7 Analysis and Design for Torsion

7.1 Introduction

Reinforced concrete members are commonly subjected to bending moments, to transverse shears associated with those bending moments, and, in the case of columns, to axial forces often combined with bending and shear. In addition, torsional forces may act, tending to twist a member about its longitudinal axis. Such torsional forces seldom act alone and are almost always concurrent with bending moment and transverse shear, and sometimes with axial force as well.

For many years, torsion was regarded as a secondary effect and was not considered explicitly in design—its influence being absorbed in the overall factor of safety of rather conservatively designed structures. Current methods of analysis and design, however, have resulted in less conservatism, leading to somewhat smaller members that, in many cases, must be reinforced to increase torsional strength. In addition, there is increasing use of structural members for which torsion is a central feature of behavior; examples include curved bridge girders, eccentrically loaded box beams, and helical stairway slabs. The design procedures in the ACI Code were first proposed in Switzerland (Refs. 7.1 and 7.2) and are also included in the European and Canadian model codes (Refs. 7.3 and 7.4).

It is useful in considering torsion to distinguish between primary and secondary torsion in reinforced concrete structures. *Primary torsion*, sometimes called *equilibrium torsion* or *statically determinate torsion*, exists when the external load has no alternative load path but must be supported by torsion. For such cases, the torsion required to maintain static equilibrium can be uniquely determined. An example is the cantilevered slab of Fig. 7.1*a*. Loads applied to the slab surface cause twisting moments m_t to act along the length of the supporting beam. These are equilibrated by the resisting torque T provided at the columns. Without the torsional moments, the structure will collapse.

In contrast to this condition, *secondary torsion*, also called *compatibility torsion* or *statically indeterminate torsion*, arises from the requirements of continuity, i.e., compatibility of deformation between adjacent parts of a structure. For this case, the torsional moments cannot be found based on static equilibrium alone. Disregard of continuity in the design will often lead to extensive cracking, but generally will not cause collapse. An internal readjustment of forces is usually possible and an alternative equilibrium of forces found. An example of secondary torsion is found in the spandrel or edge beam supporting a monolithic concrete slab, shown in Fig. 7.1*b*. If the spandrel beam is torsionally stiff and suitably reinforced, and if the columns can provide the necessary resisting torque T, then the slab moments will approximate those for a rigid exterior support as shown in Fig. 7.1*c*. However, if the beam has little torsional stiffness and

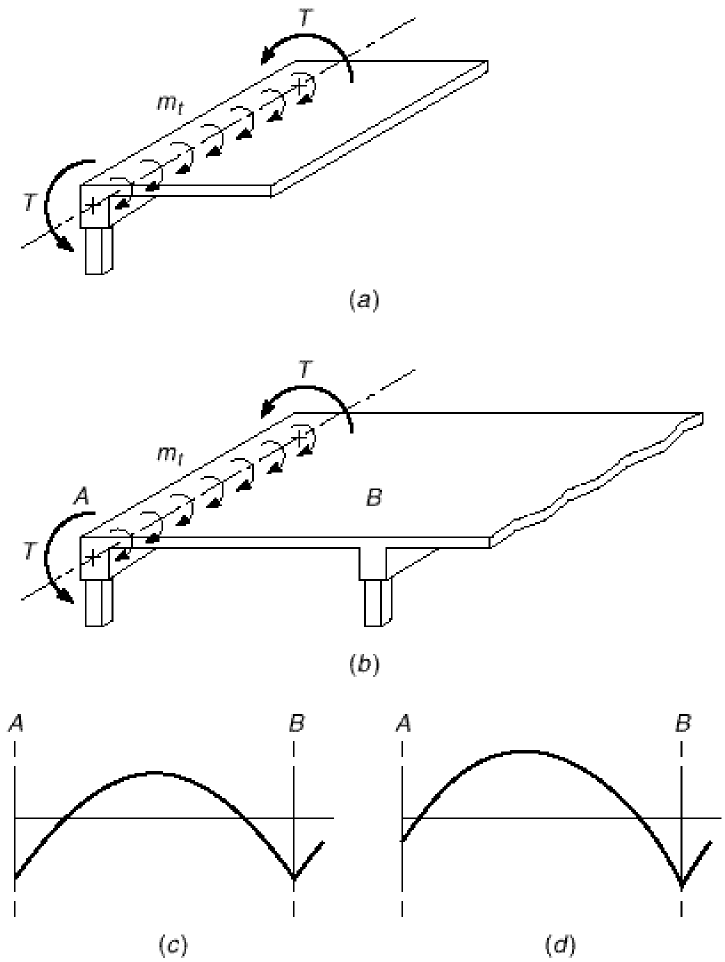


FIGURE 7.1
Torsional effects in reinforced concrete: (*a*) primary or equilibrium torsion at a cantilevered slab; (*b*) secondary or compatibility torsion at an edge beam; (*c*) slab moments if edge beam is stiff torsionally; (*d*) slab moments if edge beam is flexible torsionally.

inadequate torsional reinforcement, cracking will occur to further reduce its torsional stiffness, and the slab moments will approximate those for a hinged edge, as shown in Fig. 7.1*d*. If the slab is designed to resist the altered moment diagram, collapse will not occur (see discussion in Section 12.10).

Although current techniques for analysis permit the realistic evaluation of torsional moments for statically indeterminate conditions as well as determinate, designers often neglect secondary torsional effects when torsional stresses are low and alternative equilibrium states are possible. This is permitted according to the ACI Code and many other design specifications. On the other hand, when torsional strength is an essential feature of the design, such as for the bridge shown in Fig. 7.2, special analysis and special torsional reinforcement is required, as described in the remainder of this chapter.

7.2 Torsion in Plain Concrete Members

Figure 7.3 shows a portion of a prismatic member subjected to equal and opposite torques T at the ends. If the material is elastic, St. Venant's torsion theory indicates that

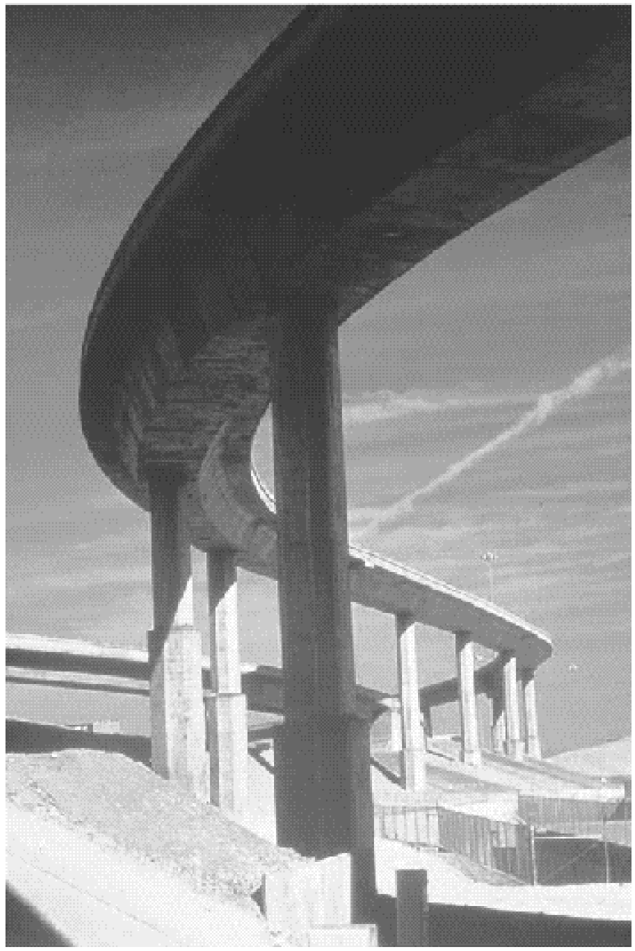


FIGURE 7.2
Curved continuous beam bridge, Las Vegas, Nevada, designed for torsional effects. (*Courtesy of Portland Cement Association.*)

