

Richard L. Burden
J. Douglas Faires

Numerical Analysis

The background features a complex, abstract geometric design. It consists of several overlapping, semi-transparent spheres and wireframe structures. The central focus is a bright blue, glowing sphere with a swirling, vortex-like pattern inside. This central sphere is surrounded by larger, darker blue and green wireframe spheres. The overall effect is a sense of depth and mathematical complexity, with various geometric shapes like circles and arcs layered on top of each other.

Ninth Edition

Numerical Analysis

NINTH EDITION

Richard L. Burden

Youngstown State University

J. Douglas Faires

Youngstown State University



Australia • Brazil • Japan • Korea • Mexico • Singapore • Spain • United Kingdom • United States

Numerical Analysis,
Ninth Edition
Richard L. Burden and J. Douglas Faires

Editor-in-Chief: *Michelle Julet*

Publisher: *Richard Stratton*

Senior Sponsoring Editor: *Molly Taylor*

Associate Editor: *Daniel Seibert*

Editorial Assistant: *Shaylin Walsh*

Associate Media Editor: *Andrew Coppola*

Senior Marketing Manager: *Jennifer Pursley Jones*

Marketing Coordinator: *Erica O'Connell*

Marketing Communications Manager: *Mary Anne Payumo*

Content Project Manager: *Jill Clark*

Art Director: *Jill Ort*

Senior Manufacturing Buyer: *Diane Gibbons*

Senior Rights Acquisition Specialist: *Katie Huha*

Production Service: *Cadmus Communications*

Text Designer: *Jay Purcell*

Cover Designer: *Wing Ngan*

Cover Image: *Spiral Vortex*

Photographer: *Akira Inoue*

Collection: *Amana images, Gettyimages.com*

Compositor: *Cadmus Communications*

© 2011, 2005, 2001 Brooks/Cole, Cengage Learning

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transmitted, stored, or used in any form or by any means graphic, electronic, or mechanical, including but not limited to photocopying, recording, scanning, digitizing, taping, Web distribution, information networks, or information storage and retrieval systems, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without the prior written permission of the publisher.

For product information and technology assistance, contact us at:

Cengage Learning Customer & Sales Support,
1-800-354-9706

For permission to use material from this text or product,
submit all requests online at

www.cengage.com/permissions.

Further permissions questions can be emailed to
permissionrequest@cengage.com.

Library of Congress Control Number: 2010922639

ISBN-13: 978-0-538-73351-9

ISBN-10: 0-538-73351-9

Brooks/Cole
20 Channel Center Street
Boston, MA 02210
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil and Japan. Locate your local office at **international.cengage.com/region**.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

For your course and learning solutions, visit

www.cengage.com.

Purchase any of our products at your local college store or at our preferred online store **www.cengagebrain.com**.

Printed in Canada

1 2 3 4 5 6 7 14 13 12 11 10

Contents

Preface ix

1 Mathematical Preliminaries and Error Analysis 1

- 1.1 Review of Calculus 2
- 1.2 Round-off Errors and Computer Arithmetic 17
- 1.3 Algorithms and Convergence 32
- 1.4 Numerical Software 41

2 Solutions of Equations in One Variable 47

- 2.1 The Bisection Method 48
- 2.2 Fixed-Point Iteration 56
- 2.3 Newton's Method and Its Extensions 67
- 2.4 Error Analysis for Iterative Methods 79
- 2.5 Accelerating Convergence 86
- 2.6 Zeros of Polynomials and Müller's Method 91
- 2.7 Survey of Methods and Software 101

3 Interpolation and Polynomial Approximation 105

- 3.1 Interpolation and the Lagrange Polynomial 106
- 3.2 Data Approximation and Neville's Method 117
- 3.3 Divided Differences 124
- 3.4 Hermite Interpolation 136
- 3.5 Cubic Spline Interpolation 144
- 3.6 Parametric Curves 164
- 3.7 Survey of Methods and Software 171

