

# Adaptive Lead–Acid Battery Modelling using RC Method

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## Abstract

The lead-acid battery in electric vehicles, constantly subjected to charging and discharging process resulting into depreciation in its performance. In this study, the battery is modeled as an RC circuit using Kalman filter method. Each element in the circuit represents certain battery characteristic. The parameters of the model are computed using parameter estimation techniques and sliding mode control. The results indicate that the proposed technique accurately estimates the battery model parameters with an error of just 0.05% in the most cases. Hence this method can be employed in battery monitoring algorithms for the online estimation of state of charge and predicted time to run for many applications.

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## 1 Introduction

The popularity of Hybrid-Electric Vehicles (HEV) in today's automotive industry is because of its higher fuel efficiency and reduced emissions of polluting gases. Since the battery forms an integral part in the operation of the vehicle it becomes essential to know the available charge left in the battery, how long will it be able to provide required energy, and also about the life of the battery. Researchers around the world developed a wide range of battery models with varying degrees of complexity [1, 2]. These days with the use of lead-acid batteries becoming more critical in HEVs, on-line modeling (adaptive) is becoming significantly important in order to compute the battery parameters, even when the vehicle is in operation, and to keep updating the model to give a more accurate value of the parameters [3, 4, 5]. Adaptive models, however, require few cycles to update the model and give a more accurate model of the battery.

In this study, the lead-acid battery is simulated with Kalman [6] filter method. The estimation parameters consist of two phases. The first phase consists of introducing step change in the input value and the second phase is a time varying input signal.

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## 2 Battery Model

The adaptive model consists of resistors and capacitors to represent both the steady-state and dynamic characteristics of the battery. Figure 1 shows the circuit diagram for Kalman filter model of battery.

In the above model,  $R_d$  represents the self-discharge resistance (which is considered to be quite high and hence can be neglected), capacitor  $C_b$  represents the main charge store (i.e. an indicator of the charge of the battery),  $R_i$  represents the resistance of the battery terminals and inter-cell connections, while  $R_t$  and  $C_s$  represent the transient performance of the battery. The voltage across the capacitor  $C_b$  is a suitable indicator of the State of Charge (SOC) of the battery while the State of Health (SOH) can be attributed to the change in the value of  $C_b$ .

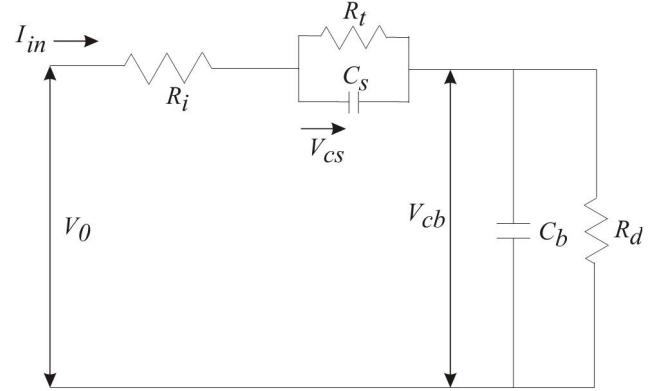


Figure 1: Circuit diagram for Kalman filter model

The differential equations representing the system are given by:

$$\frac{dV_{Cb}}{dt} = \frac{1}{C_b} I_{in} \quad (1)$$

$$\frac{dV_{Cs}}{dt} = -\frac{1}{C_b R_t} V_{Cs} + \frac{1}{C_s} I_{in} \quad (2)$$

The terminal voltage  $V_o$  can be calculated using the following equation:

$$V_o = V_{Cb} + V_{Cs} + I_{in} R_i \quad (3)$$

The above equations represent the dynamics of the battery. This relation can be used in the model of the battery to simulate the battery at different loading conditions.

## 3 Results

The dynamic behavior of the lead-acid battery is studied using the above model. The input current in the first phase is the step function and in the second phase is a time-varying function. In the following, the results for each phase are discussed.

### Estimation of $R_i$

This term represents the internal resistance of the battery. When there is a step change in the input, the values of  $V_{Cb}$  and  $V_{Cs}$  can not change immediately (since capacitor opposes the changes in voltage). Hence based on Eq. (3), the change in  $V_o$  is totally attributed to  $I_{in} R_i$ . From the observed decrease in the output voltage, and from the input step change, the value of  $R_i$  can be easily found. To verify our work, the experimental data of ref. [5] is used. By substituting  $\Delta V_o = 0.48$  V and  $I_{in} = 30$  A in Eq. (3) we have  $R_i = 16$  m $\Omega$  which is the same value of  $R_i$  in the Ref. [5].

