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Control of Natural Frequency of Steel Cantilever Beam Using Pre-tensioning Cable

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Abstract

Sometimes natural frequency of beams is not within the allowable range, regardless of the appropriate shear and bending design. Moreover, every so often structural designers are faced with cases in which it is not possible to increase the beam height due to architectural limitations, or cases in which beam frequency limitation has not been considered during design, and problem of vibration is observed after implementation. Steel cable is utilized in this research to control the natural frequency of beam due to its important advantages such as low weight, small cross-sectional area and high tensile strength. In this study, for the first time, theoretical relation is developed to calculate the increase in pre-tensioning force of steel cable under external loading based on the method of least work. Moreover, the natural frequency of steel cantilever beam without cable and with cable is calculated based on Rayleigh's method. To verify the theoretical relations, the steel cantilever beam is modeled in the finite element ABAQUS without cable and with cable. The obtained results show that the theoretical relations can appropriately predict the natural frequency of beam. In addition, it is concluded that the pre-tensioning stress of the cable has no effect on the natural frequency.

Keywords:

Natural frequency, Steel cantilever beam, Cable, Pre-tensioning, Rayleigh's method

1. Introduction

Reckoned as important components of a structure, cables are materials which can tolerate tensile force, and generally increase the stiffness and bearing capacity of a structure (Razavi and Sheidaii 2012). Nowadays,

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cables are increasingly used in structures. Hou and Tagawa (2009) applied cable-cylinder bracing in the seismic retrofitting of steel flexural frames. From their viewpoint, through this retrofitting method, the lateral strength of the storey augments without decreasing the ductility of flexural frame. Fanaie et al. (2016a) presented theoretical relations for the cable-cylinder bracing system using a rigid cylinder like steel cylinder. They verified the results by finite element ABAQUS software. Fanaie et al. (2016b) also studied seismic behavior of steel flexural frames strengthened with cable-cylinder bracing and obtained reasonable results. Pre-tensioning of steel beams through high strength cables is one of the most efficient methods so as to decrease the required steel and increase their bearing capacity. The pre-tensioning technique was primarily used in reinforced concrete structures; however, for the first time, Dischinger and Magnel utilized it in steel beams. Pre-tensioned steel structures are constructed all over the world, especially in America, Russia, and Germany. This fact shows the structural and economic merits of pre-stressed steel beams compared to non-prestressed ones. The pre-tensioning technique is appropriate to construct new structures as well as strengthening the existing ones (Troitsky 1990). Some researchers have studied pre-stressed beams using steel cables. Nie et al. (2007) presented theoretical relations to calculate the deflection as well as yield and ultimate moments of simply supported pre-stressed steel-concrete composite beam considering the slip effect. They verified the suggested formulas with the experimental results. Troitsky (1990) evaluated the behavior of pre-stressed steel beam using cables, and observed the increase in the stiffness and decrease in the deformation of the beam. Belletti and Gasperi (2010) studied the behavior of pre-stressed simply supported steel I-shaped beams by tendons with focusing on two parameters, namely, the number of deviators and the value of pre-stressing force. Park et al. (2010) analytically and experimentally evaluated the flexural behavior of steel I-beam pre-stressed with externally unbonded tendons. They observed a considerable increase in the yielding and ultimate bearing capacity of steel I-beam.

A number of researchers have investigated the dynamic behavior of pre-tensioned beams using steel cables. Miyamoto et al. (2000) have studied the dynamic behavior of the pre-tensioned simply supported composite beam with external tendon. They derived the natural frequency equation of pre-tensioned beam based on the flexural vibration equation and verified the predicted equation by comparing it with the results of the dynamic experiment. Park et al. (2005) analytically and experimentally studied the strengthening effect of bridges using external pre-tensioned tendons and concluded that strengthening reduces the mid span deflection by 10-24%.

In controlling the natural frequency of the beam, structural designers have been faced with cases in which it is not possible to increase the beam height due to architectural limitations, or cases in which beam frequency limitation was not considered during design, and problem of vibration is observed after implementation. In this study, the natural frequency of steel cantilever beam has been evaluated without cable and with cable. The increase in pre-tensioning force of steel cable subjected to external loading is determined using method of least work. Then, Rayleigh’s method is applied to develop the natural frequency relations of steel beam equipped with cable. In order to validate the obtained natural frequency relations, the results of theoretical relations are compared with those of finite element model of the beams.

