued

Table C.1 (concluded)

Symbols	Terms	Units	Equation where first used
N	total number of pinion load cycles	—	C.19
s_{ac}	allowable contact stress number	lb/in² (N/mm²)	C.2
T_P	transmitted pinion torque, per mesh	in lbs (Nm)	C.1
T_1	torque on high speed shaft	in lbs (Nm)	C.11
Subscripts/ sign convention			
1	high speed mesh		
2	low speed mesh		
±	upper sign external gearsets, lower sign internal gearsets		

C4. Derivation

This is how equation 21 was derived. From [1]*:

$$F(d)^2 = \frac{2T_P}{K}\left(\frac{m_G \pm 1}{m_G}\right) \quad \text{...(C.1)}$$

From [2] :

$$K_{ac} = \frac{I}{C_G}\left(\frac{C_v}{C_a\, C_s\, C_m\, C_f}\right)\left(\frac{s_{ac}\ C_L\ C_H}{C_p\ C_T\ C_R}\right)^2 \quad \text{...(C.2)}$$

Where:

$$C_G = \frac{m_G}{m_G \pm 1} \quad \text{...(C.3)}$$

Group variables which are approximately equal for both high and low speed meshes and call it K_k.

$$K_{ac} = \left(\frac{I(s_{ac}\, C_L)^2}{C_G\, C_m}\right)K_k \quad \text{... (C.4)}$$

Where:

$$K_k = \left(\frac{C_v}{C_a\, C_s\, C_f}\right)\left(\frac{C_H}{C_p\, C_T\, C_R}\right)^2 \quad \text{... (C.5)}$$

Setting $K = K_{ac}$

$$F(d)^2 = \left(\frac{m_G \pm 1}{m_G}\right)\left(\frac{2T_P C_m}{I(\,s_{ac} C_L\,)^2 K_k}\right)\left(\frac{m_G}{m_G \pm 1}\right) \quad \text{... (C.6)}$$

$$F(d)^2 = \frac{2T_P C_m}{I(\,s_{ac} C_L\,)^2 K_k} \quad \text{... (C.7)}$$

Substituting

$$F = m_a\,(d)$$

and

$$d = \frac{2\,C_r}{m_G \pm 1} \quad \text{... (C.8)}$$

yields:

$$m_a\,(d)^3 = \frac{2\,T_P\,C_m}{I\,(\,s_{ac}\,C_L\,)^2\,K_k} \quad \text{... (C.9)}$$

$$m_a\left(\frac{2\,C_r}{m_G \pm 1}\right)^3 = \frac{2\,T_P\,C_m}{I\,(\,s_{ac}\,C_L\,)^2\,K_k} \quad \text{... (C.10)}$$

In the case of a two stage box:

Since $T_{P1} = T_1/b$ and $T_{P2} = (T_1 m_{G1})/b$

$$m_{a1}\left(\frac{2\,C_{r1}}{m_{G1} + 1}\right)^3 = \left(\frac{2\,T_1\,C_{m1}}{b\,I_1\,(\,s_{ac1} C_{L1}\,)^2\,K_k}\right) \quad \text{... (C.11)}$$

* Numbers in brackets [ ] refer to the reference list at the end of this annex.

$$m_{a2}\left(\frac{2\,C_{r2}}{m_{G2}+1}\right)^3=\left(\frac{2\,m_{G1}\,T_1\,C_{m2}}{b\,I_2\,(s_{ac2}\,C_{L2})^2\,K_k}\right) \quad \text{... (C.12)}$$

Rearranging equation C11:

$$\left(\frac{m_{a1}\,b\,I_1}{C_{m1}}\right)(s_{ac1}\,C_{L1})^2\left(\frac{C_{r1}}{m_{G1}+1}\right)^3=\frac{2\,T_1}{(2)^3\,K_k} \quad \text{... (C.13)}$$

