Image Matching Fundamentals

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Historical Remarks
Terminology, Working Definitions

- Conjugate Entity
- Matching Entity
- Similarity Measure
- Matching Method
- Matching Strategy
## Relationship between Matching Methods and Matching Entities

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<th>Matching Method</th>
<th>Similarity Measure</th>
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<tr>
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<td>Symbolic description</td>
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Problem Statement

1- Select a matching entity in one image
2- Find its conjugate entity in the other image
3- Compute the 3d location of the matched entity in object space
4- Assess the quality of the match
Fundamental Problems of Image Matching

- Search Space, Uniqueness of Matching Entity
  - Combinatorial Explosion
  - Ambiguity
- Approximations, Constraints and Assumption

- Geometric Distortion of Matching Entities
  - Noise
  - Changing illumination
  - Reflection properties
  - Central projection
  - Relief
Approximations, Constraints and Assumption
Geometric Distortion of Matching Entities
Geometric Distortion Due to Orientation Parameters

a) Scale Difference Between the two Images

b) Difference Rotation Angles Between the two Images
Scale Difference Between the two Images

a

b
Difference Rotation Angles
Between the two Images

a

b

c
Effect of Tilted Surface on Geometric Distortion
Effect of Relief on Geometric Distortions
Solutions to the Fundamental Problems
Search Space and Approximations

- Epipolar Lines
- Vertical Line Locus
- Hierarchical Approach
Epipolar Lines
Epipolar Lines

\[ x_p'' = x_p' - px_p \]

\[ px_p = \frac{B}{H_D - Z_p} f \]

\[ x_p'' = x_p' - b_o \frac{H_D}{H_D - Z_p} \]

\[ S = x_U'' - x_L'' \]
Epipolar Lines

\[
\begin{align*}
x''_U &= x'_U - b_o \frac{H_D}{H_D - Z_U} \\
x''_L &= x'_L - b_o \frac{H_D}{H_D - Z_L} \\
s &= b_o H_D \frac{\delta Z}{(H_D - Z_L)(H_D - Z_U)} \\
s &\approx b_o \frac{H_D \delta Z}{(H_D - Z_p)^2}
\end{align*}
\]
Steps for Implementing Matching Scheme Along Epipolar Lines

- Select Matching entity in one image, $p'$
- Estimate the Elevation, $Z_p$, of the matching entity and its uncertainty range $\delta Z$
- Compute approximate location $P''$
- Compute search interval, $s$
- Perform matching within search interval
- Analyze the similarity measures obtained in previous step for determining conjugate location $p'''$
Vertical Line Locus
Concept of Matching Along the Vertical Line Loci \( v' \) and \( V'' \)
Combining Vertical Line Locus and Epipolar Line Method
Implementation procedure for VLL

1-Select the X-Y position of a point P in object space and estimate the elevation Zp and its lower and upper bounds ZL, ZR

2-Begin the matching process by projecting Xp,Yp,ZL back to both images

3-Perform a similarity measure at the image positions X’L, Y’L, and X’’L, Y’’L

4-Move along the vertical line in suitable steps \( dZ = \frac{(Zu-ZL)}{n} \)

5-Project the object points Xp,Yp,ZL+dz back to the images and repeat the similarity measure

6-Repeat step 4 until Zu is reached

7-Analyze the similarity measures obtained in every step and determine the maximum/minimum for the conjugate position
Hierarchical Approach
Uniqueness of Matching Entity

- Variance
- Autocorrelation
- Entropy
Aera-Based Matching

- Template
- Search Window
- Matching Window
Area-Based Matching

- Location of Template
- Size of Template
- Location and Size of Search Window
- Acceptance Criteria
- Quality Control
Area-Based Matching

- Correlation
  - Cross-Correlation Factor
  - Maximum Correlation Factor

- Least Squares Matching
Cross-Correlation Factor

\[ \rho = \frac{\sigma_{LR}}{\sigma_L \sigma_R} \]

- \( \sigma_{LR} \) Covariance of image patches L and R
- \( \sigma_L \) Standard deviation of image patch L
- \( \sigma_R \) Standard deviation of image patch R
Cross-Correlation Factor

\[
g_L = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} g_L(x_i, y_j)}{n \cdot m} \quad \sigma_L = \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \left( g_L(x_i, y_j) - g_L \right)^2}{n \cdot m - 1}}
\]

\[
g_R = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} g_R(x_i, y_j)}{n \cdot m} \quad \sigma_R = \sqrt{\frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \left( g_R(x_i, y_j) - g_R \right)^2}{n \cdot m - 1}}
\]

\[
\sigma_{LR} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \left( g_L(x_i, y_j) - g_L \right) \left( g_R(x_i, y_j) - g_R \right)}{n \cdot m - 1}
\]
Maximum Correlation Factor
Steps of Area-Based Matching with Correlation
Steps of Area-Based Matching with Correlation