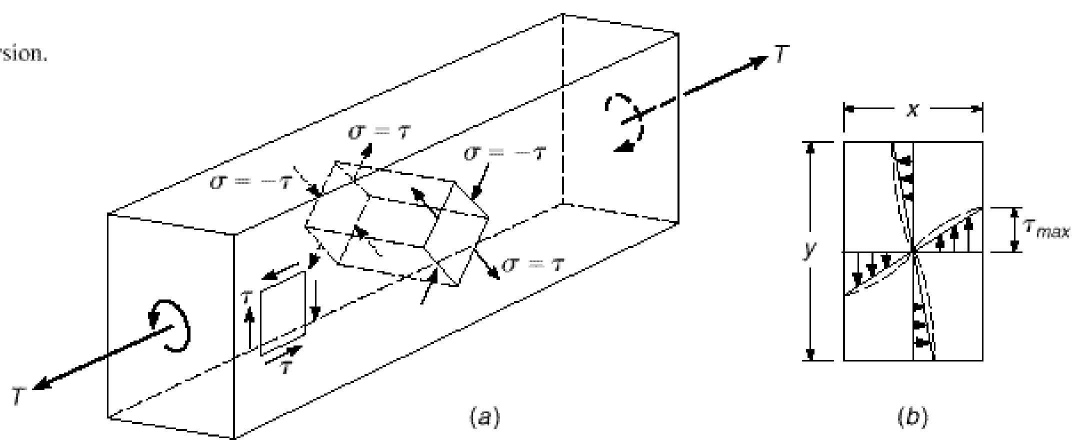
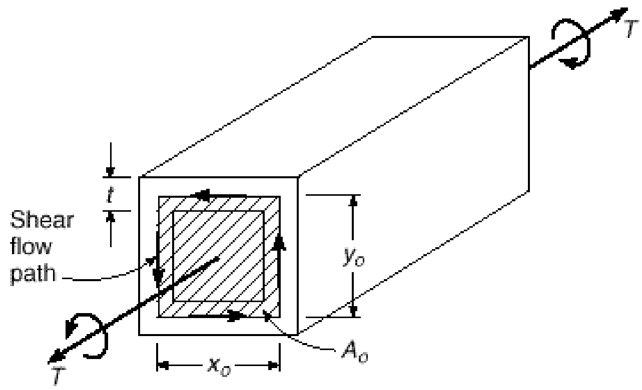


FIGURE 7.3
Stresses caused by torsion.

torsional shear stresses are distributed over the cross section, as shown in Fig. 7.3*b*. The largest shear stresses occur at the middle of the wide faces. If the material deforms inelastically, as expected for concrete, the stress distribution is closer to that shown by the dashed line.

FIGURE 7.4
Thin-walled tube under torsion.



Shear stresses in pairs act on an element at or near the wide surface, as shown in Fig. 7.3*a*. As explained in strength of materials texts, this state of stress corresponds to equal tension and compression stresses on the faces of an element at 45° to the direction of shear. These inclined tension stresses are of the same kind as those caused by transverse shear, discussed in Section 4.2. However, in the case of torsion, since the torsional shear stresses are of opposite sign on opposing sides of the member (Fig. 7.3*b*), the corresponding diagonal tension stresses are at right angles to each other (Fig. 7.3*a*).

When the diagonal tension stresses exceed the tensile resistance of the concrete, a crack forms at some accidentally weaker location and spreads immediately across the beam. The value of torque corresponding to the formation of this diagonal crack is known as the *cracking torque* T_{cr}.

There are several ways of analyzing members subjected to torsion. The nonlinear stress distribution shown by the dotted lines in Fig. 7.3*b* lends itself to the use of the *thin-walled tube, space truss analogy*. Using this analogy, the shear stresses are treated as constant over a finite thickness t around the periphery of the member, allowing the beam to be represented by an equivalent tube, as shown in Fig. 7.4. Within the walls of the tube, torque is resisted by the shear flow q, which has units of force per unit length. In the analogy, q is treated as a constant around the perimeter of the tube. As shown in Fig. 7.4, the resultants of the individual components of shear flow are located within the walls of the tube and act along lengths y_o in the vertical walls and along lengths x_o in the horizontal walls, with y_o and x_o measured at the center of the walls.

The relationship between the applied torque and the shear flow can be obtained by summing the moments about the axial centerline of the tube, giving

$$T = 2qx_o y_o / 2 + 2qy_o x_o / 2 \qquad (a)$$

where the two terms on the right-hand side represent the contributions of the horizontal and vertical walls to the resting torque, respectively. Thus,

$$T = 2qx_o y_o \qquad (b)$$

The product $x_o y_o$ represents the area enclosed by the shear flow path A_o, giving

$$T = 2qA_o \qquad (c)$$

and

$$q = \frac{T}{2A_o} \qquad (d)$$

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Note that, although A_o is an area, it derives from the moment calculation shown in Eq. (*a*) above. Thus, A_o is applicable for hollow box sections, as well as solid sections, and in such case includes the area of the central void.

For a tube wall thickness t, the unit shear stress acting within the walls of the tube is

$$\tau = \frac{q}{t} = \frac{T}{2A_o t} \tag{7.1}$$

As shown in Fig. 7.3*a*, the principal tensile stress $\sigma = \tau$. Thus, the concrete will crack only when $\tau = \sigma = f_t'$, the tensile strength of concrete. Considering that concrete is under biaxial tension and compression, f_t' can be conservatively represented by $4\sqrt{f_c'}$ rather than the value typically used for the modulus of rupture of concrete, which is taken as $f_r = 7.5\sqrt{f_c'}$ for normal-density concrete. Substituting $\tau = \tau_{cr} = 4\sqrt{f_c'}$ in Eq. (7.1) and solving for T gives the value of the cracking torque:

$$T_{cr} = 4\sqrt{f_c'}\,(2A_o t) \tag{7.2}$$

Remembering that A_o represents the area enclosed by the shear flow path, A_o must be some fraction of the area enclosed by the outside perimeter of the full concrete cross section A_{cp}. The value of t can, in general, be approximated as a fraction of the ratio A_{cp}/p_{cp}, where p_{cp} is the perimeter of the cross section. For solid members with rectangular cross sections, t is typically one-sixth to one-fourth of the minimum width. Using a value of one-fourth for a member with a width-to-depth ratio of 0.5 yields a value of A_o approximately equal to $\frac{2}{3}A_{cp}$. For the same member, $t = \frac{3}{4}A_{cp}/p_{cp}$. Using these values for A_o and t in Eq. (7.2) gives

$$T_{cr} = 4\sqrt{f_c'}\,\frac{A_{cp}^2}{p_{cp}} \quad \text{in-lb} \tag{7.3}$$

It has been found that Eq. (7.3) gives a reasonable estimate of the cracking torque of solid reinforced concrete members regardless of the cross-sectional shape. For hollow sections, T_{cr} in Eq. (7.3) should be reduced by the ratio A_g/A_{cp}, where A_g is the gross cross section of the concrete, i.e., not including the area of the voids (Ref. 7.5).

7.3 Torsion in Reinforced Concrete Members

To resist torsion for values of T above T_{cr}, reinforcement must consist of closely spaced stirrups and longitudinal bars. Tests have shown that longitudinal bars alone hardly increase the torsional strength, with test results showing an improvement of at most 15 percent (Ref. 7.5). This is understandable because the only way in which longitudinal steel can directly contribute to torsional strength is by dowel action, which is particularly weak and unreliable if longitudinal splitting along bars is not restrained by transverse reinforcement. Thus, the torsional strength of members reinforced only with longitudinal steel is satisfactorily, and somewhat conservatively, predicted by Eqs. (7.2) and (7.3).