4 Numerical Differentiation and Integration 173

- 4.1 Numerical Differentiation 174
- 4.2 Richardson's Extrapolation 185
- 4.3 Elements of Numerical Integration 193

v

- 4.4 Composite Numerical Integration 203
- 4.5 Romberg Integration 213
- 4.6 Adaptive Quadrature Methods 220
- 4.7 Gaussian Quadrature 228
- 4.8 Multiple Integrals 235
- 4.9 Improper Integrals 250
- 4.10 Survey of Methods and Software 256

5 Initial-Value Problems for Ordinary Differential Equations 259

- 5.1 The Elementary Theory of Initial-Value Problems 260
- 5.2 Euler's Method 266
- 5.3 Higher-Order Taylor Methods 276
- 5.4 Runge-Kutta Methods 282
- 5.5 Error Control and the Runge-Kutta-Fehlberg Method 293
- 5.6 Multistep Methods 302
- 5.7 Variable Step-Size Multistep Methods 315
- 5.8 Extrapolation Methods 321
- 5.9 Higher-Order Equations and Systems of Differential Equations 328
- 5.10 Stability 339
- 5.11 Stiff Differential Equations 348
- 5.12 Survey of Methods and Software 355

6 Direct Methods for Solving Linear Systems 357

- 6.1 Linear Systems of Equations 358
- 6.2 Pivoting Strategies 372
- 6.3 Linear Algebra and Matrix Inversion 381
- 6.4 The Determinant of a Matrix 396
- 6.5 Matrix Factorization 400
- 6.6 Special Types of Matrices 411
- 6.7 Survey of Methods and Software 428

7 Iterative Techniques in Matrix Algebra 431

- 7.1 Norms of Vectors and Matrices 432
- 7.2 Eigenvalues and Eigenvectors 443
- 7.3 The Jacobi and Gauss-Siedel Iterative Techniques 450
- 7.4 Relaxation Techniques for Solving Linear Systems 462
- 7.5 Error Bounds and Iterative Refinement 469
- 7.6 The Conjugate Gradient Method 479
- 7.7 Survey of Methods and Software 495

8 Approximation Theory 497

- 8.1 Discrete Least Squares Approximation 498
- 8.2 Orthogonal Polynomials and Least Squares Approximation 510
- 8.3 Chebyshev Polynomials and Economization of Power Series 518
- 8.4 Rational Function Approximation 528
- 8.5 Trigonometric Polynomial Approximation 538
- 8.6 Fast Fourier Transforms 547
- 8.7 Survey of Methods and Software 558

9 Approximating Eigenvalues 561

- 9.1 Linear Algebra and Eigenvalues 562
- 9.2 Orthogonal Matrices and Similarity Transformations 570
- 9.3 The Power Method 576
- 9.4 Householder's Method 593
- 9.5 The QR Algorithm 601
- 9.6 Singular Value Decomposition 614
- 9.7 Survey of Methods and Software 626

10 Numerical Solutions of Nonlinear Systems of Equations 629

- 10.1 Fixed Points for Functions of Several Variables 630
- 10.2 Newton's Method 638
- 10.3 Quasi-Newton Methods 647
- 10.4 Steepest Descent Techniques 654
- 10.5 Homotopy and Continuation Methods 660
- 10.6 Survey of Methods and Software 668

11 Boundary-Value Problems for Ordinary Differential Equations 671

- 11.1 The Linear Shooting Method 672
- 11.2 The Shooting Method for Nonlinear Problems 678
- 11.3 Finite-Difference Methods for Linear Problems 684
- 11.4 Finite-Difference Methods for Nonlinear Problems 691
- 11.5 The Rayleigh-Ritz Method 696
- 11.6 Survey of Methods and Software 711

12 Numerical Solutions to Partial Differential Equations 713

- 12.1 Elliptic Partial Differential Equations 716
- 12.2 Parabolic Partial Differential Equations 725
- 12.3 Hyperbolic Partial Differential Equations 739
- 12.4 An Introduction to the Finite-Element Method 746
- 12.5 Survey of Methods and Software 760

Bibliography 763

Answers to Selected Exercises 773

Index 863