- Select the centre of the template in one image
- Determine approximate locations for the conjugate position in the other image
- For both the template and the correlation window, determine the minimum size which passes the uniqueness criterion, select the larger of the two values as the window size for the current matching location
- Compute the correlation coefficient $Pr,c$ for all positions $r,c$ of the correlation window within the search window
- Analyze the correlation factors. A minimum threshold must be reached for a valid match. Apart from the maximum, determine the distinctness as a quality measure
- Repeat above steps for new template location until all positions to be matched are visited
- Analyze the matching results on a global base for consistency and/or compatibility with a priori scene knowledge
Least Squares Matching

To minimize the gray level differences between the template and the matching windows whereby the position and the shape of the matching window are parameters to be determined in adjustment process.
Least Squares Matching

- Mathematical Model
- Adjustment Procedure
Least Squares Matching

- Factors affecting gray level differences between template and matching windows
  - Illumination and reflectance differences between the two images
  - the photographic development process and scanner in case of digitized photographs
  - Geometric distortions
Mathematical Model

To compensate for differences in brightness and contrast, a transformation of the matching window is introduced

\[ T_R \{m(i, j)\} = r_0 + r_1 \cdot m(i, j) \]
Mathematical Model

\[ m(x, y) = m \left[ T_G(i, j) \right] \]

\[ x = T_x(i, j) \quad y = T_y(i, j) \]
Mathematical Model

- Since the two image patches are related via the common surface patch by the central projection, central projection model should be used.
- Matching windows and surface are small.
- It may be approximated by a plane.
Mathematical Model
(Affine Transformation)

- Affine Transformation:
  
  \[ x = t_0 + t_1 \cdot i + t_2 \cdot j \]
  
  \[ y = t_3 + t_4 \cdot i + t_5 \cdot j \]

- Observation Equation:

  \[ r(i, j) = t(i, j) - m(x, y) \]
Mathematical Model
(Affine Transformation)

- Linearization of the model:

\[
m(x, y) = m^0(x, y) + \frac{\partial m(x, y)}{\partial T_x} \left[ \frac{\partial T_x}{\partial t_0} \delta t_0 + \frac{\partial T_x}{\partial t_1} \delta t_1 + \frac{\partial T_x}{\partial t_2} \delta t_2 \right] + \ldots
\]

\[
\frac{\partial m(x, y)}{\partial T_y} \left[ \frac{\partial T_y}{\partial t_3} \delta t_3 + \frac{\partial T_y}{\partial t_4} \delta t_4 + \frac{\partial T_x}{\partial t_5} \delta t_5 \right]
\]
Mathematical Model
(Linearization process)

\[
\frac{\partial m(x, y)}{\partial T_x} = g_x \quad \frac{\partial T_x}{\partial t_0} = 1 \\
\frac{\partial m(x, y)}{\partial T_y} = g_y \quad \frac{\partial T_x}{\partial t_3} = 1
\]

\[
\frac{\partial T_x}{\partial t_1} = x \quad \frac{\partial T_x}{\partial t_4} = x \\
\frac{\partial T_x}{\partial t_5} = y
\]

\[
m^0(x, y) = m^0(i, j) \quad c = t(i, j) - m(i, j)
\]
Mathematical Model
(Linearization process)

<table>
<thead>
<tr>
<th>Pixel</th>
<th>$\delta t_0$</th>
<th>$\delta t_1$</th>
<th>$\delta t_2$</th>
<th>$\delta t_3$</th>
<th>$\delta t_4$</th>
<th>$\delta t_5$</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td>$g_{x1}$</td>
<td>$g_{x1} \cdot x_1$</td>
<td>$g_{x1} \cdot y_1$</td>
<td>$g_{y1}$</td>
<td>$g_{y1} \cdot x_1$</td>
<td>$g_{y1} \cdot y_1$</td>
<td>t(1,1)-m(1,1)</td>
</tr>
<tr>
<td>2,1</td>
<td>$g_{x2}$</td>
<td>$g_{x2} \cdot x_2$</td>
<td>$g_{x2} \cdot y_1$</td>
<td>$g_{y1}$</td>
<td>$g_{y1} \cdot x_2$</td>
<td>$g_{y1} \cdot y_1$</td>
<td>t(2,1)-m(2,1)</td>
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<tr>
<td>n,m</td>
<td>$g_{xn}$</td>
<td>$g_{xn} \cdot x_n$</td>
<td>$g_{xn} \cdot y_m$</td>
<td>$g_{ym}$</td>
<td>$g_{ym} \cdot x_n$</td>
<td>$g_{ym} \cdot y_m$</td>
<td>t(n,m)-(n,m)</td>
</tr>
</tbody>
</table>
Adjustment procedure

\[ r = \hat{A} t + c \]

\[ \hat{t} = \left( A^T P A \right)^{-1} A^T c \]

\[ \sigma^2 = \frac{r^T P r}{n.m - 6} \]
Adjustment Procedure (Resampling)
LSM procedure

- Select the centre of the template in one image (Rt,Ct)
- Determine approximate locations for the matching window (RM,CM)
- Determine minimum size of template and matching window which passes the uniqueness criteria, select the larger of the two values as the window size
- Start first iterations with matching window at location RM,CM
- Transform matching window and determine the gray values for the tessellation (Resampling)
- Repeat adjustment and resampling sequence until termination criteria is reached.
- Assess quality of conjugate point
- Repeat above steps for new position of template
References