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Problem set 6

1. In the design space, plot the following constrained problem and sketch the Pareto optimal set:

minimize:

$$f_1 = (x_1 - 3)^2 + (x_2 - 7)^2$$

$$f_2 = (x_1 - 9)^2 + (x_2 - 8)^2$$

subjected to:

$$g_1 = 70 - 4x_2 - 8x_1 \le 0$$

$$g_2 = -2.5x_2 + 3x_1 \le 0$$

$$g_1 = -6.8 + x_1 \le 0$$

2. Solve the following problem using KKT optimality conditions with the weighted sum method:

minimize:

$$f_1 = 20(x_1 - 0.75)^2 + (2x_2 - 2)^2$$

$$f_2 = 5(x_1 - 1.6)^2 + 2x_2$$

subjected to:

$$g_1 = -x_2 \le 0$$

First, use $w_1 = 0.1$ and $w_2 = 0.9$. Then, resolve the problem using $w_1 = 0.9$ and $w_2 = 0.1$.

3. Plot the objective functions contours for the following problem (on the same graph) and solve the problem using the lexicographic method:

minimize

$$f_1 = (x - 1)^2 (x - 4)^2$$

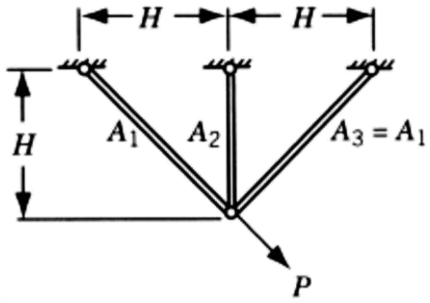
$$f_2 = 4(x - 2)^2$$

$$f_3 = 8(x - 3)^2$$

Indicate the final solution point on the graph. Assume the functions are prioritized in the following order: f_1, f_2, f_3, with f_1 being the most important.

4. ***(Optional)*** **Design of a Three-Bar Truss**

The optimal design of the three-bar truss shown in Figure is considered using two different objectives with the cross-sectional areas of members 1 (and 3) and 2 as design variables.



Design vector:

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix}$$

Objective functions:

$$f_1(\mathbf{X}) = \text{weight} = 2\sqrt{2}x_1 + x_2$$

$$f_2(\mathbf{X}) = \text{vertical deflection of loaded joint} = \frac{PH}{E}\frac{1}{x_1 + \sqrt{2}x_2}$$

Constraints:

$$\sigma_1(\mathbf{X}) - \sigma^{(u)} \le 0$$

$$\sigma_2(\mathbf{X}) - \sigma^{(u)} \le 0$$

$$\sigma_3(\mathbf{X}) - \sigma^{(l)} \le 0$$

$$x_i^{(l)} \le x_i \le x_i^{(u)}, \quad i = 1, 2$$

where σ_i is the stress induced in member *i*, $\sigma^{(u)}$ the maximum permissible stress in tension, $\sigma^{(l)}$ the maximum permissible stress in compression, $x_i^{(l)}$ the lower bound on x_i , and $x_i^{(u)}$ the upper bound on x_i . The stresses are given by

$$\sigma_1(\mathbf{X}) = P\frac{x_2 + \sqrt{2}x_1}{\sqrt{2}x_1^2 + 2x_1x_2}$$

$$\sigma_2(\mathbf{X}) = P\frac{1}{x_1 + \sqrt{2}x_2}$$

$$\sigma_3(\mathbf{X}) = -P\frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}$$

Data: $\sigma^{(u)} = 20$, $\sigma^{(l)} = -15$, $x_i^{(l)} = 0.1 (i = 1, 2)$, $x_i^{(u)} = 5.0 (i = 1, 2)$, $P = 20$, and $E = 1$.

Optimum design:

$$\mathbf{X}_1^* = \begin{Bmatrix} 0.78706 \\ 0.40735 \end{Bmatrix}, \quad f_1^* = \begin{matrix} 2.6335, \text{ stress constraint of} \\ \text{member 1 is active at } \mathbf{X}_1^* \end{matrix}$$

$$\mathbf{X}_2^* = \begin{Bmatrix} 5.0 \\ 5.0 \end{Bmatrix}, \quad f_2^* = 1.6569$$