

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix}, u_{ij} = a_{ij}, l_{ii} = \frac{a_{ii}}{u_{ii}} \rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & l_{32} & 1 \end{bmatrix}, U = \begin{bmatrix} 3 & 2 & 0 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$u_{22} = a_{22} - \sum_{k=1}^1 l_{2k} u_{k2} = 3 - \left(\frac{2}{3}\right)(2) = \frac{5}{3}, u_{23} = a_{23} - \sum_{k=1}^1 l_{2k} u_{k3} = 0 - \left(\frac{2}{3}\right)(0) = 0$$

$$l_{32} = \frac{a_{32} - \sum_{k=1}^1 l_{3k} u_{k2}}{u_{22}} = \frac{0 - \left(\frac{2}{3}\right)(2)}{\frac{5}{3}} = -0.8, u_{33} = a_{33} - \sum_{k=1}^2 l_{3k} u_{k3} = 3 - \left(\frac{2}{3}\right)(0) - (-0.8)(0) = 3$$

$$\rightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & -0.8 & 1 \end{bmatrix}, U = \begin{bmatrix} 3 & 2 & 0 \\ 0 & \frac{5}{3} & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow |A| = |L| |U| = (1)(15) = 15$$

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{3} & 1 & 0 \\ \frac{2}{3} & -0.8 & 1 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{cases} 1y_{11} = 1 \rightarrow y_{11} = 1 \\ \frac{2}{3}y_{11} + 1y_{21} = 0 \rightarrow y_{21} = -\frac{2}{3} \\ \frac{2}{3}y_{11} - 0.8y_{21} + 1y_{31} = 0 \rightarrow y_{31} = -1/2 \end{cases}, \dots \rightarrow$$

$$Y = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ -1/2 & 0.8 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 2 & 0 \\ 0 & \frac{5}{3} & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ -1/2 & 0.8 & 1 \end{bmatrix} \rightarrow \begin{cases} 3x_{11} = -1/2 \rightarrow x_{11} = -0.1667 \\ \frac{5}{3}x_{21} = -\frac{2}{3} \rightarrow x_{21} = -0.4 \\ 3x_{31} + 2x_{21} = 1 \rightarrow x_{31} = 0.6 \end{cases}$$

$$A^{-1} = X = \begin{bmatrix} 0.6 & -0.4 & 0 \\ -0.4 & 0.6 & 0 \\ -0.4 & \frac{4}{15} & \frac{1}{3} \end{bmatrix}$$

در تعیین ماتریسهای X و Y بایستی، سه معادله نوشته شود.

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 \\ 0 & 5 & 0 \\ 0 & -4 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & -4 & 3 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & -4 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & -4 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A^{-1}$$

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$$A - \lambda I = \begin{bmatrix} 3-\lambda & 2 & 0 \\ 2 & 3-\lambda & 0 \\ 2 & 0 & 3-\lambda \end{bmatrix} \rightarrow \begin{bmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & -2 & 0 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \rightarrow$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

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$$A_1 = A \rightarrow p_1 = \frac{1}{3} \text{tr}(A_1) = 9, A_2 = A(A_1 - p_1 I) = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} -9 & 2 & 0 \\ 2 & -9 & 0 \\ 2 & 0 & -9 \end{bmatrix} \rightarrow$$

$$A_2 = \begin{bmatrix} -14 & -6 & 0 \\ -6 & -14 & 0 \\ -6 & 4 & -18 \end{bmatrix} \rightarrow p_2 = \frac{1}{3} \text{tr}(A_2) = -23, A_3 = A(A_2 - p_2 I) = \begin{bmatrix} -14 & -6 & 0 \\ -6 & -14 & 0 \\ -6 & 4 & -18 \end{bmatrix} \begin{bmatrix} 9 & -6 & 0 \\ -6 & 9 & 0 \\ -6 & 4 & 5 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix} \rightarrow p_3 = \frac{1}{3} \text{tr}(A_3) = 15 \rightarrow \lambda^3 - 9\lambda^2 + 23\lambda - 15 = 0$$

$$\begin{cases} x^r + y^r - e^{y-1} = \cdot = f(x, y) \rightarrow \frac{\partial f}{\partial x} = r x, \frac{\partial f}{\partial y} = r y - e^{y-1}, \frac{\partial g}{\partial x} = -\cdot / \Delta x^{-1/\Delta}, \frac{\partial g}{\partial y} = 1 \\ x^{-1/\Delta} + y = \cdot = g(x, y) \end{cases}$$