Since:

$$m_{G2}=\frac{M_o}{m_{G1}}$$

Rearranging equation C12:

$$m_{a2}\left(\frac{2\,C_{r2}}{(M_o/m_{G1})+1}\right)^3=\left(\frac{2\,m_{G1}\,T_1\,C_{m2}}{b\,I_2\,(s_{ac2}\,C_{L2})^2\,K_k}\right) \quad \text{... (C.14)}$$

$$\left(\frac{m_{a2}\,b\,I_2}{m_{G1}\,C_{m2}}\right)(s_{ac2}\,C_{L2})^2\left(\frac{C_{r2}}{(M_o/m_{G1})+1}\right)^3=\frac{2\,T_1}{(2)^3\,K_k} \quad \text{... (C.15)}$$

$$\left(\frac{m_{a2}\,b\,I_2}{m_{G1}\,C_{m2}}\right)(s_{ac2}\,C_{L2})^2\left(\frac{C_{r2}\,m_{G1}}{M_o+m_{G1}}\right)^3=\frac{2\,T_1}{(2)^3\,K_k} \quad \text{... (C.16)}$$

Equating equations C13 and C16:

$$\left(\frac{m_{a1}\,b\,I_1}{C_{m1}}\right)(s_{ac1}\,C_{L1})^2\left(\frac{C_{r1}}{m_{G1}+1}\right)^3=\left(\frac{m_{a2}\,b\,I_2}{m_{G1}\,C_{m2}}\right)(s_{ac2}\,C_{L2})^2\left(\frac{C_{r2}\,m_{G1}}{M_o+m_{G1}}\right)^3 \quad \text{... (C.17)}$$

$$\left(\frac{M_o+m_{G1}}{m_{G1}+1}\right)^3 m_{G1}^{-2}\left(\frac{C_{L1}}{C_{L2}}\right)^2=\left(\frac{m_{a2}\,I_2\,C_{m1}}{m_{a1}\,I_1\,C_{m2}}\right)\left(\frac{C_{r2}}{C_{r1}}\right)^3\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \quad \text{... (C.18)}$$

In order to eliminate C_L and realizing that

$$N_1=N_2\,b\,m_{G1} \quad \text{... (C.19)}$$

$$C_{L1}=2.466(N_1)^{-0.056}=2.466(N_2\,bm_{G1})^{-0.056} \quad \text{... (C.20)}$$

$$C_{L2}=2.466(N_2)^{-0.056} \quad \text{... (C.21)}$$

$$\left(\frac{C_{L1}}{C_{L2}}\right)=\left(\frac{N_2\,bm_{G1}}{N_2}\right)^{-0.056}$$

Reduces to:

$$\left(\frac{C_{L1}}{C_{L2}}\right)^2=(bm_{G1})^{-0.112} \quad \text{... (C.22)}$$

Substituting:

$$\left(\frac{M_o+m_{G1}}{m_{G1}+1}\right)^3 m_{G1}^{-2}\,(b\,m_{G1})^{-0.112}=\left(\frac{m_{a2}}{m_{a1}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{C_{r2}}{C_{r1}}\right)^3\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \quad \text{... (C.23)}$$

$$\left(\frac{M_o+m_{G1}}{m_{G1}+1}\right)^3 m_{G1}^{-2.112}=b^{0.112}\left(\frac{m_{a2}}{m_{a1}}\right)\left(\frac{I_2}{I_1}\right)\left(\frac{C_{m1}}{C_{m2}}\right)\left(\frac{C_{r2}}{C_{r1}}\right)^3\left(\frac{s_{ac2}}{s_{ac1}}\right)^2 \quad \text{... (C.24)}$$

Equation C.24 is identical to equation 21. This completes the derivation.

Annex C References

1. Willis Jr., R. J., Lightest-Weight Gears, *Product Engineering*, Jan. 21, 1963.

2. ANSI/AGMA 2001-B88, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.*

Annex D
(informative)
Example problems

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 901–A92, *A Rational Procedure for Designing Minimum Volume Gears.*]

D.1 Example problems.

These example problems are intended to illustrate ways to use the formulas in the main body of AGMA 901–A92. They can also be used to check computer programs written for AGMA 901–A92. All variable names and equation numbers are from the main body of AGMA 901–A92, to show how each value was arrived at.

The first example deals primarily with the formulas in clause 5. The second and third examples deal with 3.7.1 and clause 5, where there are two stages and the volume of the gearsets are to be minimized. The third example is identical to the second, except that it is done using metric units. The fourth example deals with 3.7.2 and clause 5, where there are two gear stages and the center distances are already known. Here the aim is to maximize the overall rating of the gearsets by balancing the high speed and the low speed ratings. The fifth example is a single stage double helical gearset with a fixed center distance and the sixth example involves a single stage spur gearset with a fixed center distance. These examples also aim to achieve a maximum power rating.