When members are adequately reinforced, as in Fig. 7.5*a*, the concrete cracks at a torque that is equal to or only somewhat larger than in an unreinforced member, as given by Eq. (7.3). The cracks form a spiral pattern, as shown in Fig. 7.5*b*. Upon cracking, the torsional resistance of the concrete drops to about half of that of the uncracked member, the remainder being now resisted by reinforcement. This redistribution of internal resistance is reflected in the torque-twist curve (Fig. 7.6), which at

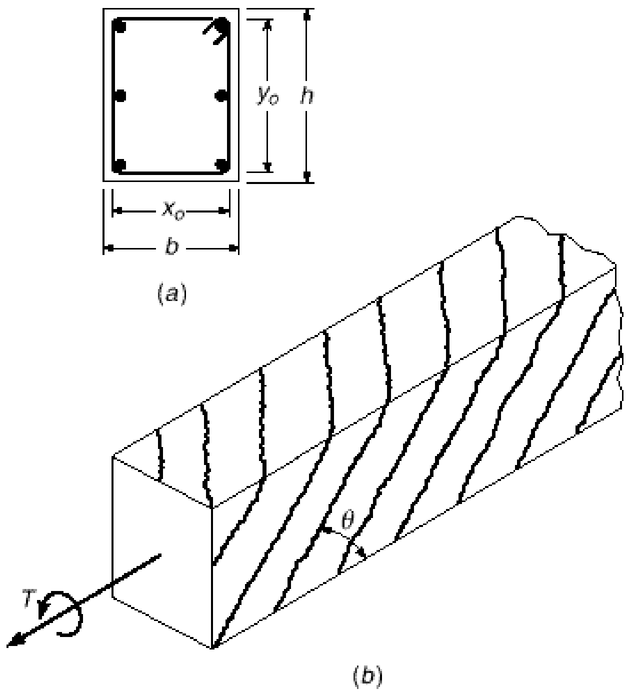


FIGURE 7.5
Reinforced concrete beam in torsion: (*a*) torsional reinforcement; (*b*) torsional cracks.

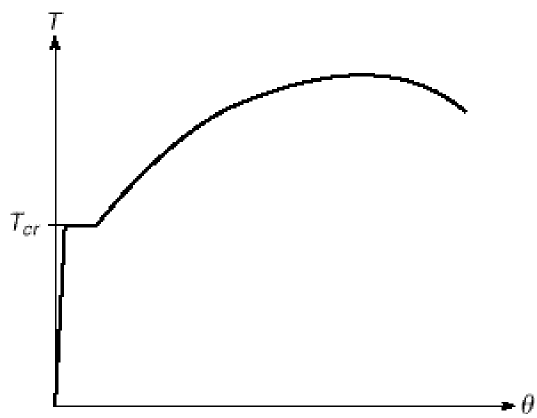


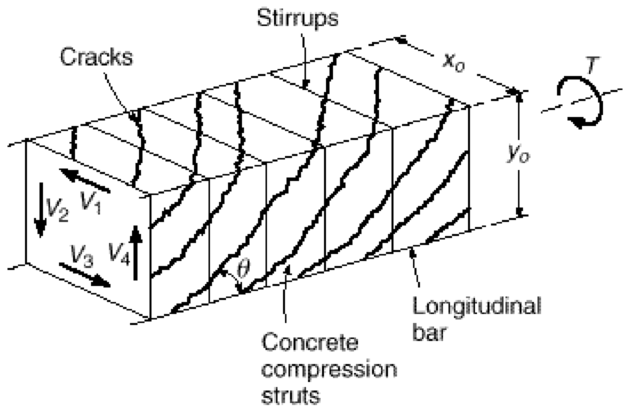
FIGURE 7.6
Torque-twist curve in reinforced concrete member.

the cracking torque shows continued twist at constant torque until the internal forces have been redistributed from the concrete to the steel. As the section approaches the ultimate load, the concrete outside the reinforcing cage cracks and begins to spall off, contributing progressively less to the torsional capacity of the member.

Tests show that, after cracking, the area enclosed by the shear path is defined by the dimensions x_o and y_o measured to the centerline of the outermost closed transverse reinforcement, rather than to the center of the tube walls as before. These dimensions define the gross area $A_{oh} = x_o y_o$ and the shear perimeter $p_h = 2(x_o + y_o)$ measured at the steel centerline.

Analysis of the torsional resistance of the member is aided by treating the member as a space truss consisting of spiral *concrete diagonals* that are able to take load parallel but not perpendicular to the torsional cracks, transverse *tension tie members*

FIGURE 7.7
Space truss analogy.



that are provided by closed stirrups or ties, and *tension chords* that are provided by longitudinal reinforcement. The hollow-tube, space truss analogy represents a simplification of actual behavior, since, as will be demonstrated, the calculated torsional strength is controlled by the strength of the transverse reinforcement, independent of concrete strength. Such a simplification will be used here because it aids understanding, although it greatly underestimates torsional capacity and does not reflect the higher torsional capacities obtained with higher concrete strengths (Refs. 7.6 and 7.7).

With reference to Fig. 7.7, the torsional resistance provided by a member with a rectangular cross section can be represented as the sum of the contributions of the shears in each of the four walls of the equivalent hollow tube. The contribution of the shear acting in the right-hand vertical wall of the tube to the torsional resistance, for example, is

$$T_4 = \frac{V_4 x_o}{2} \qquad (a)$$

Following a procedure similar to that used for analyzing the variable angle truss shear model discussed in Section 4.8 and shown in Figs. 4.19 and 4.20, the equilibrium of a section of the vertical wall—with one edge parallel to a torsional crack with angle · —can be evaluated using Fig. 7.8*a*. Assuming that the stirrups crossing the crack are yielding, the shear in the wall under consideration is

$$V_4 = A_t f_{yv} n \qquad (b)$$

where A_t = area of one leg of a closed stirrup
f_{yv} = yield strength of transverse reinforcement
n = number of stirrups intercepted by torsional crack

Since the horizontal projection of the crack is y_o cot · and $n = y_o$ cot · · s where · is the slope angle of the strut and s is the spacing of the stirrups,

$$V_4 = \frac{A_t f_{yv} y_o}{s} \cot \cdot \qquad (c)$$

Combining Eqs. (*c*) and (*a*) gives

$$T_4 = \frac{A_t f_{yv} y_o x_o}{2s} \cot \cdot \qquad (d)$$

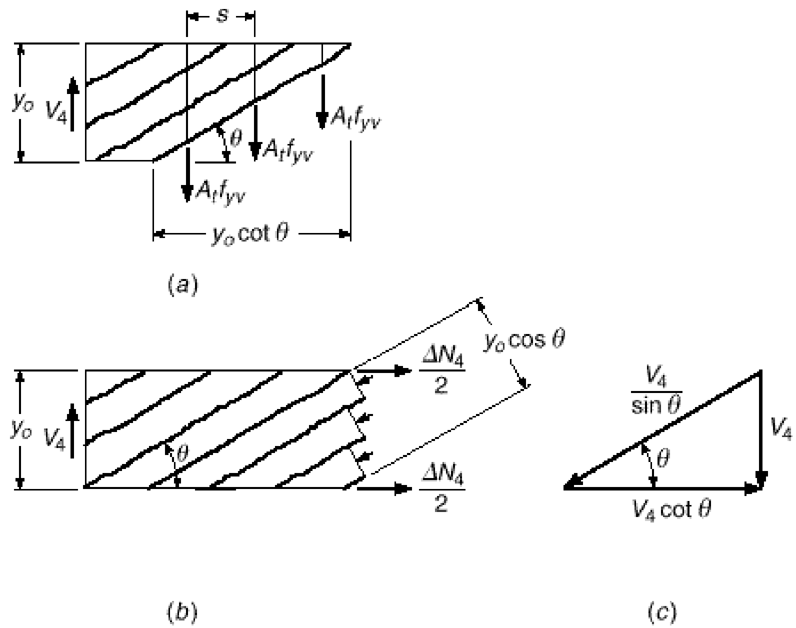


FIGURE 7.8
Basis for torsional design: (*a*) vertical tension in stirrups; (*b*) diagonal compression in vertical wall of beam; (*c*) equilibrium diagram of forces due to shear in vertical wall.