$$\begin{cases} (r x_n) h_n + (r y_n - e^{y_n-1}) k_n = -(x_n^r + y_n^r - e^{y_n-1}) \rightarrow \\ (-\cdot / \Delta x_n^{-1/\Delta}) h_n + (1) k_n = -(x_n^{-1/\Delta} + y_n) \end{cases}$$

$$h_n = \frac{\begin{vmatrix} -(x_n^r + y_n^r - e^{y_n-1}) & r y_n - e^{y_n-1} \\ -(x_n^{-1/\Delta} + y_n) & 1 \end{vmatrix}}{\begin{vmatrix} r x_n & r y_n - e^{y_n-1} \\ -\cdot / \Delta x_n^{-1/\Delta} & 1 \end{vmatrix}}, k_n = \frac{\begin{vmatrix} r x_n & -(x_n^r + y_n^r - e^{y_n-1}) \\ -\cdot / \Delta x_n^{-1/\Delta} & -(x_n^{-1/\Delta} + y_n) \end{vmatrix}}{\begin{vmatrix} r x_n & r y_n - e^{y_n-1} \\ -\cdot / \Delta x_n^{-1/\Delta} & 1 \end{vmatrix}}$$

$$h_n = \frac{-x_n^r + y_n^r + e^{y_n-1} + r y_n x_n^{-1/\Delta} - x_n^{-1/\Delta} e^{y_n-1} - y_n e^{y_n-1}}{r x_n + y_n x_n^{-1/\Delta} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1}},$$

$$x_{n+1} = x_n + h_n = \frac{x_n^r + y_n^r + e^{y_n-1} + r y_n x_n^{-1/\Delta} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1} - y_n e^{y_n-1}}{r x_n + y_n x_n^{-1/\Delta} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1}}$$

$$k_n = \frac{-r / \Delta x_n^{-1/\Delta} - r x_n y_n - \cdot / \Delta x_n^{-1/\Delta} y_n^r + \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1}}{r x_n + y_n x_n^{-1/\Delta} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1}},$$

$$y_{n+1} = y_n + k_n = \frac{-r / \Delta x_n^{-1/\Delta} + \cdot / \Delta x_n^{-1/\Delta} y_n^r + \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1} y_n}{r x_n + y_n x_n^{-1/\Delta} - \cdot / \Delta x_n^{-1/\Delta} e^{y_n-1}}$$

$$\begin{cases} x_1 = 2/4 \rightarrow \begin{cases} x_1 = 1/1.3 \rightarrow \begin{cases} x_r = -/273 \rightarrow \begin{cases} x_r = -\cdot / 129 + \cdot / 199i \rightarrow \begin{cases} x_f = -1/324 - 1/215i \rightarrow \\ y_1 = -\cdot / 6 \rightarrow \begin{cases} y_1 = -\cdot / 12. \rightarrow \begin{cases} y_r = -1/546 \rightarrow \begin{cases} y_r = -\cdot / 698 - \cdot / 34i \rightarrow \begin{cases} y_f = \cdot / 124 + 1/92i \rightarrow \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases}$$

$$\begin{cases} x_2 = -1/143 - 1/24.i \rightarrow \begin{cases} x_p = -\cdot / 555 - \cdot / 249i \rightarrow \begin{cases} x_v = -1/287 - \cdot / 92i \rightarrow \begin{cases} x_\lambda = -1/11 + \cdot / 41i \rightarrow \\ y_2 = -\cdot / 3.7 - \cdot / 7.7i \rightarrow \begin{cases} y_p = -\cdot / 362 - \cdot / 963i \rightarrow \begin{cases} y_v = \cdot / 321 - \cdot / 73.i \rightarrow \begin{cases} y_\lambda = \cdot / 1.8 - \cdot / 955i \rightarrow \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases}$$