2. Pre-tensioning symmetric I-shaped steel cantilever beam with steel cable

As shown in Fig. 1, pre-stressed cables have been used in both sides of beam web, and subjected to external loading. Moreover, in the frequency analysis, beam movement is of back and forth type; therefore, the cable pattern should be considered according to Fig. 1 for cantilever beam.

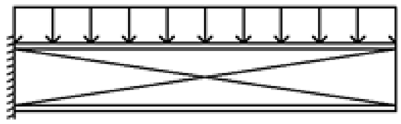

Figure 1. Pre-stressed symmetric I-shaped steel cantilever beam with steel cable under external loading

The following assumptions are taken into account to analyze pre-stressed symmetric I-shaped steel cantilever beam with steel cable:

1- The materials of steel beam and cable are linearly elastic;

2-The deformations are small;

3- Shear deformation is neglected;

4- The friction loss in the region of the cable deformation and the relaxation of steel cable are ignored;

5- Steel beam section is rolled; therefore, it is compact;

3. Natural Frequency

The natural frequency of system with distributed mass and rigidity which is usually considered as single degree of freedom system and known as generalized single degree of freedom system, can be calculated using the Rayleigh's method. This method is based on the principle of conservation of energy. The principle of conservation of energy states that the total energy in a freely vibrating undamped system is constant (i.e., it does not vary with time).

4. Calculating the Natural Circular Frequency of Beam by Rayleigh's Method

The simple harmonic motion of a beam under free vibration can be defined as follows:

$$u(x,t) = z_0 \psi(x) \sin \omega_n t' \tag{1}$$

where $\psi(x)$ is an assumed shape function defining the form of deflections and satisfying the displacement boundary conditions. The shape function can be determined from deflections thanks to a selected set of static forces. One common selection for these forces is the weight of structure applied in an appropriate direction. Also, z_0 is the amplitude of generalized coordinate z(t). ω_n is the natural circular frequency of beam. The velocity of beam is equal to:

$$\dot{u}(x,t)=\omega_n z_0 \psi(x)\cos\omega_n t' \tag{2}$$

The maximum potential energy of the system over a vibration cycle is equal to its strain energy associated with the maximum displacement $u_o(x)$:

$$E_{So}=\int_0^L \frac{1}{2} EI(x)\left[u_o''(x)\right]^2 dx \tag{3}$$

The maximum kinematic energy of the system over a vibration cycle is associated with the maximum velocity $\dot{u}_0(x)$:

$$E_{Ko}=\int_0^L \frac{1}{2} m(x)\left[\dot{u}_o(x)\right]^2 dx \tag{4}$$

Using Eqs. (1) and (2), the maximum displacement $u_0(x)$ and maximum velocity $\dot{u}_0(x)$ are defined as follows:

$$u_o(x)=z_o\psi(x) \tag{5}$$

$$\dot{u}_o(x)=\omega_n z_o\psi(x)=\omega_n u_o(x) \tag{6}$$

The natural circular frequency of beam is obtained by replacing Eqs. (5) and (6) in Eqs. (3) and (4), and using the principle of conservation of energy, equating maximum strain energy E_{So} to maximum kinematic energy E_{Ko} gives:

$$\omega_n^2=\frac{\int_0^L EI(x)\left[\psi''(x)\right]^2 dx}{\int_0^L m(x)\left[\psi(x)\right]^2 dx} \tag{7}$$

In which $m(x)$ is the mass per unit length of the beam, $EI(x)$ is flexural rigidity and L is the beam length.

Eq. (7) is the Rayleigh's quotient for a system with distributed mass and rigidity.

Rayleigh's method can be used to calculate the natural frequency of beam without cable and with cable.