It is easily shown that an identical expression is obtained for each horizontal and vertical wall. Thus, summing over all four sides, the nominal capacity of the section is

$$T_n = \sum_{i=1}^{4} T_i = \frac{2A_t f_{yv} y_o x_o}{s} \cot\theta \qquad (e)$$

Noting that $y_o x_o = A_{oh}$, and rearranging slightly gives

$$T_n = \frac{2A_{oh} A_t f_{yv}}{s} \cot\theta \qquad (7.4)$$

The diagonal compression struts that form parallel to the torsional cracks are necessary for the equilibrium of the cross section. As shown in Fig. 7.8*b* and *c*, the horizontal component of compression in the struts in the vertical wall must be equilibrated by an axial tensile force ΔN_4. Based on the assumed uniform distribution of shear flow around the perimeter of the member, the diagonal stresses in the struts must be uniformly distributed, resulting in a line of action of the resultant axial force that coincides with the midheight of the wall. Referring to Fig. 7.8*c*, the total contribution of the right-hand vertical wall to the change in axial force of the member due to the presence of torsion is

$$\Delta N_4 = V_4 \cot\theta = \frac{A_t f_{yv} y_o}{s} \cot^2\theta$$

Again, summing over all four sides, the total increase in axial force for the member is

$$\Delta N = \sum_{i=1}^{4} \Delta N_i = \frac{A_t f_{yv}}{s} 2(x_o + y_o) \cot^2\theta \qquad (7.5a)$$

$$\Delta N = \frac{A_t f_{yv} p_h}{s} \cot^2\theta \qquad (7.5b)$$

where p_h is the perimeter of the centerline of the closed stirrups.

Longitudinal reinforcement must be provided to carry the added axial force ΔN. If that steel is designed to yield, then

$$A_l f_{yl} = \frac{A_t f_{yv} p_h}{s} \cot^2 \theta \qquad (7.6)$$

and

$$A_l = \frac{A_t}{s} p_h \frac{f_{yv}}{f_{yl}} \cot^2 \theta \qquad (7.7)$$

where A_l = total area of longitudinal reinforcement to resist torsion, in^2
f_{yl} = yield strength of longitudinal torsional reinforcement, psi

It has been found experimentally that, after cracking, the effective area enclosed by the shear flow path is somewhat less than the value of A_{oh} used in the previous development. It is recommended in Ref. 7.7 that the reduced value be taken as $A_o = 0.85A_{oh}$, where, it will be recalled, A_{oh} is the area enclosed by the centerline of the transverse reinforcement. This recommendation is incorporated in the ACI Code (see Section 7.5) and in a modified form of Eq. (7.4) with A_o substituted for A_{oh}. It has further been found experimentally that the thickness of the equivalent tube at loads near ultimate is closely approximated by $t = A_{oh}/p_h$, where p_h is the perimeter of A_{oh}.

7.4 Torsion Plus Shear

Members are rarely subjected to torsion alone. The prevalent situation is that of a beam subject to the usual flexural moments and shear forces, which, in addition, must also resist torsional moments. In an uncracked member, shear forces as well as torque produce shear stresses. In a cracked member, both shear and torsion increase the forces in the diagonal struts (Figs. 4.20*d* and 7.8*b*), they increase the width of diagonal cracks, and they increase the forces required in the transverse reinforcement (Figs. 4.20*e* and 7.8*a*).

Using the usual representation for reinforced concrete, the nominal shear stress caused by an applied shear force V is $\tau_v = V/b_w d$. The shear stress caused by torsion, given in Eq. (7.1) is $\tau_t = T/(2A_o t)$. As shown in Fig 7.9*a* for *hollow sections*, these stresses are directly additive on one side of the member. Thus, for a cracked concrete cross section with $A_o = 0.85A_{oh}$ and $t = A_{oh}/p_h$, the maximum shear stress can be expressed as

$$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7A_{oh}^2} \qquad (7.8)$$

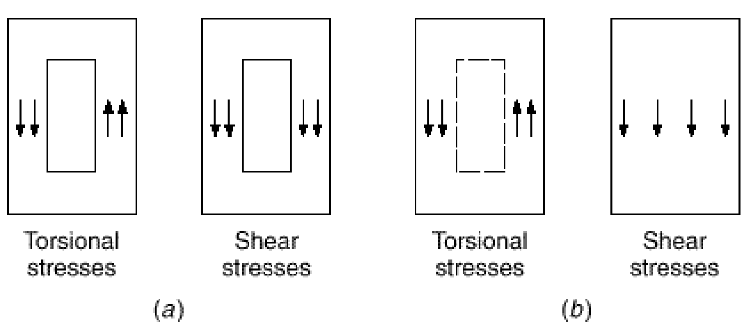


FIGURE 7.9
Addition of torsional and shear stresses: (*a*) hollow section; (*b*) solid section. (*Adapted from Ref. 7.7.*)

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For a member with a *solid section*, Fig. 7.9*b*, τ_t is predominately distributed around the perimeter, as represented by the hollow tube analogy, but the full cross section contributes to carrying τ_v. Comparisons with experimental results show that Eq. (7.8) is somewhat overconservative for solid sections and that a better representation for maximum shear stress is provided by the square root of the sum of the squares of the nominal shear stresses:

$$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{Tp_h}{1.7A_{oh}^2}\right)^2} \tag{7.9}$$

Equations (7.8) and (7.9) serve as a measure of the shear stresses in the concrete under both service and ultimate loading.

7.5 ACI Code Provisions for Torsion Design

The basic principles upon which ACI Code design provisions are based have been presented in the preceding sections. ACI Code 11.6.3.5 safety provisions require that

$$T_u \le \phi T_n \tag{7.10}$$

where T_u = required torsional strength at factored loads
T_n = nominal torsional strength of member

The strength reduction factor $\phi = 0.75$ applies for torsion. T_n is based on Eq. (7.4) with A_o substituted for A_{oh}, thus

$$T_n = \frac{2A_oA_tf_{yv}}{s}\cot\theta \tag{7.11}$$

In accordance with ACI Code 11.6.2, sections located less than a distance d from the face of a support may be designed for the same torsional moment T_u as that computed at a distance d, recognizing the beneficial effects of support compression. However, if a concentrated torque is applied within this distance, the critical section must be taken at the face of the support. These provisions parallel those used in shear design. For beams supporting slabs such as are shown in Fig. 7.1, the torsional loading from the slab may be treated as being uniformly distributed along the beam.

a. T Beams and Box Sections

For T beams, a portion of the overhanging flange contributes to the cracking torsional capacity and, if reinforced with closed stirrups, to the torsional strength. According to ACI Code 11.6.1, the contributing width of the overhanging flange on either side of the web is equal to the smaller of (1) the projection of the beam above or below the slab, whichever is greater, and (2) four times the slab thickness. These criteria are the same as those used for two-way slabs with beams, illustrated in Fig. 13.10. As with solid sections, A_{cp} for box sections, with or without flanges, represents the area enclosed by the outside perimeter of the concrete section.

After torsional cracking, the applied torque is resisted by the portion of the section represented by A_{oh}, the area enclosed by the centerline of the outermost closed transverse torsional reinforcement. A_{oh} for rectangular, box, and T sections is illus-

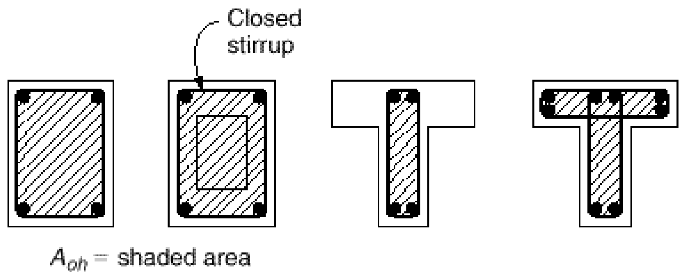

FIGURE 7.10
Definition of A_{oh}. *(Adapted from Ref. 7.7.)*

trated in Fig. 7.10. For sections with flanges, the Code does not require that the section used to establish A_{cp} coincide with that used to establish A_{oh}.

b. Minimal Torsion

If the factored torsional moment T_u does not exceed $\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})$, torsional effects may be neglected, according to ACI Code 11.6.1. This lower limit is 25 percent of the cracking torque, given by Eq. (7.3), reduced by the factor ϕ, as usual, for design purposes. The presence of torsional moment at or below this limit will have a negligible effect on the flexural and shear strength of the member.