$$\begin{cases} x_3 = 1/1.85 + \cdot / 211i \rightarrow \begin{cases} x_{11} = \cdot / 57.0 + 2/211i \rightarrow \begin{cases} x_{11} = \cdot / 38.0 + 1/46i \rightarrow \begin{cases} x_{1r} = \cdot / 541 + \cdot / 124i \rightarrow \\ y_3 = -\cdot / 1.77 + 1/922i \rightarrow \begin{cases} y_{11} = -1/153 + \cdot / 9.5i \rightarrow \begin{cases} y_{11} = -\cdot / 687 + \cdot / 5.4i \rightarrow \begin{cases} y_{1r} = -\cdot / 116 + \cdot / 5.2i \rightarrow \end{cases} \end{cases} \end{cases} \end{cases} \end{cases} \end{cases}$$

$$\begin{cases} x_{1r} = \cdot / 543 + \cdot / 139i \rightarrow \begin{cases} x_{1f} = \cdot / 543 + \cdot / 139i \rightarrow \\ y_{1r} = -\cdot / 179 + \cdot / 471i \rightarrow \begin{cases} y_{1f} = -\cdot / 179 + \cdot / 471i \rightarrow \end{cases} \end{cases}$$

$$\begin{cases} x_1^r + x_1^r - e^{x_1-1} = 0 = f_1(x_1, x_r) \rightarrow \frac{\partial f_1}{\partial x_1} = 2x_1, \frac{\partial f_1}{\partial x_r} = 2x_r - e^{x_1-1}, \frac{\partial f_1}{\partial x_1} = -1/\delta x_1^{-1/\delta}, \frac{\partial f_1}{\partial y} = 1 \\ x_1^{-1/\delta} + x_r = 0 = f_r(x_1, x_r) \end{cases}$$

$$x^{(0)} = \begin{bmatrix} 2/4 \\ -1/6 \end{bmatrix}, F = \begin{bmatrix} 5/918 \\ 1/45 \end{bmatrix} \rightarrow g^{(0)} = f_1^r + f_r^r = 35/26, J = \begin{bmatrix} 4/80 & -1/42 \\ -1/33 & 1 \end{bmatrix},$$

$$\nabla g = rJ^T F = r \begin{bmatrix} 4/80 & -1/33 \\ -1/42 & 1 \end{bmatrix} \begin{bmatrix} 5/918 \\ 1/45 \end{bmatrix} = \begin{bmatrix} 56/784 \\ -16/54 \end{bmatrix} \rightarrow \|\nabla g(x^{(0)})\|_r = 59/134,$$

$$z = \frac{\nabla g(x^{(0)})}{\|\nabla g(x^{(0)})\|_r} = \begin{bmatrix} 1/96 \\ -1/279 \end{bmatrix}, \alpha_1 = 0 \rightarrow h_1 = g(x^{(0)} - \alpha_1 z) = 35/26$$

$$\alpha_r = 1 \rightarrow h_r = g(x^{(0)} - \alpha_r z) = g\left(\begin{bmatrix} 1/44 \\ -1/32 \end{bmatrix}\right) = 3/91 < h_1 \rightarrow$$

$$\alpha_r = 1/\delta \rightarrow h_r = g(x^{(0)} - \alpha_r z) = g\left(\begin{bmatrix} 1/92 \\ -1/46 \end{bmatrix}\right) = 13/56 \rightarrow$$

$$L_1(\alpha) = \prod_{j=1, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_1 - \alpha_j)} = \frac{(\alpha - 1/\delta)(\alpha - 1)}{(1 - 1/\delta)(1 - 1)} = 2(\alpha - 1/\delta)(\alpha - 1)$$

$$L_r(\alpha) = \prod_{j=r, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_r - \alpha_j)} = \frac{(\alpha - 1)(\alpha - 1)}{(1/\delta - 1)(1/\delta - 1)} = -4\alpha(\alpha - 1)$$

$$L_r(\alpha) = \prod_{j=r, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_r - \alpha_j)} = \frac{(\alpha - 1)(\alpha - 1/\delta)}{(1 - 1)(1 - 1/\delta)} = 2\alpha(\alpha - 1/\delta)$$

$$h = 7 \cdot 1/\delta 2(\alpha - 1/\delta)(\alpha - 1) - 54/26 \alpha(\alpha - 1) + 7/82 \cdot \alpha(\alpha - 1/\delta) \rightarrow$$

$$h' = 14 \cdot 1/\delta \alpha - 10 \cdot 1/\delta 2 \alpha - 10 \cdot 1/\delta 2 \alpha + 54/26 + 15/64 \alpha - 3/91 = 0 \rightarrow \hat{\alpha} = 1/152$$