4.1. Calculating natural frequency of the cantilever beam without cable

To determine the shape function of the cantilever beam, as shown in Fig. 2(a), boundary conditions are as follows:

$$u_o(0)=0 \quad , \quad u_o'(0)=0$$

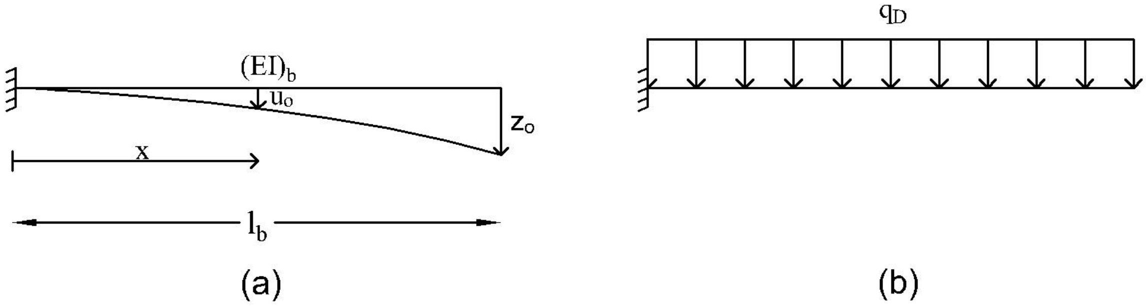

Figure 2. (a). Deflection curve of the cantilever beam; (b) Cantilever beam subjected to vertical structural weight.

One possible way to select the shape function is based on the deflection curve under static force. The bending moment equation of the cantilever beam due to vertical structural weight (as in Fig. 2(b)) in order to determine the deflection curve, is as follows:

$$M(x)=-\frac{q_D l_b^2}{2}+q_D l_b x-\frac{q_D x^2}{2} \tag{8}$$

The internal bending moment is equal to:

$$M(x)=-(EI)_b u_o''(x) \tag{9}$$

By replacing Eq. (8) in Eq. (9) and imposing boundary conditions, the deflection curve is obtained as follows:

$$u_o(x)=\frac{q_D l_b^4}{8(EI)_b}\left[2(\frac{x}{l_b})^2-\frac{4}{3}(\frac{x}{l_b})^3+\frac{1}{3}(\frac{x}{l_b})^4\right] \tag{10}$$

If the deflection at the end of the cantilever beam (as shown in Fig. 2(a)) is assumed as an amplitude of generalized coordinate $z_o=u_o(l_b)=\frac{q_D l_b^4}{8(EI)_b}$, the shape function is obtained as follows:

$$\psi(x)=2(\frac{x}{l_b})^2-\frac{4}{3}(\frac{x}{l_b})^3+\frac{1}{3}(\frac{x}{l_b})^4 \tag{11}$$

The natural circular frequency of the cantilever beam using the Rayleigh's quotient formula (Eq. (7)) by replacing the shape function obtained from the deflection curve according to Eq. (11), is obtained as follows:

$$\omega_n^2 = \frac{\int_0^{l_b} EI(x)\left[\psi''(x)\right]^2 dx}{\int_0^{l_b} m(x)\left[\psi(x)\right]^2 dx} = \frac{(EI)_b \int_0^{l_b} (\frac{4}{l_b^2} - \frac{8x}{l_b^3} + \frac{4x^2}{l_b^4})^2 dx}{\frac{q_D}{g} \int_0^{l_b} \left(2(\frac{x}{l_b})^2 - \frac{4}{3}(\frac{x}{l_b})^3 + \frac{1}{3}(\frac{x}{l_b})^4 \right)^2 dx} = \frac{162(EI)_b g}{13 l_b^4 q_D} \tag{12}$$

$$\rightarrow \omega_n = \frac{9}{l_b^2}\sqrt{\frac{2(EI)_b g}{13 q_D}}$$

In which m(x) is the mass per unit length equal to $\frac{q_D}{g}$ (q_D is uniform dead load per unit length and g is the gravity acceleration), l_b is the beam length and $(EI)_b$ is the flexural rigidity of the beam.

Then the natural frequency of the cantilever beam is obtained as follows:

$$f = \frac{\omega_n}{2\pi} = \frac{9}{\pi l_b^2}\sqrt{\frac{(EI)_b g}{26 q_D}} \tag{13}$$

4.2. Calculating the natural frequency of cantilever beam with cable

One possible way to select the shape function in order to calculate natural frequency of the beam with cable using the Rayleigh's method is based on the deflection curve due to static force. One common selection for these forces is the weight of structure applied in vertical direction.