For members subjected to an axial load N_u (positive in compression), torsional effects may be neglected when T_u does not exceed $\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$. For hollow sections (with or without axial load), A_{cp} must be replaced by the gross area of the concrete A_g to determine if torsional effects may be neglected. This has the effect of multiplying 25 percent of the cracking torque by the ratio A_g/A_{cp} twice—once to account for the reduction in cracking torque for hollow sections from the value shown in Eq. (7.3) and a second time to account for the transition from the circular interaction of combined shear and torsion stresses in Eq. (7.9) to the linear interaction represented by Eq. (7.8).

c. Equilibrium versus Compatibility Torsion

A distinction is made in the ACI Code between equilibrium (primary) torsion and compatibility (secondary) torsion. For the first condition, described earlier with reference to Fig. 7.1*a*, the supporting member *must* be designed to provide the torsional resistance required by static equilibrium. For secondary torsion resulting from compatibility requirements, shown in Fig. 7.1*b*, it is assumed that cracking will result in a redistribution of internal forces; and according to ACI Code 11.6.2, the maximum torsional moment T_u may be reduced to $4\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})$, or $4\phi\sqrt{f'_c}\,(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$ for members subjected to axial load. In the case of hollow sections, A_{cp} is *not* replaced by A_g.The design moments and shears in the supported member must be adjusted accordingly. The reduced value of T_u permitted by the ACI Code is intended to approximate the torsional cracking strength of the supporting beam, for combined torsional and flexural loading. The large rotations that occur at essentially constant torsional load would result in significant redistribution of internal forces, justifying use of the reduced value for design of the torsional member and the supported elements.

d. Limitations on Shear Stress

Based largely on empirical observations, the width of diagonal cracks caused by combined shear and torsion under *service loads* can be limited by limiting the calculated shear stress under *factored shear and torsion* (Ref. 7.4) so that

$$v_{max} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \tag{7.12}$$

v_{max} in Eq. (7.12) corresponds to the upper limits on shear capacity described in Section 4.5d. Combining Eq. (7.12) with Eq. (7.8) provides limits on the cross-sectional dimensions of *hollow sections*, in accordance with ACI Code 11.6.3.

$$\frac{V_u}{b_w d} + \frac{T_u p_h}{1.7A_{oh}^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \tag{7.13}$$

Likewise, for *solid sections*, combining Eq. (7.12) with Eq. (7.9) gives

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right) \tag{7.14}$$

Either member dimensions or concrete strength must be increased if the criteria in Eq. (7.13) or (7.14) are not satisfied.

ACI Code 11.6.3 requires that, if the wall thickness varies around the perimeter of a hollow section, Eq. (7.13) must be evaluated at the location where the left-hand side of the expression is a maximum. If the wall thickness is less than the assumed value of t used in the development of Eq. (7.8) A_{oh}/p_h, the actual value of t must be used in the calculation of torsional shear stress. As a result, the second term on the left-hand side of Eq. (7.13) must be taken as

$$\frac{T_u}{1.7A_{oh}t}$$

where t is the thickness of the wall of the hollow section at the location where the stresses are being checked.

e. Reinforcement for Torsion

The nominal torsional strength is given by Eq. (7.11).

$$T_n = \frac{2A_o A_t f_{yv}}{s}\cot\theta \tag{7.11}$$

According to ACI Code 11.6.3, the angle θ may assume any value between 30° and 60°, with a value of $\theta = 45°$ suggested. The area enclosed by the shear flow A_o may be determined by analysis using procedures such as suggested in Ref. 7.8, or A_o may be taken as equal to $0.85A_{oh}$. Combining Eq. (7.11) with Eq. (7.10), the required cross-sectional area of one stirrup leg for torsion is

$$A_t = \frac{T_u s}{2\phi A_o f_{yv}\cot\theta} \tag{7.15}$$

The Code limits f_{yv} to a maximum of 60,000 psi for reasons of crack control.

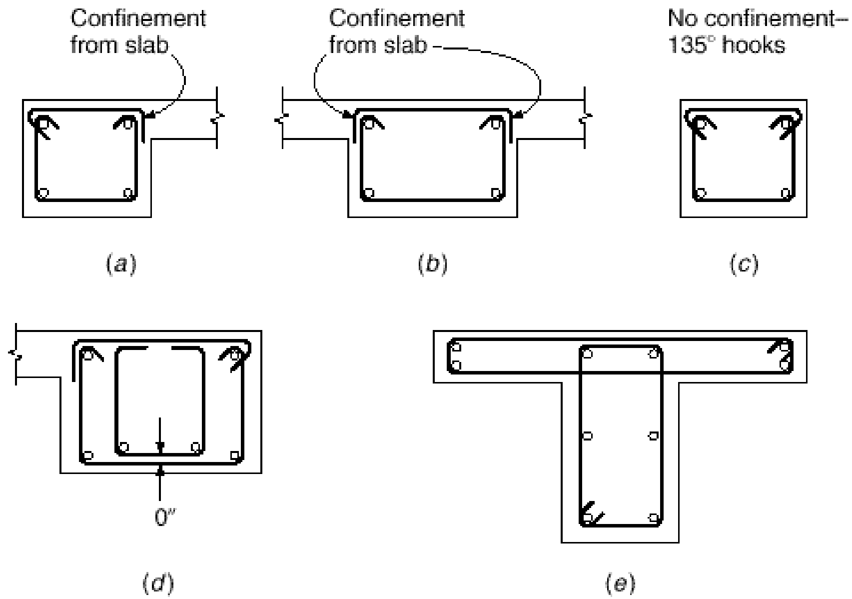

FIGURE 7.11
Stirrup-ties and longitudinal reinforcement for torsion: (*a*) spandrel beam with flanges on one side; (*b*) interior beam; (*c*) isolated rectangular beam; (*d*) wide spandrel beam; (*e*) T beam with torsional reinforcement in flanges.

The reinforcement provided for torsion must be combined with that required for shear. Based on the typical two-leg stirrup, this may be expressed as

$$\left(\frac{A_{v+t}}{s}\right) = \frac{A_v}{s} + 2\frac{A_t}{s} \tag{7.16}$$

As described in Section 7.3, the transverse stirrups used for torsional reinforcement must be of a closed form to provide the required tensile capacity across the diagonal cracks of all faces of the beam. U-shaped stirrups commonly used for transverse shear reinforcement are not suitable for torsional reinforcement. On the other hand, one-piece closed stirrups make field assembly of beam reinforcement difficult, and for practical reasons torsional stirrups are generally two-piece stirrup-ties, as shown in Fig. 7.11. A U-shaped stirrup is combined with a horizontal top bar, suitably anchored.

Because concrete outside the reinforcing cage tends to spall off when the member is subjected to high torque, transverse torsional reinforcement must be anchored within the concrete core (Ref. 7.9). ACI Code 11.6.4 requires that stirrups or ties used for transverse longitudinal reinforcement must be anchored with a 135° standard hook around a longitudinal bar, unless the concrete surrounding the anchorage is restrained against spalling by a flange or a slab, in which case 90° standard hooks may be used, as shown in Fig. 7.11*a, b*, and *d*. Overlapping U-shaped stirrups, such as shown in Fig. 5.12*d*, may not be used. If flanges are included in the computation of torsional strength for T or L-shaped beams, closed torsional stirrups must be provided in the flanges, as shown in Fig. 7.11*e*.

The required spacing of closed stirrups, satisfying Eq. (7.16), is selected for the trial design based on standard bar sizes.

To control spiral cracking, the maximum spacing of torsional stirrups should not exceed $p_h/8$ or 12 in., whichever is smaller. In addition, for members requiring both shear and torsion reinforcement, the minimum area of closed stirrups is equal to

$$(A_v + 2A_t) = 0.75\sqrt{f'_c}\,\frac{b_w s}{f_{yv}} \geq 50\,\frac{b_w s}{f_{yv}} \tag{7.17}$$

according to ACI Code 11.6.5.

The area of longitudinal bar reinforcement A_l required to resist torsion is given by Eq. (7.7), where θ must have the same value used to calculate A_t. The term A_t/s in Eq. (7.7) should be taken as the value calculated using Eq. (7.15), not modified based on minimum transverse steel requirements. ACI Code 11.6.3 permits the portion of A_l in the flexural compression zone to be reduced by an amount equal to $M_u/(0.9df_{yl})$, where M_u is the factored moment acting at the section in combination with T_u.