D.2 Example 1 – Single stage spur gearset with unrestricted center distance

This example takes the data directly from example 1 of [1]* and [2] and applies it to the algorithm presented in clause 5. This procedure calculates the preferred number of pinion teeth of 27 which is essentially the same result as in [2] after an extensive computer search.

The input data is:

s_{nc} = 200 000 lb/in²
s_{nt} = 60 000 lb/in²
P = 20 horsepower
n_p = 1260 rpm
ϕ_n = 20°
ψ = 0°
m_G = 5.00
m_a = 0.25
$n_c = n_t$ = 1.0
$C_d = K_d$ = 1.0
b = 1
K_B = 1.0

The design algorithm gives:

I = 0.134 (equation 11)
J = 0.450 (equation 12)
K_c = 1.973 in³ (equation 32)
K_t = 0.074 in³ (equation 33)
$N_{P\,pre}$ = 27 (equation 34)
d = 1.991 inches (equation 35)
F = 0.498 inches (equation 36)

D.3 Example 2 – Two stage spur gearset with unrestricted center distances.

This example illustrates the results from a two stage spur gearset with unrestricted center distances. The application is for a variable density mixer driven by an electric motor. Figures D.1 and D.2 illustrate how figures representative to figure 2 look for this example.

The input data is:

M_o = 25.0
Both stages carburized and hardened (grade 1)
P = 50 horsepower
n_{p1} = 1750 rpm
L = 3400 hours
$\phi_{n1} = \phi_{n2}$ = 20°
$\psi_1 = \psi_2$ = 0°
b = 1
$K_{B1} = K_{B2}$ = 1.0

The design algorithm gives:

$s_{ac1} = s_{ac2}$ = 180 000 lb/in² (table 2)
$s_{at1} = s_{at2}$ = 55 000 lb/in² (table 3)
$N_1 = 3.570 \times 10^8$ cycles (equation 3)
T_{p1} = 1800 in lbs (equation 7)
$C_{a1} = C_{a2} = K_{a1} = K_{a2}$ = 1.50 (table 4)
$C_{v1} = C_{v2} = K_{v1} = K_{v2}$ = 0.7

* Numbers in brackets [ ] refer to the reference list at the end of this annex.