The cable length increased by Δl, and its pre-tensioning force, F_{pt}, increased by ΔF, in beam with cable under uniform dead load. As the structure is statically indeterminate, the static equilibrium equations are not enough to calculate ΔF. The increase in the force in the cable can be calculated using the method of least work. Regarding the cantilever beam with cable, as shown in Fig. 3, As the slope of steel cable is constant in the cantilever beam with cable, the increase in pre-tensioning force of steel cable is equal to ΔF. Therefore, the axial force of the beam is equal to $\Delta F \cos\theta$. The maximum strain energy of the cantilever beam with cable for the whole beam is obtained as follows:

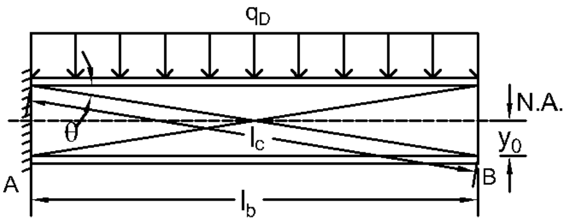

Fig. 3. Cantilever beam along with cable

$$E_{So}=\frac{1}{2(EI)_b}\int_0^{l_b}M(x)^2dx+\frac{\Delta F^2 l_c}{2(AE)_c}+\frac{(\Delta F\cos\theta)^2 l_b}{2(AE)_b} \tag{14}$$

In which l_b and l_c are the lengths of beam and inclined cable, A_b and A_c are cross section area of beam and cable on both sides of the web, E_b and E_c are elasticity modulus of beam and cable, respectively, and I_b is the moment of inertia of beam, θ is the angle of inclined cable with the horizontal axis, and $M(x)$ is the bending moment of the cantilever beam with cable.

The bending moment of the cantilever beam with cable under uniform distributed dead load is obtained as follows:

For the $0\le x\le l_b$ range;

$$M(x)=-\frac{q_D l_b^2}{2}+\Delta F\cos\theta y_0-\Delta F\sin\theta x+q_D l_b x-\frac{q_D x^2}{2} \tag{15}$$

In which q_D is the uniform distributed dead load per unit length, y_0 is the distance of neutral axis to the connection point of steel cable to the beam flange.

Replacing Eq. (15) in Eq. (14), the maximum strain energy formula is obtained as follows:

$$E_{So}=\frac{1}{2(EI)_b}\int_0^{l_b}\left(-\frac{q_D l_b^2}{2}+\Delta F\cos\theta y_0-\Delta F\sin\theta x+q_D l_b x-\frac{q_D x^2}{2}\right)^2dx+\frac{\Delta F^2 l_c}{2(AE)_c}$$
$$+\frac{(\Delta F\cos\theta)^2 l_b}{2(AE)_b}=\frac{1}{2(EI)_b}\left\{\begin{matrix}\frac{q_D^2 l_b^5}{20}+\frac{\Delta F^2 l_b^3\sin^2\theta}{3}+\Delta F^2 l_b y_0^2\cos^2\theta\\-\Delta F^2 l_b^2 y_0\sin\theta\cos\theta+\frac{q_D\Delta F l_b^4\sin\theta}{12}-\frac{q_D\Delta F l_b^3 y_0\cos\theta}{3}\end{matrix}\right\} \tag{16}$$
$$+\frac{\Delta F^2 l_c}{2(AE)_c}+\frac{\Delta F^2 l_b\cos^2\theta}{2(AE)_b}$$

Calculating the increase in pre-tensioning force of the cable (ΔF) through the method of least work, the relation of whole strain energy is differentiated with respect to ΔF and the obtained result is equated to zero:

$$\frac{\partial E_{So}}{\partial(\Delta F)} = 0 \tag{17}$$

The relation for calculating the increase of pre-tensioning force of the cable (ΔF) is obtained as follows:

$$\Delta F = \frac{-q_D l_b^{\,4}\sin\theta + 4q_D l_b^{\,3} y_0\cos\theta}{8\left(l_b^{\,3}\sin^2\theta + 3l_b y_0^{\,2}\cos^2\theta - 3l_b^{\,2} y_0\sin\theta\cos\theta + \dfrac{3(EI)_b l_c}{(AE)_c} + \dfrac{3I_b l_b\cos^2\theta}{A_b}\right)} \tag{18}$$
$$= q_D\mu$$

