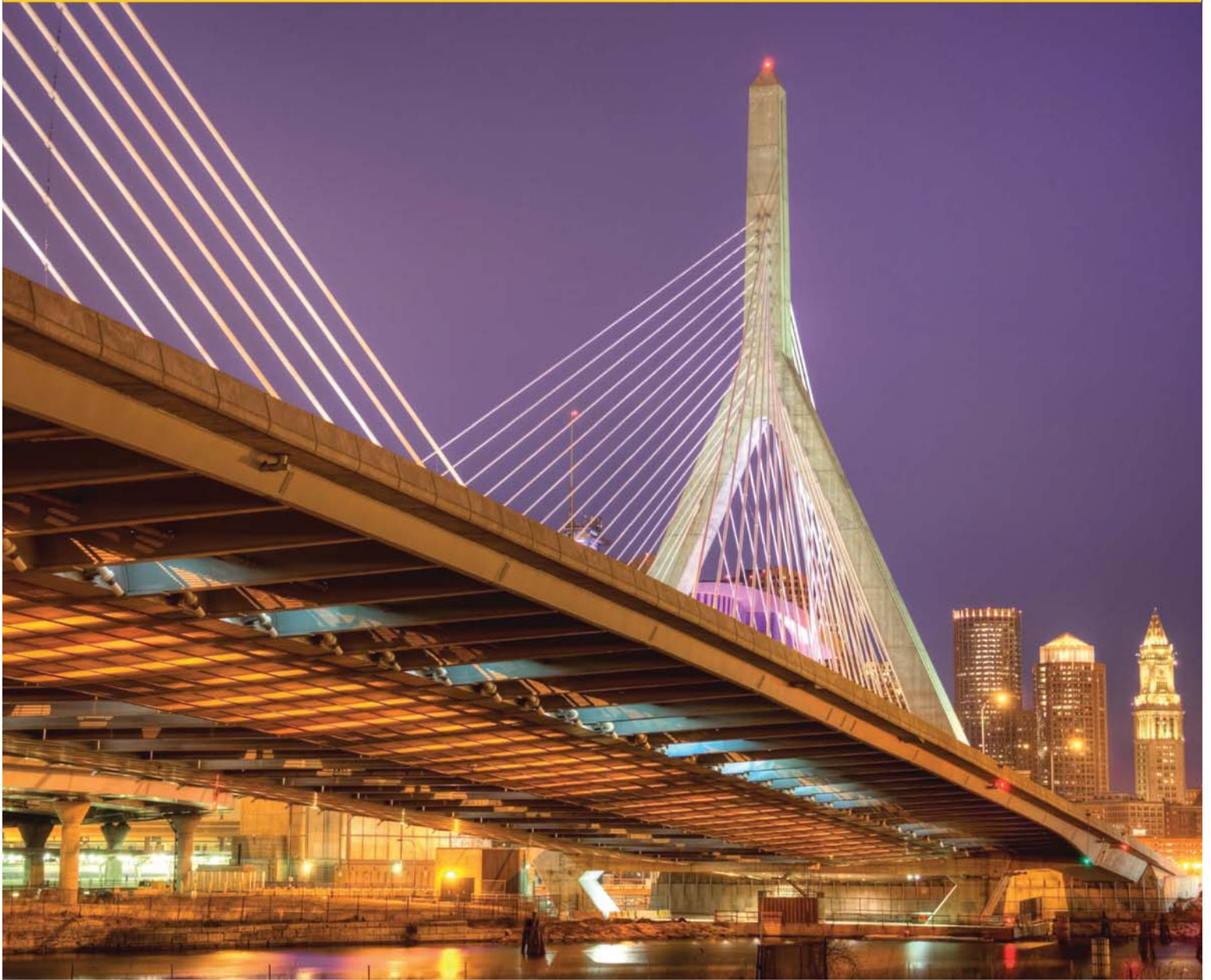


**Tenth Edition**



**ERWIN KREYSZIG**  
**ADVANCED ENGINEERING**  
**MATHEMATICS**

10<sup>TH</sup> EDITION

# ADVANCED ENGINEERING MATHEMATICS

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