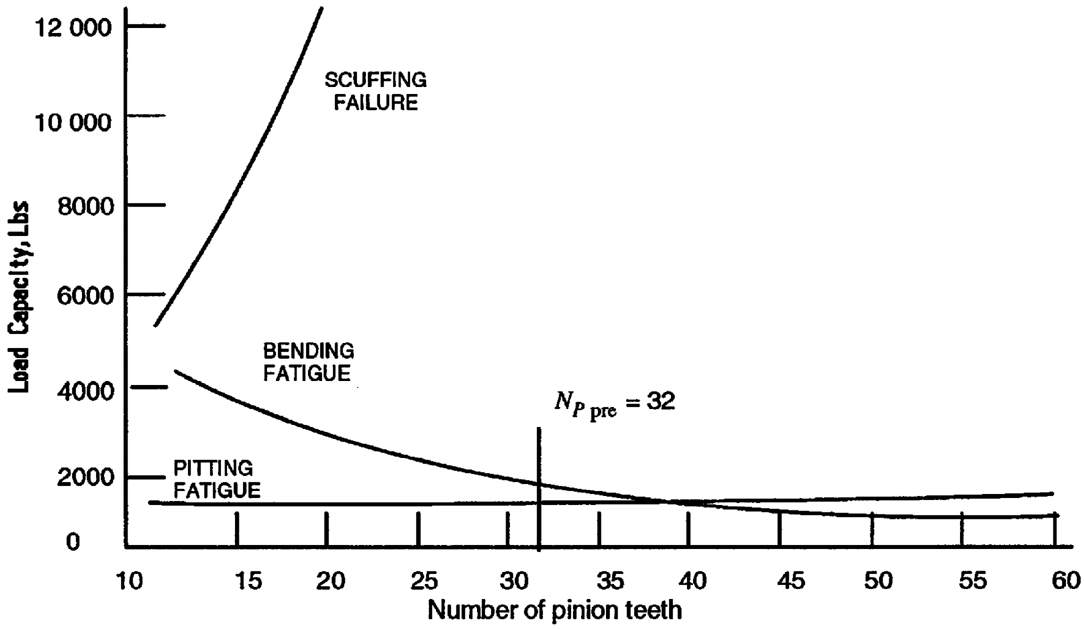

Figure D.1 – Example 2, first stage

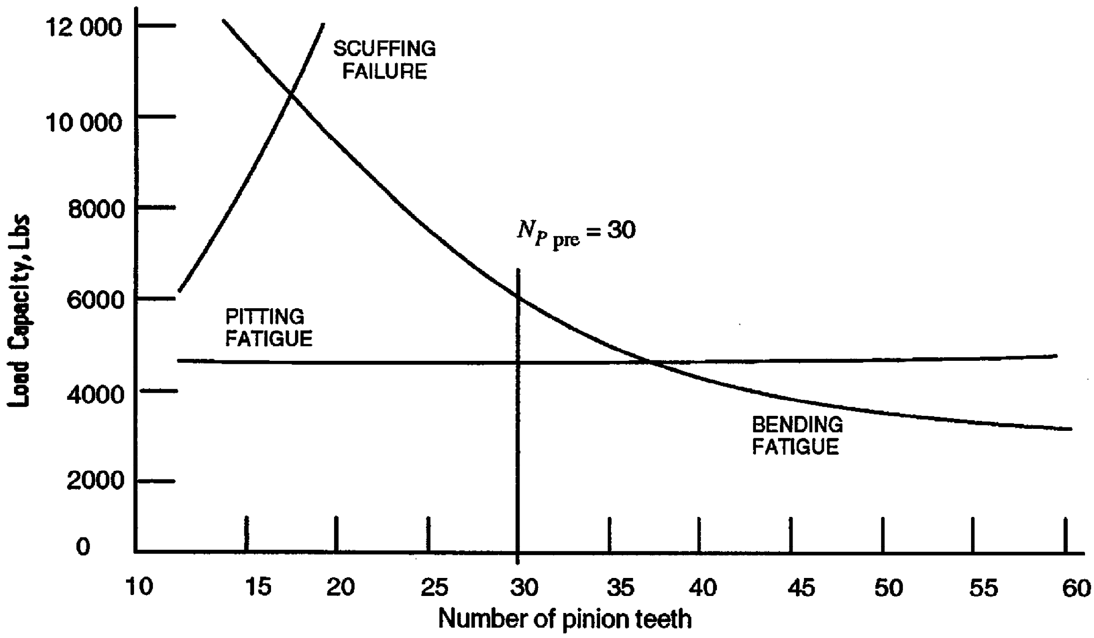

Figure D.2 – Example 2, second stage

Assuming $m_{G1} = M_o^{0.5}$ for an initial guess:

m_{G1} = 5.000
m_{G2} = 5.000
$m_{a1} = m_{a2}$ = 0.833 (equation 4)
$C_{m1} = K_{m1}$ = 1.23 (equation 9)
$C_{m2} = K_{m2}$ = 1.28 (equation 9)
$I_1 = I_2$ = 0.134 (equation 11)
$T_{p2} = T_{p1}(m_{G1})$ = 9000 in lbs

Iteratively solving equation 15:

m_{G1} = 6.176; $m_{G2} = M_o/m_{G1}$ = 4.048

Now recalculate inputs to equation 15 which are dependent on m_{G1} or m_{G2}:

m_{a1} = 0.861 m_{a2} = 0.802 (equation 4)
$C_{m1} = K_{m1}$ = 1.24 (equation 9)
$C_{m2} = K_{m2}$ = 1.28 (equation 9)
I_1 = 0.138 I_2 = 0.129 (equation 11)
$T_{p2} = T_{p1}(m_{G1})$ = 11 117 in lbs

Again iteratively solving equation 15:

m_{G1} = 6.290; $m_{G2} = M_o/m_{G1}$ = 3.975

Therefore:

$N_2 = N_1/(b m_{G1})$ = 5.676 x 10^7 cycles
$n_{p2} = n_{p1}/m_{G1}$ = 278.2 rpm
m_{a1} = 0.863 m_{a2} = 0.799 (equation 4)
$n_{c1} = n_{c2}$ = 1.0
$n_{t1} = n_{t2}$ = 1.2 (for this example, extra conservatism was taken in bending.)
C_{L1} = 0.8185 C_{L2} = 0.9073 (equation 26)
K_{L1} = 0.8910 K_{L2} = 0.9455 (equation 27)
s_{nc1} = 147 330 lb/in² s_{nc2} = 163 314 lb/in² (equation 28)
s_{nt1} = 49 005 lb/in² s_{nt2} = 52 002 lb/in² (equation 29)
$C_{d1} = K_{d1}$ = 2.657 (equation 30, equation 31)
$C_{d2} = K_{d2}$ = 2.743 (equation 30, equation 31)
$J_1 = J_2$ = 0.45 (equation 12)
K_{c1} = 16.89 in³ K_{c2} = 95.50 in³ (equation 32)
K_{t1} = 0.5205 in³ K_{t2} = 3.185 in³ (equation 33)
$N_{P\,\text{pre}1}$ = 32.4 = 32 $N_{P\,\text{pre}2}$ = 30.0 = 30 (equation 34)
d_1 = 2.695 inches d_2 = 4.926 inches (equation 35)
F_1 = 2.33 inches F_2 = 3.94 inches (equation 36)
C_{r1} = 9.822 inches C_{r2} = 12.253 inches (rearranging equation 37)

See figures D.1 and D.2. The analysis used to draw the scuffing failure lines is from appendix A of [3] and is beyond the scope of this annex.

D.4 Example 3 – Two stage spur gearset with unrestricted center distances (metric units).

This example is identical to example 2 above, except that it uses metric units.

The input data is:

M_o = 25.0
Both stages carburized and hardened (grade 1)
P = 37.29 kW
n_{p1} = 1750 rpm
L = 3400 hours
$\phi_{n1} = \phi_{n2}$ = 20°
$\psi_1 = \psi_2$ = 0°
b = 1
$K_{B1} = K_{B2}$ = 1.0

The design algorithm gives:

$s_{ac1} = s_{ac2}$ = 1250 N/mm² (table 2)
$s_{at1} = s_{at2}$ = 380 N/mm² (table 3)
N_1 = 3.570 x 10^8 cycles (equation 3)
T_{p1} = 203.4 Nm (equation 7M)
$C_{a1} = C_{a2} = K_{a1} = K_{a2}$ = 1.50 (table 4)
$C_{v1} = C_{v2} = K_{v1} = K_{v2}$ = 0.7

Assuming $m_{G1} = M_o^{0.5}$ for an initial guess:

m_{G1} = 5.000
m_{G2} = 5.000
$m_{a1} = m_{a2}$ = 0.833 (equation 4)
$C_{m1} = K_{m1}$ = 1.23 (equation 9M)
$C_{m2} = K_{m2}$ = 1.28 (equation 9M)
$I_1 = I_2$ = 0.134 (equation 11)
$T_{p2} = T_{p1}(m_{G1})$ = 1017 Nm

Iteratively solving equation 15:

m_{G1} = 6.176; $m_{G2} = M_o/m_{G1}$ = 4.048

Now recalculate inputs to equation 15 which are dependent on m_{G1} or m_{G2}:

m_{a1} = 0.861 m_{a2} = 0.802 (equation 4)
$C_{m1} = K_{m1}$ = 1.24 (equation 9M)
$C_{m2} = K_{m2}$ = 1.28 (equation 9M)
I_1 = 0.138 I_2 = 0.129 (equation 11)
$T_{p2} = T_{p1}(m_{G1})$ = 1256 Nm

Again iteratively solving equation 15:

m_{G1} = 6.290; $m_{G2} = M_o/m_{G1}$ = 3.975

Therefore:

$N_2 = N_1/(b m_{G1})$ = 5.676 x 10^7 cycles
$n_{p2} = n_{p1}/m_{G1}$ = 278.2 rpm