Based on an evaluation of the performance of reinforced concrete beam torsional test specimens, ACI Code 11.6.5 requires that A_l not be less than

$$A_{l,min} = \frac{5\sqrt{f'_c}\,A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}} \qquad (7.18)$$

where $A_t/s \geq 25b_w/f_{yv}$, with f_{yv} in psi.

The spacing of the longitudinal bars should not exceed 12 in., and they should be distributed around the perimeter of the cross section to control cracking. The bars may not be less than No. 3 (No. 10) in size nor have a diameter less than 0.042 times the spacing of the transverse stirrups. At least one longitudinal bar must be placed at each corner of the stirrups. Careful attention must be paid to the anchorage of longitudinal torsional reinforcement so that it is able to develop its yield strength at the face of the supporting columns, where torsional moments are often maximum.

Reinforcement required for torsion may be combined with that required for other forces, provided that the area furnished is the sum of the individually required areas and that the most restrictive requirements of spacing and placement are met. According to ACI Code 11.6.6, torsional reinforcement must be provided at least a distance $b_t + d$ beyond the point theoretically required, where b_t is the width of that part of the cross section containing the closed stirrups resisting torsion. According to the provisions of the ACI Code, the point at which the torsional reinforcement is no longer required is the point at which $T_u < \phi\sqrt{f'_c}(A_{cp}^2/p_{cp})$, or $T_u < \phi\sqrt{f'_c}(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$ for members subjected to axial load. The value is 25 percent of the cracking torque, reduced by the factor ϕ, as given in Section 7.5b.

The subject of torsional design of prestressed concrete is not treated here, but, as presented in ACI Code 11.6, it differs only in certain details from the above presentation for nonprestressed reinforced concrete beams.

f. Lightweight Concrete

As discussed in Section 4.5a, the ACI Code recognizes that lightweight concrete possesses lower tensile strength than normal-weight concrete of the same compressive strength. The provisions in ACI Code 11.2 apply the same criteria to members loaded in torsion as to members loaded in shear: $f_{ct}/6.7$ is substituted for $\sqrt{f'_c}$ in all applicable equations, with the additional restriction that $f_{ct}/6.7$ shall not exceed $\sqrt{f'_c}$. If the split-cylinder strength f_{ct} is not available, $\sqrt{f'_c}$ must be multiplied by 0.75 for all-lightweight concrete and by 0.85 for sand-lightweight concrete.

g. Design for Torsion

Designing a reinforced concrete flexural member for torsion involves a series of steps. The following sequence ensures that each is covered:

1. Determine if the factored torque is less than $\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})$, or $\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$ for members subjected to axial load. If so, torsion may be

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neglected. If not, proceed with the design. Note that in this step, portions of overhanging flanges, as defined in Section 7.5a, must be included in the calculation of A_{cp} and p_{cp}.

2. If the torsion is compatibility torsion, rather than equilibrium torsion, as described in Sections 7.1 and 7.5c, the maximum factored torque may be reduced to $4\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})$, or $4\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$ for members subjected to axial load, with the moments and shears in the supported members adjusted accordingly. Equilibrium torsion cannot be adjusted.
3. Check the shear stresses in the section under combined torsion and shear using the criteria of Section 7.5d.
4. Calculate the required transverse reinforcement for torsion using Eq. (7.15) and shear using Eq. (4.14*a*). Combine A_t and A_v using Eq. (7.16).
5. Check that the minimum transverse reinforcement requirements are met for both torsion and shear. These include the maximum spacing, as described in Sections 7.5e and 4.5d, and minimum area, as given in Eq. (7.17).
6. Calculate the required longitudinal torsional reinforcement A_l using the larger of the values given in Eqs. (7.7) and (7.18), and satisfy the spacing and bar size requirements given in Section 7.5e. The portion of A_l in the flexural compression zone may be reduced by $M_u/(0.9df_{yl})$, providing that Eq. (7.18) and the spacing and bar size requirements are satisfied.
7. Continue torsional reinforcement $b_t + d$ past the point where T_u is less than $\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})$, or $\phi\sqrt{f'_c}(A_{cp}^2/p_{cp})\sqrt{1 + N_u/(4A_g\sqrt{f'_c})}$ for members subjected to axial load.

EXAMPLE 7.1

Design for torsion with shear. The 28 ft span beam shown in Fig. 7.12*a* and *b* carries a monolithic slab cantilevering 6 ft past the beam centerline. The resulting L beam supports a live load of 900 lb/ft along the beam centerline plus 50 psf uniformly distributed over the upper slab surface. The effective depth to the flexural steel centroid is 21.5 in., and the distance from the beam surfaces to the centroid of stirrup steel is $1\frac{3}{4}$ in. Material strengths are $f'_c = 5000$ psi and $f_y = 60{,}000$ psi. Design the torsional and shear reinforcement for the beam.

Solution. Applying ACI load factors gives the slab load as

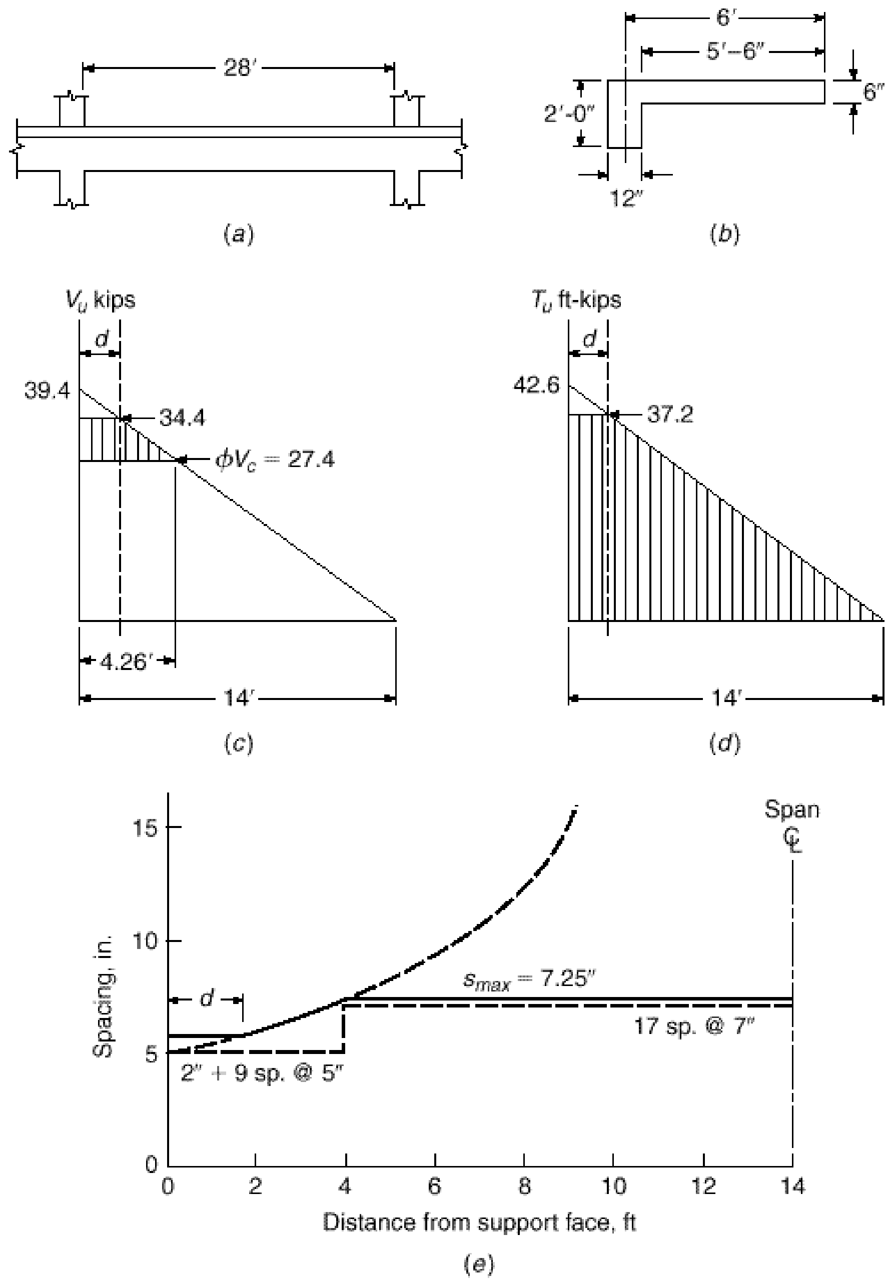
$$1.2w_d = 1.2 \times 75 \times 5.5 = 495 \text{ lb/ft}$$
$$1.6w_l = 1.6 \times 50 \times 5.5 = \underline{440 \text{ lb/ft}}$$
$$\text{Total} = 935 \text{ lb/ft at 3.25 ft eccentricity}$$

while the beam carries directly

$$1.2w_d = 1.2 \times 300 = 360 \text{ lb/ft}$$
$$1.6w_l = 1.6(900 + 50) = \underline{1520 \text{ lb/ft}}$$
$$\text{Total} = 1880 \text{ lb/ft}$$