In Eq. (18), μ is as follows,

$$\mu = \frac{-l_b^{\,4}\sin\theta + 4l_b^{\,3} y_0\cos\theta}{8\left(l_b^{\,3}\sin^2\theta + 3l_b y_0^{\,2}\cos^2\theta - 3l_b^{\,2} y_0\sin\theta\cos\theta + \dfrac{3(EI)_b l_c}{(AE)_c} + \dfrac{3I_b l_b\cos^2\theta}{A_b}\right)} \tag{19}$$

Replacing Eq. (18) in Eq. (16), the maximum strain energy of the beam E_{So} is obtained as follows:

$$E_{So} = \frac{q_D^{\,2}}{120(EI)_b}\left\{\begin{aligned} &3l_b^{\,5} + 20\mu^2 l_b^{\,3}\sin^2\theta + 60\mu^2 l_b y_0^{\,2}\cos^2\theta - 60\mu^2 l_b^{\,2} y_0\sin\theta\cos\theta \\ &+5\mu l_b^{\,4}\sin\theta - 20\mu l_b^{\,3} y_0\cos\theta + \frac{30\mu^2(EI)_b l_c}{(AE)_c} + \frac{30\mu^2 I_b l_b\cos^2\theta}{A_b}\end{aligned}\right\} \tag{20}$$
$$= \frac{q_D^{\,2}\beta}{120(EI)_b}$$

In Eq. (20), β is as follows:

$$\begin{aligned}\beta &= 3l_b^{\,5} + 20\mu^2 l_b^{\,3}\sin^2\theta + 60\mu^2 l_b y_0^{\,2}\cos^2\theta - 60\mu^2 l_b^{\,2} y_0\sin\theta\cos\theta + 5\mu l_b^{\,4}\sin\theta \\ &-20\mu l_b^{\,3} y_0\cos\theta + \frac{30\mu^2(EI)_b l_c}{(AE)_c} + \frac{30\mu^2 I_b l_b\cos^2\theta}{A_b}\end{aligned} \tag{21}$$

The maximum kinematic energy of the cantilever beam with cable for the whole beam is obtained as follows:

$$E_{Ko} = \int_0^{l_b}\frac{1}{2}m(x)\left(\dot{u}_o(x)\right)^2 dx = \int_0^{l_b}\frac{1}{2}m(x)\left(\omega_n u_o(x)\right)^2 dx \tag{22}$$

In which $m(x)$ is mass per unit length of the beam equal to $\frac{q_D}{g}$ (mass of cable is neglected) and $u_o(x)$ is the deflection curve of the cantilever beam with cable.

To determine the deflection curve, the internal bending moment is equal to:

$$M(x) = -(EI)_b u_o''(x) \tag{23}$$

The deflection curve in Eq. (23) should satisfy the displacement boundary conditions. For the cantilever beam with cable, the boundary conditions are as follows:

$$u_o(0) = 0 \quad , \quad u_o'(0) = 0$$

Replacing Eq. (15) in Eq. (23) and imposing the above boundary conditions, the deflection curve of the beam is obtained as follows:

$$u_o(x) = \frac{1}{(EI)_b}\left[\frac{q_D l_b^2 x^2}{4} - \frac{\Delta F y_0 \cos\theta x^2}{2} + \frac{\Delta F \sin\theta x^3}{6} - \frac{q_D l_b x^3}{6} + \frac{q_D x^4}{24}\right] \tag{24}$$