$m_{a1} = 0.863$ $m_{a2} = 0.799$ (equation 4)
$n_{c1} = n_{c2} = 1.0$
$n_{t1} = n_{t2} = 1.2$ (for this example, extra conservatism was taken in bending.)
$C_{L1} = 0.8185$ $C_{L2} = 0.9073$ (equation 26)
$K_{L1} = 0.8910$ $K_{L2} = 0.9455$ (equation 27)
$s_{nc1} = 1023$ N/mm² $s_{nc2} = 1134$ N/mm² (equation 28)
$s_{nt1} = 338.6$ N/mm² $s_{nt2} = 359.3$ N/mm² (equation 29)
$C_{d1} = K_{d1} = 2.657$ (equation 30, equation 31)
$C_{d2} = K_{d2} = 2.743$ (equation 30, equation 31)
$J_1 = J_2 = 0.45$ (equation 12)
$K_{c1} = 273\,060$ mm³ $K_{c2} = 1.544 \times 10^6$ mm³ (equation 32M)
$K_{t1} = 8517$ mm³ $K_{t2} = 52\,122$ mm³ (equation 33M)
$N_{P\,\mathrm{pre1}} = 32.1 = 32$ $N_{P\,\mathrm{pre2}} = 29.6 = 30$ (equation 34)
$d_1 = 68.1$ mm $d_2 = 124.6$ mm (equation 35)
$F_1 = 59$ mm $F_2 = 100$ mm (equation 36)
$C_{r1} = 248.4$ mm $C_{r2} = 309.8$ mm (rearranging equation 37)

D.5 Example 4 – Two stage gearset with fixed center distances.

This example illustrates the results from a two stage gearset with fixed center distances. The application is for a uniformly loaded conveyor driven by an electric motor.

The input data is:

$M_o = 20.0$
$C_{r1} = 7.0$ in
$C_{r2} = 18.0$ in
High speed stage carburized and hardened (grade 1)
Low speed stage 300 BHN through hardened (grade 1)
$P = 125$ horsepower
$n_{p1} = 1750$ rpm
$L = 10\,000$ hours
$\phi_{n1} = \phi_{n2} = 20°$
$\psi_1 = 15°$ $\psi_2 = 0°$
$b = 2$
$K_{B1} = K_{B2} = 1.0$

The design algorithm gives:

$s_{ac1} = 180\,000$ lb/in² (table 2)
$s_{ac2} = 124\,100$ lb/in² (equation 1)
$s_{at1} = 55\,000$ lb/in² (table 3)
$s_{at2} = 36\,146$ lb/in² (equation 2)
$N_1 = 2.100 \times 10^9$ cycles ($q = 2$) (equation 3)
$C_{a1} = C_{a2} = K_{a1} = K_{a2} = 1.25$ (table 4)
$C_{v1} = C_{v2} = K_{v1} = K_{v2} = 0.7$

Assuming $m_{G1} = M_o^{0.5}$ for an initial guess:

$m_{G1} = 4.4721$
$m_{G2} = 4.4721$
$m_{a1} = m_{a2} = 0.817$ (equation 4)
$I_1 = 0.229$ $I_2 = 0.131$ (equation 13, equation 11)
$d_1 = 2.558$ in $d_2 = 6.579$ in (equation 37)
$C_{m1} = K_{m1} = 1.23$ (equation 10)
$C_{m2} = K_{m2} = 1.32$ (equation 10)

Iteratively solving equation 21:

$m_{G1} = 4.246$; $m_{G2} = M_o/m_{G1} = 4.710$

Now recalculate inputs to equation 21 which are dependent on m_{G1} or m_{G2}:

$m_{a1} = 0.809$ $m_{a2} = 0.825$ (equation 4)
$I_1 = 0.227$ $I_2 = 0.133$ (equation 13, equation 11)
$d_1 = 2.669$ inches $d_2 = 6.305$ inches (equation 37)
$C_{m1} = K_{m1} = 1.23$ (equation 10)
$C_{m2} = K_{m2} = 1.32$ (equation 10)

Again iteratively solving equation 21:

$m_{G1} = 4.200$; $m_{G2} = M_o/m_{G1} = 4.762$

Therefore:

$N_2 = N_1/(b m_{G1}) = 2.500 \times 10^8$ cycles
$n_{p2} = n_{p1}/m_{G1} = 416.7$ rpm
$m_{a1} = 0.808$ $m_{a2} = 0.826$ (equation 4)
$n_{c1} = n_{c2} = 1.0$
$n_{t1} = n_{t2} = 1.0$
$C_{L1} = 0.7412$ $C_{L2} = 0.8350$ (equation 26)
$K_{L1} = 0.8414$ $K_{L2} = 0.9013$ (equation 27)
$s_{nc1} = 133\,420$ lb/in² (equation 28)
$s_{nc2} = 103\,629$ lb/in² (equation 28)
$s_{nt1} = 46\,277$ lb/in² $s_{nt2} = 32\,577$ lbs/in² (equation 29)
$C_{d1} = K_{d1} = 2.196$ (equation 30, equation 31)
$C_{d2} = K_{d2} = 2.357$ (equation 30, equation 31)
$J_1 = 0.50$ $J_2 = 0.45$ (equation 14, equation 12)
$K_{c1} = 12.94$ in³ $K_{c2} = 164.98$ in³ (equation 32)
$K_{t1} = 0.4271$ in³ $K_{t2} = 3.038$ in³ (equation 33)