Thus, the uniformly distributed load on the beam is 2815 lb/ft, acting together with a uniformly distributed torque of $935 \times 3.25 = 3040$ ft-lb/ft. At the face of the column, the design shear force is $V_u = 2.815 \times 28/2 = 39.4$ kips. At the same location, the design torsional moment is $T_u = 3.040 \times 28/2 = 42.6$ ft-kips.

FIGURE 7.12
Shear and torsion design example.



The variation of V_u and T_u with distance from the face of the supporting column is given by Fig. 7.12*c* and *d*, respectively. The values of V_u and T_u at the critical design section, a distance *d* from the column face, are

$$V_u = 39.4 \times \frac{12.21}{14} = 34.4 \text{ kips}$$

$$T_u = 42.6 \times \frac{12.21}{14} = 37.2 \text{ ft-kips}$$

For the effective beam, $A_{cp} = 12 \times 24 + 6 \times 18 = 396$ in² and $p_{cp} = 2 \times 24 + 2 \times 30 = 108$ in. According to the ACI Code, torsion may be neglected if $T_u \leq 0.75 \times \cdot \sqrt{5000} \cdot 396^2 \cdot 108 \cdot \cdot 12{,}000 = 6.4$ ft-kips. Torsion must clearly be considered in the present case. Since the torsional resistance of the beam is required for equilibrium, no reduction in T_u may be made.

Before designing the torsional reinforcement, the section will be checked for adequacy in accordance with Eq. (7.14). Although A_{cp} was calculated considering the flange to check if torsion could be neglected (as required by ACI Code 11.6.1), subsequent calculations for serviceability and strength will neglect the flange and no torsional reinforcement will be provided in the flange. For reference, $b_w d = 12 \times 21.5 = 258$ in². With $1\frac{3}{4}$ in. cover to the center of the stirrup bars from all faces, $x_o = 12 - 3.5 = 8.5$ in. and $y_o = 24.0 - 3.5 = 20.5$ in. Thus, $A_{oh} = 8.5 \times 20.5 = 174$ in², $A_o = 0.85 \times 174 = 148$ in², and $p_h = 2(8.5 + 20.5) = 58$ in. Using Eq. (7.14),

$$\sqrt{\left(\frac{34.4}{258}\right)^2 + \left(\frac{37.2 \times 12 \times 58}{1.7 \times 174^2}\right)^2} \leq \frac{0.75}{1000}\left(2\sqrt{5000} + 8\sqrt{5000}\right)$$

$$0.520 \text{ ksi} \leq 0.530 \text{ ksi}$$

Therefore, the cross section is of adequate size for the given concrete strength.

The values of A_t and A_v will now be calculated at the column face (for reference only). Using Eq. (7.15) and choosing $\theta = 45°$,

$$A_t = \frac{T_u s}{2\phi A_o f_{yv} \cot \theta}$$

$$= \frac{42.6 \times 12s}{2 \times 0.75 \times 148 \times 60 \times 1} = 0.0384s$$

for one leg of a closed vertical stirrup, or $0.0768s$ for two legs.

The shear capacity of the concrete alone, obtained using Eq. (4.12b), is

$$\phi V_c = 0.75 \times 2\sqrt{f'_c}\, b_w d$$

$$= \frac{0.75 \times 2\sqrt{5000} \times 258}{1000} = 27.4 \text{ kips}$$

From Eq. (4.14a), the web reinforcement for transverse shear, again computed at the column face, is

$$A_v = \frac{(V_u - \phi V_c)s}{\phi f_{yv} d} = \frac{(39.4 - 27.4)s}{0.75 \times 60 \times 21.5} = 0.0124s$$

to be provided in two vertical legs.

The calculated value of A_t will decrease linearly to zero at the midspan, and the calculated value of A_v will decrease linearly to zero 4.26 ft from the face of the support, the point at which $V_u = \phi V_c$. Thus, the total area to be provided by the two vertical legs is

$$2A_t + A_v = 0.0768s\left(1 - \frac{x}{14}\right) + 0.0124s\left(1 - \frac{x}{4.26}\right)$$

for $0 \leq x \leq 4.26$ ft., where x is the distance from the face of the support, and

$$2A_t + A_v = 0.0768s\left(1 - \frac{x}{14}\right)$$

for $4.26 \leq x \leq 14$ ft.

Number 4 (No. 13) closed stirrups will provide a total area in the two legs of 0.40 in^2. For $2A_t + A_v = 0.40$ in^2, the required spacing at d and at 2 ft intervals along the span can be found using the given relationships between stirrup area and spacing:

$$\begin{aligned} s_d &= 5.39 \text{ in.} \\ s_2 &= 5.52 \text{ in.} \\ s_4 &= 7.19 \text{ in.} \\ s_6 &= 9.11 \text{ in.} \\ s_8 &= 12.2 \text{ in.} \\ s_{10} &= 18.2 \text{ in.} \end{aligned}$$

These values of s are plotted in Fig. 7.12*e*. ACI provisions for maximum spacing should now be checked. For torsion reinforcement, the maximum spacing is the smaller of

$$\frac{p_h}{8} = \frac{58}{8} = 7.25 \text{ in.}$$

or 12 in., whereas for shear reinforcement, the maximum spacing is $d \cdot 2 = 10.75$ in. ≤ 24 in. The most restrictive provision is the first, and the maximum spacing of 7.25 in. is plotted in Fig. 7.12*e*. Stirrups between the face of the support and the distance d can be spaced at s_d. The resulting spacing requirements are shown by the solid line in the figure. These requirements are met in a practical way by No. 4 (No. 13) closed stirrups, the first placed 2 in. from the face of the column, followed by 9 at 5 in. spacing and 17 at 7 in. spacing. According to the ACI Code, stirrups may be discontinued at the point where $V_u < \cdot\, V_c \cdot 2$ (4.9 ft from the span centerline) or $(b_t + d) = 2.8$ ft past the point at which $T_u < \cdot\ \cdot\ \overline{f_c'} \cdot A_{cp}^2 \cdot p_{cp} \cdot$. The latter point is past the centerline of the member; therefore, minimum stirrups are required throughout the span. The minimum web steel provided, 0.40 in^2, satisfies the ACI Code minimum $= 0.75 \cdot \overline{f_c'}\, b_w s \cdot f_{yv} = 0.75 \cdot \overline{5000}\ 12 \times 7 \cdot 60{,}000 = 0.074$ in^2 $\geq 50 b_w s \cdot f_{yv} = 50 \times 12 \times 7 \cdot 60{,}000 = 0.070$ in^2.

