پاسخ کوییز دوم گروه ۱

۱-

$$\nabla = hD - \frac{h^2}{2!}D^2 + \frac{h^3}{3!}D^3 - + \ldots \rightarrow \nabla \propto h \rightarrow \nabla^5 \propto h^5$$

$$D^3 = \frac{1}{h^3}\left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \ldots\right)^3 = \frac{1}{h^3}\left[\nabla^2 + 2(\nabla)(\frac{\nabla^2}{2})\right]\left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \ldots\right)$$

$$D^3 = \frac{1}{h^3}\left[\nabla^2 + \nabla^3\right]\left(\nabla + \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \ldots\right) = \frac{1}{h^3}\left[\nabla^3 + \nabla^4 + \frac{\nabla^4}{2}\right] = \frac{1}{h^3}\left[\nabla^3 + 1/5\nabla^4\right]$$

$$\nabla^3 = y_i - 3y_{i-1} + 3y_{i-2} - y_{i-3} \ , \ \nabla^4 = y_i - 4y_{i-1} + 6y_{i-2} - 4y_{i-3} + y_{i-4}$$

$$y_i''' = \frac{1/5y_{i-4} - 7y_{i-3} + 12y_{i-2} - 9y_{i-1} + 2/5y_i}{h^3}$$

۲-

$$I = \int_0^{\frac{\pi}{2}} \cos^2(2x)dx \rightarrow f(x) = \cos^2(2x) \rightarrow f' = -4\cos(2x)\sin(2x) = -2\sin(4x) \rightarrow f'' = -8\cos(4x)$$

$$\rightarrow f''' = 32\sin(4x) \rightarrow f^{(4)}(x) = 128\cos(4x) \rightarrow M_4 = 128$$

$$E = -\frac{(\Delta x)^5}{180} n\, f^{(4)}(\varepsilon) \ , \ \Delta x = \frac{b-a}{n} \rightarrow n \geq \sqrt[4]{\frac{(b-a)^5 M_4}{180\Delta I}} = \sqrt[4]{\frac{128\pi^5}{32 \times 180(0/05)}} = 3/4 \rightarrow n \geq 4$$

$$\Delta x = \frac{b-a}{n} = \frac{\pi}{8} \rightarrow A = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

$$A = \frac{\pi}{24}[f(0) + 4f(\frac{\pi}{8}) + 2f(\frac{\pi}{4}) + 4f(\frac{3\pi}{8}) + f(\frac{\pi}{2})]$$

$$A = \frac{\pi}{24}[1 + 4\cos^2(\frac{\pi}{4}) + 2\cos^2(\frac{\pi}{2}) + 4\cos^2(\frac{3\pi}{4}) + \cos^2(\pi)] = \frac{\pi}{24}[1 + 2 + 0 + 2 + 1] = \frac{\pi}{4} = 0/79 \simeq 0/8$$

$$k=۳\rightarrow A=\frac{\pi}{۶۴۸}[۰+۴\sin(۳\frac{\pi}{۸})+۲\sin(۳\frac{\pi}{۴})+۴\sin(۳\frac{۳\pi}{۸})+\sin(۳\frac{\pi}{۲})]$$

$$=\frac{\pi}{۶۴۸}[۰+۴(۰/۹۲۴)+۲(۰/۷۰۷)+۴(-۰/۳۸۳)-۱]=۰/۰۱۲\rightarrow ۰/۰۱$$

$$k=۴\rightarrow A=\frac{\pi}{۱۵۳۶}[۰+۴\sin(۴\frac{\pi}{۸})+۲\sin(۴\frac{\pi}{۴})+۴\sin(۴\frac{۳\pi}{۸})+\sin(۴\frac{\pi}{۲})]$$

$$=\frac{\pi}{۱۵۳۶}[۰+۴(۱)+۲(۰)+۴(-۱)+۰]=۰\rightarrow ۰/۰۰$$

$$k=۵\rightarrow A=\frac{\pi}{۳۰۰۰}[۰+۴\sin(۵\frac{\pi}{۸})+۲\sin(۵\frac{\pi}{۴})+۴\sin(۵\frac{۳\pi}{۸})+\sin(۵\frac{\pi}{۲})]$$

$$=\frac{\pi}{۳۰۰۰}[۰+۴(۰/۹۲۴)+۲(-۰/۷۰۷)+۴(-۰/۳۸۳)+۱]=۰/۰۰۱۸\rightarrow ۰/۰۰$$

$$k=۶\rightarrow A=\frac{\pi}{۵۱۸۴}[۰+۴\sin(۶\frac{\pi}{۸})+۲\sin(۶\frac{\pi}{۴})+۴\sin(۶\frac{۳\pi}{۸})+\sin(۶\frac{\pi}{۲})]$$

$$=\frac{\pi}{۵۱۸۴}[۰+۴(۰/۷۰۷)+۲(-۱)+۴(۰/۷۰۷)+۰]=۰/۰۰۲\rightarrow ۰/۰۰$$

$$k=۷\rightarrow A=\frac{\pi}{۸۲۳۲}[۰+۴\sin(۷\frac{\pi}{۸})+۲\sin(۷\frac{\pi}{۴})+۴\sin(۷\frac{۳\pi}{۸})+\sin(۷\frac{\pi}{۲})]$$

$$=\frac{\pi}{۸۲۳۲}[۰+۴(۰/۳۸۳)+۲(-۰/۷۰۷)+۴(۰/۹۲۴)-۱]=۰/۰۰۱\rightarrow ۰/۰۰$$

$$k=۸\rightarrow A=\frac{\pi}{۱۲۲۸۸}[۰+۴\sin(۸\frac{\pi}{۸})+۲\sin(۸\frac{\pi}{۴})+۴\sin(۸\frac{۳\pi}{۸})+\sin(۸\frac{\pi}{۲})]$$

$$=\frac{\pi}{۱۲۲۸۸}[۰+۴(۰)+۲(۰)+۴(۰)+۰]=۰\rightarrow ۰/۰۰$$

$$k=۹\rightarrow A=\frac{\pi}{۱۷۴۹۶}[۰+۴\sin(۹\frac{\pi}{۸})+۲\sin(۹\frac{\pi}{۴})+۴\sin(۹\frac{۳\pi}{۸})+\sin(۹\frac{\pi}{۲})]$$

$$=\frac{\pi}{۱۷۴۹۶}[۰+۴(-۰/۳۸۳)+۲(۰/۷۰۷)+۴(-۰/۹۲۴)+۱]=-۰/۰۰۰۵\rightarrow ۰/۰۰$$

$$k=۱۰\rightarrow A=\frac{\pi}{۲۴۰۰۰}[۰+۴\sin(۱۰\frac{\pi}{۸})+۲\sin(۱۰\frac{\pi}{۴})+۴\sin(۱۰\frac{۳\pi}{۸})+\sin(۱۰\frac{\pi}{۲})]$$

$$=\frac{\pi}{۲۴۰۰۰}[۰+۴(-۰/۷۰۷)+۲(۱)+۴(-۰/۷۰۷)+۰]=-۰/۰۰۰۵\rightarrow ۰/۰۰$$