Replacing Eq. (24) in Eq. (22), the maximum kinematic energy of the beam is obtained as follows:

$$\begin{aligned} E_{Ko} &= \omega_n^2 \frac{q_D}{2g}\int_0^{l_b}\left(\frac{1}{(EI)_b}\left[\frac{q_D l_b^2 x^2}{4} - \frac{\Delta F y_0 \cos\theta x^2}{2} + \frac{\Delta F \sin\theta x^3}{6} - \frac{q_D l_b x^3}{6} + \frac{q_D x^4}{24}\right]\right)^2 dx \\ &= \omega_n^2 \frac{q_D}{2(EI)_b^2 g}\left\{\begin{aligned} &\frac{13 q_D^2 l_b^9}{3240} + \frac{\Delta F^2 l_b^7 \sin^2\theta}{252} + \frac{\Delta F^2 l_b^5 y_0^2 \cos^2\theta}{20} - \frac{\Delta F^2 l_b^6 y_0 \sin\theta\cos\theta}{36} \\ &+ \frac{31 q_D \Delta F l_b^8 \sin\theta}{4032} - \frac{71 q_D \Delta F l_b^7 y_0 \cos\theta}{2520} \end{aligned}\right\} \end{aligned} \tag{25}$$

Replacing Eq. (18) in Eq. (25), the maximum kinematic energy of the beam E_{Ko} is obtained as follows:

$$\begin{aligned} E_{Ko} &= \omega_n^2 \frac{q_D^3}{362880(EI)_b^2 g}\left\{\begin{aligned} &1456 l_b^9 + 1440\mu^2 l_b^7 \sin^2\theta + 18144\mu^2 l_b^5 y_0^2 \cos^2\theta \\ &-10080\mu^2 l_b^6 y_0 \sin\theta\cos\theta + 2790\mu l_b^8 \sin\theta \\ &-10224\mu l_b^7 y_0 \cos\theta \end{aligned}\right\} \\ &= \omega_n^2 \frac{q_D^3 \gamma}{362880(EI)_b^2 g} \end{aligned} \tag{26}$$

In Eq. (26), γ is as follows:

$$\begin{aligned} \gamma &= 1456 l_b^9 + 1440\mu^2 l_b^7 \sin^2\theta + 18144\mu^2 l_b^5 y_0^2 \cos^2\theta - 10080\mu^2 l_b^6 y_0 \sin\theta\cos\theta \\ &+ 2790\mu l_b^8 \sin\theta - 10224\mu l_b^7 y_0 \cos\theta \end{aligned} \tag{27}$$

The natural circular frequency of the cantilever beam with cable using the Rayleigh's method and the principle of conservation of energy, is obtained as follows:

$$\omega_n^{\ 2} = \frac{3024(EI)_b g\beta}{q_D\gamma} \rightarrow \omega_n = 12\sqrt{\frac{21(EI)_b g\beta}{q_D\gamma}} \quad (28)$$

The natural frequency of the cantilever beam with cable is obtained as follows:

$$f_n = \frac{\omega_n}{2\pi} = \frac{6}{\pi}\sqrt{\frac{21(EI)_b g\beta}{q_D\gamma}} \quad (29)$$

5. Finite element modeling of steel cantilever beam pre-stressed with steel cable

Cantilever beam has been designed based on Load and Resistance Factor Design (LRFD) method using AISC360-10 code (2010).Then the natural frequency of the cantilever beam is obtained based on the shape function resulting from the deflection curve corresponding to Eq. (13). The cantilever beam is designed such that its natural frequency is smaller than the minimum permissible frequency of 5 Hz. Table (1) shows the cantilever beam property with its natural frequency based on shape function. It should be noted that the length of loading span is 1.5 m for the beam; dead and live loads are 450 and 200 kg/m^2 respectively.

Table 1. Property and natural frequency of cantilever beam

Type of beam	Beam span length(m)	Cross-section of beam	Natural frequency based on shape function of the deflection curve (Hz)	Allowable natural frequency (Hz)
Cantilever beam	3.5	IPE240	4.88	5

The cantilever beam without cables and with cables were modeled in ABAQUS finite element software. Fig. 4 shows the finite element model of the cantilever beam with cables. The beams and cables have been modeled in 3-dimensional space with shell and truss elements (as wire) respectively. The weld's connector is used to connect the cable to the top flange of the beam at end providing a perfect connection between two nodes. Moreover, coupling constraint is used to connect the cable to the bottom flange of the beam. Uniform distributed load is applied as a surface traction type on the top flange. Predefined field tool is used to create the initial pre-tensioning stress in the cable as well. Mesh size was used 0.05. Fig. 5 shows the position of cables in cantilever beam.