$N_{P\ pre1}$ = 30.3 = 30 $N_{P\ pre2}$ = 54.3 = 54 (equation 34)
d_1 = 2.692 inches d_2 = 6.248 inches (equation 37)
F_1 = 1.79 inches F_2 = 4.23 inches (equation 38)
$m_{a1\ actual}$ = 0.663 $m_{a2\ actual}$ = 0.676 (equation 39)

D.6 Example 5 – Single stage double helical gearset with a fixed center distance and two power paths

This example considers a single stage double helical gearset with a fixed center distance and two power paths. The application is a marine main propulsion drive.

The input data is:

L = 300 000 hours
C_a = K_a = 1.20
m_G = 2.574
C_r = 25.172 inch
P = 13 125 horsepower
n_p = 2940 rpm
ϕ_n = 20°
ψ = 30°
Carburized and hardened gearing (grade 1)
n_c = n_t = 1.5
b = 2
K_B = 1.0

The design algorithm gives:

s_{ac} = 180 000 lb/in² (table 2)
s_{at} = 55 000 lb/in² (table 3)
d = 14.0862 in (equation 37)
N = 1.058 x 10^{11} cycles (q = 2) (equation 3)
m_a = 1.440 (equation 5)
C_m = K_m = 1.9 (equation 10)
C_v = K_v = 0.7
I = 0.2017 (equation 13)
J = 0.50 (equation 14)
C_L = 0.5951 (equation 26)
K_L = 0.7413 (equation 27)
s_{nc} = 107 124 lb/in² (equation 28)
s_{nt} = 40 773 lb/in² (equation 29)
C_d = 3.257 (equation 30)
K_d = 3.257 (equation 31)
K_c = 4710 in³ (equation 32)
K_t = 67.4 in³ (equation 33)
$N_{P\ pre}$ = 69.9 = 70 (equation 34)
F = 23.74 in (equation 38)
$m_{a\ actual}$ = 1.685 (equation 39)

Note that $m_{a\ actual}$ is greater than recommended by equation 5, so either C_r should be increased or K_c decreased.

D.7 Example 6 – Single stage spur gearset with a fixed center distance

This example illustrates a single stage spur gearset with a fixed center distance. The application is an aerospace accessory drive gear mesh.

The input data is:

L = 200 hours
C_a = K_a = 1.25
m_G = 2.1
C_r = 1.55 inch
P = 5.7 horsepower
n_p = 4100 rpm
m_a = 0.25
ϕ_n = 22.5°
ψ = 0°
Carburized and hardened gearing (grade 2)
n_c = n_t = 1.0
b = 1
K_B = 1.0

The design algorithm gives:

s_{ac} = 225 000 lb/in² (table 2)
s_{at} = 65 000 lb/in² (table 3)
d = 1.0 in (equation 37)
N = 4.92 x 10^7 cycles (equation 3)
C_m = K_m = 1.058 (equation 10)
C_v = K_v = 0.7
I = 0.1198 (equation 11)
J = 0.45 (equation 12)
C_L = 0.915; (equation 26)
K_L = 0.950; (equation 27)
s_{nc} = 205 875 lb/in² (equation 28)
s_{nt} = 61 750 lb/in² (equation 29)
C_d = 1.888 (equation 30)
K_d = 1.888 (equation 31)
K_c = 0.344 in³ (equation 32)
K_t = 0.0119 in³ (equation 33)
$N_{P\ pre}$ = 28.9 = 29 (equation 34)
F = 0.344 inch (equation 38)
$m_{a\ actual}$ = 0.344 (equation 39)

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Annex E
(informative)
References and bibliography

[This annex is provided for informational purposes only and should not be construed as a part of AGMA 901–A92, *A Rational Procedure for Designing Minimum Volume Gears.*]

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