بنام خدا

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Faculty of Mechanical Engineering, *Semester: 1397-1*

Problem set 5

A. Prepares the computer code for Algorithms as required in Class (at least 3 cases).

B.

1. The total number of function evaluations required to find the optimum solution is usually taken as a measure of the ***efficiency of the algorithm***. Present a comparative study of the various unconstrained optimization techniques using the common test functions given below.

1.1- Rosenbrock's parabolic valley:

$$f(x_1, x_2) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

$$\mathbf{X}_1 = \begin{Bmatrix} -1.2 \\ 1.0 \end{Bmatrix}, \quad \mathbf{X}^* = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$f_1 = 24.0, \quad f^* = 0.0$$

1.2- A nonlinear function of three variables:

$$f(x_1, x_2, x_3) = \frac{1}{1 + (x_1 - x_2)^2} + \sin\left(\frac{1}{2}\pi x_2 x_3\right) + \exp\left[-\left(\frac{x_1 + x_3}{x_2} - 2\right)^2\right]$$

$$\mathbf{X}_1 = \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix}, \quad \mathbf{X}^* = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

$$f_1 = 1.5, \quad f^* = f_{max} = 3.0$$

1.3- Brown's badly scaled function:

$$f(x_1, x_2) = (x_1 - 10^6)^2 + (x_2 - 2 \times 10^{-6})^2 + (x_1 x_2 - 2)^2$$

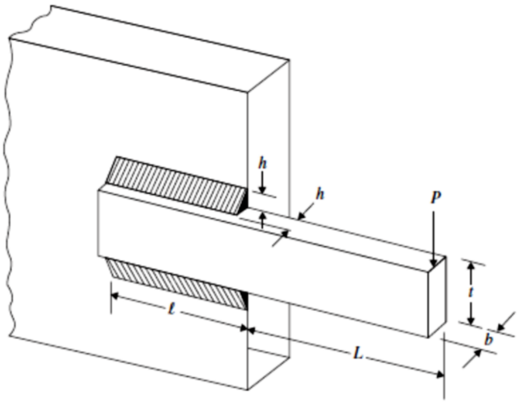
$$\mathbf{X}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}, \quad \mathbf{X}^* = \begin{Bmatrix} 10^6 \\ 2 \times 10^{-6} \end{Bmatrix}$$

$$f_1 \approx 10^{12}, \quad f^* = 0.0$$

2. Use each of available algorithms for solving **one** of constrained nonlinear programming problem. (Solution of one problem is enough.)

2.1 - Welded Beam Design

The welded beam shown in Figure is designed for minimum cost subject to constraints on shear stress in weld (τ), bending stress in the beam (σ), buckling load on the bar (Pc), end deflection of the beam (δ), and side constraints .



Design vector:

$$\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} h \\ l \\ t \\ b \end{Bmatrix}$$

Objective function: $f(\mathbf{X}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$

Constraints:

$$g_1(\mathbf{X}) = \tau(\mathbf{X}) - \tau_{\max} \le 0$$
$$g_2(\mathbf{X}) = \sigma(\mathbf{X}) - \sigma_{\max} \le 0$$
$$g_3(\mathbf{X}) = x_1 - x_4 \le 0$$
$$g_4(\mathbf{X}) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \le 0$$
$$g_5(\mathbf{X}) = 0.125 - x_1 \le 0$$
$$g_6(\mathbf{X}) = \delta(\mathbf{X}) - \delta_{\max} \le 0$$
$$g_7(\mathbf{X}) = P - P_c(\mathbf{X}) \le 0$$
$$g_8(\mathbf{X}) \text{ to } g_{11}(\mathbf{X}): \ 0.1 \le x_i \le 2.0, \qquad i = 1, 4$$
$$g_{12}(\mathbf{X}) \text{ to } g_{15}(\mathbf{X}): \ 0.1 \le x_i \le 10.0, \qquad i = 2, 3$$

where

$$\tau(\mathbf{X}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$$

$$\tau' = \frac{P}{\sqrt{2}x_1x_2}, \quad \tau'' = \frac{MR}{J}, \quad M = P\left(L + \frac{x_2}{2}\right)$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$

$$J = 2\left\{\frac{x_1x_2}{\sqrt{2}}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$

$$\sigma(\mathbf{X}) = \frac{6PL}{x_4x_3^2}$$

$$\delta(\mathbf{X}) = \frac{4PL^3}{Ex_3^3x_4}$$

$$P_c(\mathbf{X}) = \frac{4.013\sqrt{EG(x_3^2x_4^6/36)}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$

Data: $P = 6000$ lb, $L = 14$ in., $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, $\tau_{\max} =$ 13,600 psi, $\sigma_{\max} = 30{,}000$ psi, and $\delta_{\max} = 0.25$ in.
Starting and optimum solutions:

$$\mathbf{X}^{\text{start}} = \begin{Bmatrix} h \\ l \\ t \\ b \end{Bmatrix} = \begin{Bmatrix} 0.4 \\ 6.0 \\ 9.0 \\ 0.5 \end{Bmatrix} \text{ in.,} \quad f^{\text{start}} = \$5.3904, \quad \mathbf{X}^* = \begin{Bmatrix} h \\ l \\ t \\ b \end{Bmatrix}^* = \begin{Bmatrix} 0.2444 \\ 6.2177 \\ 8.2915 \\ 0.2444 \end{Bmatrix} \text{ in.,}$$

$$f^* = \$2.3810$$

2.2 –Heat Exchanger Design

Objective function: Minimize $f(\mathbf{X}) = x_1 + x_2 + x_3$

Constraints:

$$g_1(\mathbf{X}) = 0.0025(x_4 + x_6) - 1 \le 0$$
$$g_2(\mathbf{X}) = 0.0025(-x_4 + x_5 + x_7) - 1 \le 0$$
$$g_3(\mathbf{X}) = 0.01(-x_5 + x_8) - 1 \le 0$$
$$g_4(\mathbf{X}) = 100x_1 - x_1x_6 + 833.33252x_4 - 83{,}333.333 \le 0$$
$$g_5(\mathbf{X}) = x_2x_4 - x_2x_7 - 1250x_4 + 1250x_5 \le 0$$
$$g_6(\mathbf{X}) = x_3x_5 - x_3x_8 - 2500x_5 + 1{,}250{,}000 \le 0$$

$$g_7 : 100 \le x_1 \le 10{,}000 : g_8$$
$$g_9 : 1000 \le x_2 \le 10{,}000 : g_{10}$$
$$g_{11} : 1000 \le x_3 \le 10{,}000 : g_{12}$$
$$g_{13} \text{ to } g_{22} : 10 \le x_i \le 1000, \quad i = 4, 5, \ldots, 8$$

Optimum solution: $\mathbf{X}^* = \{567 \quad 1357 \quad 5125 \quad 181 \quad 295 \quad 219 \quad 286 \quad 395\}^{\mathrm{T}}$, $f^* = 7049$