$$x^{(1)} = x^{(0)} - \hat{\alpha} z = \begin{bmatrix} 2/4 \\ -1/6 \end{bmatrix} - 1/152 \begin{bmatrix} 1/96 \\ -1/279 \end{bmatrix} = \begin{bmatrix} 1/294 \\ -1/279 \end{bmatrix}$$

$$F = \begin{bmatrix} 1/474 \\ 1/6 \end{bmatrix} \rightarrow g^{(1)} = f_1^r + f_r^r = 2/533, J = \begin{bmatrix} 2/588 & -1/136 \\ -1/44 & 1 \end{bmatrix},$$

$$\nabla g = rJ^T F = r \begin{bmatrix} 2/588 & -1/44 \\ -1/136 & 1 \end{bmatrix} \begin{bmatrix} 5/918 \\ 1/45 \end{bmatrix} = \begin{bmatrix} 7/101 \\ -1/265 \end{bmatrix} \rightarrow \|\nabla g(x^{(1)})\|_r = 7/231,$$

$$z = \frac{\nabla g(x^{(1)})}{\|\nabla g(x^{(1)})\|_r} = \begin{bmatrix} 1/98 \\ -1/175 \end{bmatrix}, \alpha_1 = 0 \rightarrow h_1 = g(x^{(1)} - \alpha_1 z) = 2/533$$

$$\alpha_r = 1 \rightarrow h_r = g(x^{(1)} - \alpha_r z) = g\left(\begin{bmatrix} 1/31 \\ -1/14 \end{bmatrix}\right) = 2/91 > h_1 \rightarrow$$

$$\alpha_r = 1/\delta \rightarrow h_r = g(x^{(1)} - \alpha_r z) = g\left(\begin{bmatrix} 1/82 \\ -1/192 \end{bmatrix}\right) = 1/99 < h_1 \rightarrow$$

$$\alpha_r = 1/25 \rightarrow h_r = g(x^{(1)} - \alpha_r z) = g\left(\begin{bmatrix} 1/48 \\ -1/235 \end{bmatrix}\right) = 1/295 \rightarrow$$

$$L_1(\alpha) = \prod_{j=1, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_r - \alpha_j)} = \frac{(\alpha - 0/25)(\alpha - 0/5)}{(0 - 0/25)(0 - 0/5)} = \lambda(\alpha - 0/25)(\alpha - 0/5)$$

$$L_2(\alpha) = \prod_{j=1, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_r - \alpha_j)} = \frac{(\alpha - 0)}{(0/25 - 0)} \frac{(\alpha - 0/5)}{(0/25 - 0/5)} = -16\alpha(\alpha - 0/5)$$

$$L_3(\alpha) = \prod_{j=1, j \neq i}^3 \frac{(\alpha - \alpha_j)}{(\alpha_r - \alpha_j)} = \frac{(\alpha - 0)}{(0/5 - 0)} \frac{(\alpha - 0/25)}{(0/5 - 0/25)} = \lambda\alpha(\alpha - 0/25)$$

$$h = 20/264(\alpha - 0/25)(\alpha - 0/5) - 20/72\alpha(\alpha - 0/5) + 7/976\alpha(\alpha - 0/25) \rightarrow$$

$$h' = 40/528\alpha - 15/198 - 41/44\alpha + 10/36 + 15/952\alpha - 1/994 = 0 \rightarrow \hat{\alpha} = 2/201$$

$$x^{(2)} = x^{(1)} - \hat{\alpha}z = \begin{bmatrix} 1/294 \\ -0/279 \end{bmatrix} - 2/201 \begin{bmatrix} 0/984 \\ -0/175 \end{bmatrix} = \begin{bmatrix} -0/872 \\ 0/106 \end{bmatrix}$$

بین نتایج مرحله دوم و اول ، هیچ دقتی خاصی وجود ندارد.

با نوشتن فانکشن ذیل :

function F = root2d(x)

F(1) = x(1)^2 + x(2)^2 - exp(x(2) - 1);

F(2) = x(1)^(-0.5) + x(2);

با اجرای دستورالعملهای ذیل :

```
fun=@root2d;
x0=[2.4,-0.6];
x=fsolve(fun,x0)
```

پاسخی حاصل نمی شود.