For better presenting the behavior of cantilever beam with cable, first it has been modeled in the software without cable, and then with cable; and the obtained results have been compared with each other.

The steel material of beam considered in this research is ST-37; yield stress is 240MPa; modulus of elasticity of steel is 200 GPa; and Poisson's ratio is 0.3. The material of steel cable is in accordance with the ASTM A416M standard. 7-wire strand (grade 270 (1860)) is considered for steel cable with low relaxation, minimum ultimate strength (f_{pu}) of 270 ksi (1860 MPa), minimum yield strength at 1%

extension of 52.74 kip (234.6 kN), elasticity module of 28.5×10^6psi (196501.8 MPa) and Poisson's ratio of 0.3.

Adding cable to the beam converts it to beam-column because the horizontal component of the cable force creates an axial force in the beam. To prevent the beam buckling about the longitudinal axis, the cantilever beam with 1 lateral brace at the end of the beam. The beams are designed according to AISC360-10 so as to take the simultaneous effects of axial force and bending moment into account.

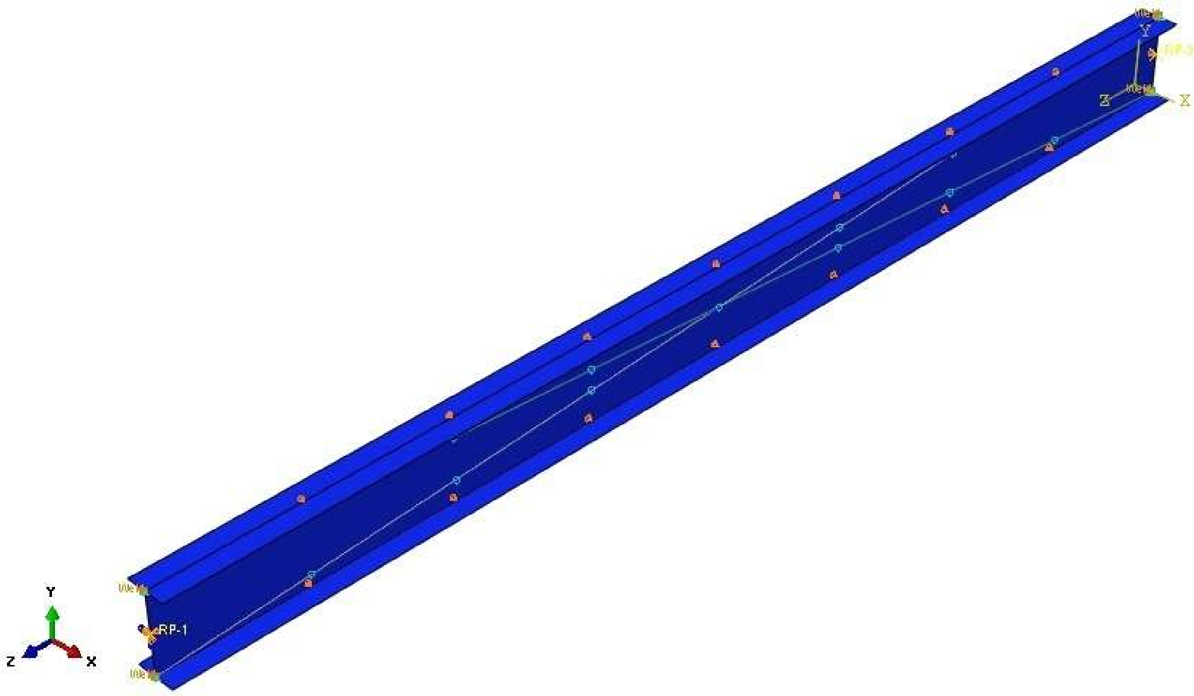

Figure 4. Finite element model of the cantilever beam along with cable

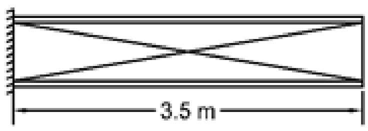

Figure 5. The locations of cables in the cantilever beam along with cable

6. Verification of theoretical relations of natural frequency with results of ABAQUS models

Frequency analysis of ABAQUS software has been employed so as to analyze the cantilever beam (Table 1), without cable and with cable. The 7-wire strand steel cable with low relaxation is considered for cantilever beam as two cables on each side of the beam web with a cross section area of 140 mm^2 in accordance with ASTM A416M standards. As a result, the entire steel cable cross-section is equal to 560 mm^2. Pre-tensioning of the steel cable is considered as 600 MPa. Controlling the accuracy of theoretical relations, natural frequency obtained from modeling is compared to those of the theoretical relation for the cantilever beam without cable and with cable. The results of natural frequency obtained from modeling are compared with those of Eqs. 13 and 29 for cantilever beam without cable and with cable, Table 2.