The longitudinal steel required for torsion at a distance d from the column face is computed next. At that location

$$\frac{A_t}{s} = 0.0384 \cdot\ 1 - \frac{1.79}{14} \cdot\ = 0.0335$$

and from Eq. (7.7)

$$A_l = 0.0335 \times 58 \times \frac{60}{60} \times 1^2 = 1.94 \text{ in}^2$$

with a total not less than given by Eq. (7.18), in which $A_t \cdot s$ is not to be taken less than $25 \times 12 \cdot 60{,}000 = 0.005$.

$$A_{l,min} = \frac{5 \cdot\ \overline{5000} \times 396}{60 \times 1000} - 0.0335 \times 58 \times \frac{60}{60} = 0.39 \text{ in}^2$$

According to the ACI Code, the spacing must not exceed 12 in., and the bars may not be less than No. 3 (No. 10) in size nor have a diameter less than $0.042s = 0.29$ in. Reinforcement will be placed at the top, middepth, and bottom of the member—each level to provide not less than $1.94 \cdot 3 = 0.65$ in^2. Two No. 6 (No. 19) bars will be used at middepth, and reinforcement to be placed for flexure will be increased by 0.65 in^2 at the top and bottom of the member.

Although A_l reduces in direct proportion to A_t and, hence, decreases linearly starting at d from the face of the column to the midspan, for simplicity of construction the added bars and the increment in the flexural steel will be maintained throughout the length of the member. Although ACI Code 11.6.3 states that A_l may be decreased in flexural compression zones by an amount equal to $M_u \cdot (0.9 d f_{yl})$, that reduction will not be made here. Adequate embedment must be provided past the face of the column to fully develop f_{yl} in the bars at that location.

REFERENCES

7.1. P. Lampert and B. Thurlimann, "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending," *Int. Assoc. Bridge and Struct. Eng.* Publ. 31-I, Zurich, 1971, pp. 107–131.
7.2. B. Thurlimann, "Torsional Strength of Reinforced and Prestressed Beams—CEB Approach," in *Concrete Design: U.S. and European Practices*, SP-59, 1979, American Concrete Institute, Detroit, MI, pp. 117–143.
7.3. *CEB-FIP Model Code 1990*, Thomas Telford, London, 1991.
7.4. CSA Committee A23.3, *Design of Concrete Structures*, Canadian Standards Association, Etobicoke, Ontario, 1994, p. 199.
7.5. T. T. C. Hsu, "Torsion of Structural Concrete—Behavior of Reinforced Concrete Rectangular Members," in *Torsion of Structural Concrete*, SP-18, 1968, American Concrete Institute, Detroit, MI, pp. 261–306.
7.6. A. H. Mattock, Disc. of "Design of Torsion" by J. G. MacGregor and M. G. Ghoneim (Ref 7.7), *ACI Struct. J.*, vol. 93, no. 1, 1996, pp. 142–143.
7.7. J. G. MacGregor and M. G. Ghoneim, "Design of Torsion," *ACI Struct. J.*, vol. 92, no. 2, 1995, pp. 218–221.
7.8. T. T. C. Hsu, "Shear Flow Zone in Torsion of Reinforced Concrete," *J. Struct. Eng.*, vol. 116, no. 11, 1990, pp. 3206–3226.
7.9. D. Mitchell and M. P. Collins, "Detailing for Torsion," *J. ACI*, vol. 73, no. 9, 1976, pp. 506–511.

PROBLEMS

7.1. A beam of rectangular cross section having $b = 22$ in. and $h = 15$ in. is to carry a total factored load of 3500 lb/ft uniformly distributed over its 26 ft span, and in addition the beam will be subjected to a uniformly distributed torsion of 1750 ft-lb/ft at factored loads. Closed stirrup-ties will be used to provide for flexural shear and torsion, placed with the stirrup steel centroid 1.75 in. from each concrete face. The corresponding flexural effective depth will be approximately 12.5 in. Design the transverse reinforcement for this beam and calculate the increment of longitudinal steel area needed to provide for torsion using $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

7.2. Architectural and clearance requirements call for the use of a transfer girder, shown in Fig. P7.2, spanning 20 ft between supporting column faces. The girder must carry from above a concentrated column load of 17.5 kips at midspan, applied with eccentricity 2 ft from the girder centerline. (Load factors are already included, as is an allowance for girder self-weight.) The mem-

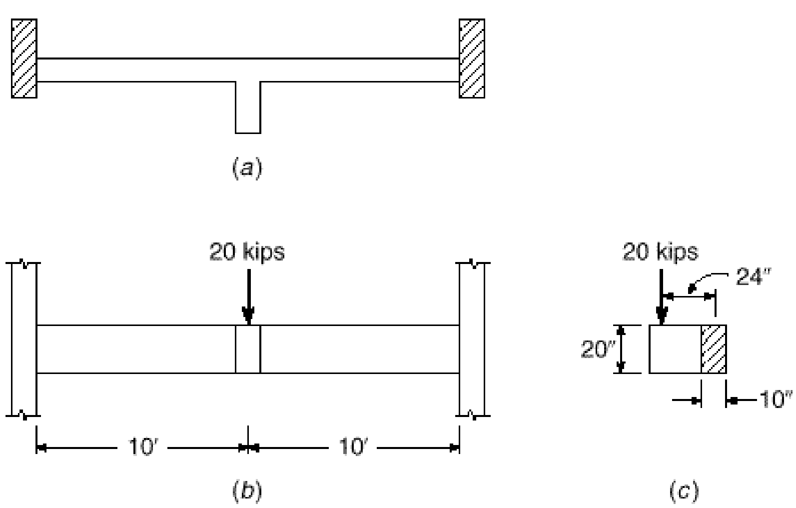
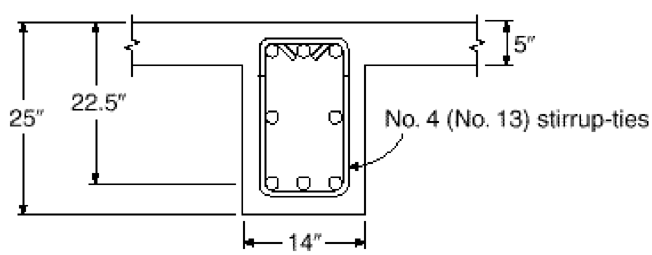


FIGURE P7.2
Transfer girder: (*a*) top view; (*b*) front view; (*c*) side view.

ber is to have dimensions $b = 10$ in., $h = 20$ in., $x_o = 6.5$ in., $y_o = 16.5$ in., and $d = 17$ in. Supporting columns provide full torsional rigidity; flexural rigidity at the ends of the span can be assumed to develop 40 percent of the maximum moment that would be obtained if the girder were simply supported. Design both transverse and longitudinal steel for the beam. Material strengths are $f'_c = 5000$ psi and $f_y = 60{,}000$ psi.

7.3. The beam shown in cross section in Fig. P7.3 is a typical interior member of a continuous building frame, with span 30 ft between support faces. At factored loads, it will carry a uniformly distributed vertical load of 3100 lb/ft, acting simultaneously with a uniformly distributed torsion of 2600 ft-lb/ft. Transverse reinforcement for shear and torsion will consist of No. 4 (No. 13) stirrup-ties, as shown, with 1.5 in. clear to all concrete faces. The effective depth to flexural steel is taken equal to 22.5 in. for both negative and positive bending regions. Design the transverse reinforcement for shear and torsion, and calculate the longitudinal steel to be added to the flexural requirements to provide for torsion. Torsional reinforcement will be provided only in the web, not in the flanges. Material strengths are $f'_c = 4000$ psi and $f_y = 60{,}000$ psi.

FIGURE P7.3



7.4. The single-span T beam bridge described in Problem 3.14 is reinforced for flexure with four No. 10 (No. 32) bars in two layers, which continue uninterrupted into the supports, permitting a service live load of 1.50 kips/ft to be carried, in addition to the dead load of 0.93 kip/ft, including self-weight. Assume now that only half that live load acts but that it is applied over only half the width of the member, entirely to the right of the section centerline. Design the transverse reinforcement for shear and torsion, and calculate the modified longitudinal steel needed for this eccentric load condition. Torsional reinforcement can be provided in the slab if needed, as well as in the web. Stirrup-ties will be No. 3 or No. 4 (No. 10 or No. 13) bars, with 1.5 in. clear to all concrete faces. Supports provide no restraint against flexural rotations but do provide full restraint against twist. Show a sketch of your final design, detailing all reinforcement. Material strengths are as given for Problem 3.14.