Table 2. Natural frequency values obtained from modeling and theoretical equations for the cantilever beam with cable

Type of beam		Natural frequency of beam obtained from modeling (Hz)	Natural frequency of beam obtained from theoretical equations (Hz)	Allowable natural frequency (Hz)
Cantilever beam	Without cable	4.72	4.88	5
	With cable	4.84	4.92	

As it is observed from Table 2, the theoretical relations well predict the natural frequency of the cantilever beam without cables and with cable. Also, it is observed that the natural frequency of the beam has increased when the pre-tensioned steel cable is used compared to that of the beam without cable; therefore, using the cable increases the natural frequency of the cantilever beam.

7. Sensitivity analysis on the cross-section of steel cable

In sensitivity analysis on the cross-section of the steel cable, different amounts of the 7-wire strand steel cable cross-section with low relaxation have been considered for cantilever beam as an equal number of cables on both sides of the beam web with an area of 140 mm^2 in accordance with ASTM A416 standard and stable pre-tensioned stress of 600 MPa. Tables 3 present the natural frequency in the cantilever beam with cable modeled in ABAQUS software for different cross-sections of the steel cable.

According to Tables 3, natural frequency is increased in the cantilever beam with cable with the increase in steel cable cross-section area due to the increase in stiffness in the beam along with cable.

Table 3. Natural frequency results of cantilever beam along with cable in sensitivity analysis on the cross-section area of steel cable

Total cross-section area of steel cable (mm^2)	Natural frequency of cantilever beam along with cable (Hz)	Allowable natural frequency (Hz)
560	4.84	5
840	4.89	5
1120	4.94	5
1400	5.0	5

8. Sensitivity analysis on the pre-tensioning stress of the steel cable

In sensitivity analysis on the pre-tensioning stress of the steel cable, 7-wire strand steel cable with low relaxation for cantilever beam is used in the form of four cables on each side of the beam web with an area of 140 mm^2 in accordance to the ASTM A16M standard. As a result, the overall steel cable cross-section is equal to 1120 mm^2. Table 4 presents the values of natural frequency of the cantilever beam cable modeled in ABAQUS software, for different values of pre-tensioning of the steel cable.

Table 4. Natural frequency results of the cantilever beam with cable in sensitivity analysis on the cable pre-tensioning stress

Type of beam		cable pre-tensioning stress (MPa)			Allowable natural frequency (Hz)
		400	600	800	
Cantilever beam	Along with cable	4.94	4.94	4.94	5

As it is observed in Table 4, the natural frequency of cantilever beam with cable remain stable with an increase in the pre-tensioning stress of steel cable.

9. Conclusion

Cables, due to their low weights, small cross sections and high tensile strengths, are reckoned as proper alternatives for pre-tensioning the steel beams subjected to external loads. In this research, cables are employed to pre-stress the cantilever beam in which the natural frequency is not within the allowable range, despite appropriate design under bending and shear. Theoretical equation has been derived to calculate the increase in pre-tensioning force of the cable as well as the natural frequency of cantilever beam with and without cable. The results obtained from the finite element model and theoretical equations, are briefly summarized as follows:

1. Comparing the results obtained from theoretical equations and those of finite element model demonstrates that the theoretical equations developed in this article can properly predict the natural frequency of cantilever beam without cable and along with cable;
2. Adding cable to the beam results in increasing the natural frequency of cantilever beam with cable;
3. In cantilever beam with cable, the natural frequency is increased by increasing the cross section of steel cable, considering equal pre-tensioning.
4. By increasing in pre-tensioning in the steel cables of equal cross-sections, the natural frequency is constant in the cantilever beam with cable.

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