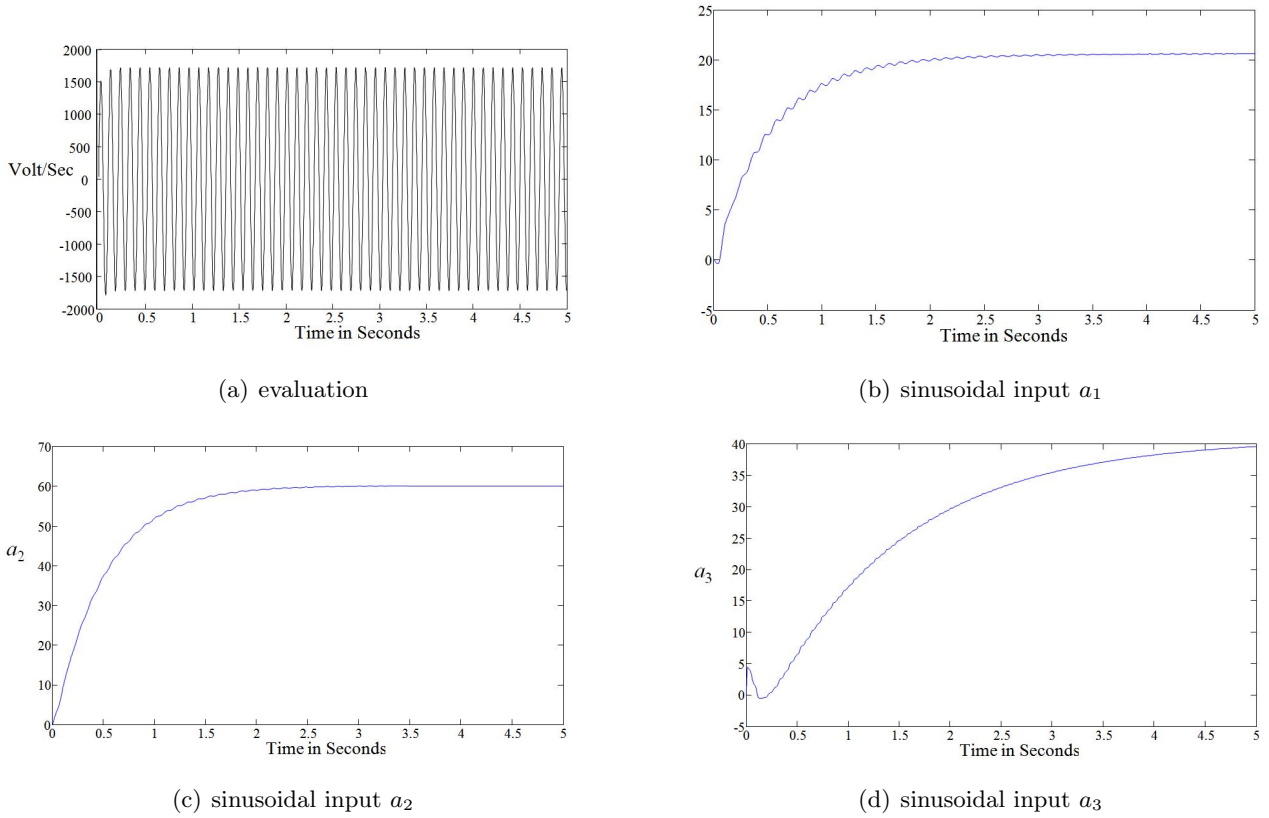


Figure 2: Simulation results

### Estimation of $C_b$

This term represents charge-store capacitor of the battery. The voltage across  $C_b$  represents the SOC of the battery. In the present study, not having experimental data, the value of  $C_b$  is taken to be constant. Taking the derivative of Eq. (3) with respect to time, one obtains:

$$\frac{dV_o}{dt} = \frac{dV_{C_b}}{dt} + \frac{dV_{C_s}}{dt} \quad (4)$$

In Eq. (4), if  $\Delta t$  is large then  $V_{C_s} \rightarrow 0$  because  $V_{C_s}$  is more predominant during short intervals of time and can be neglected when the time interval under consideration is large ( $dV_{C_s} \rightarrow 0$ ). Substituting  $\frac{dV_o}{dt}$  for  $\frac{dV_{C_b}}{dt}$  in Eq. (1), the value of  $C_b$  can be found. According to the data of ref. [5],  $\frac{dV_o}{dt} = \frac{\Delta V_o}{\Delta t} = \frac{\Delta V_{C_b}}{\Delta t} = 8.943 \times 10^{-5} \text{ V sec}^{-1}$  and  $I_{in} = 30 \text{ mA}$ . Therefore, one can find  $C_b = 335427.9 \text{ Farads}$ . The high value of  $C_b$  is because the battery stores huge amount of charge.

### Estimation of $R_t, C_s$ and $V_{C_b}$

In Fig. 1,  $R_t$  and  $C_s$  model the transient behavior of the battery and  $V_{C_b}(0)$  represents the initial condition of the voltage across the capacitor  $C_b$ . In terms of battery parameters, it represents the initial SOC. Hence, this estimation technique offers the estimation of initial SOC, so that the algorithm does not need to store the values from prior battery cycles. The only drawback in this estimation technique is that the input ( $I_{in}$ ) should be a time-varying function and can not be constant.

After taking derivative of Eq. (3) using sliding model method [7] one can find:

$$\frac{dV_o}{dt} = -\frac{1}{R_t C_s} \left( V_o - \frac{1}{C_b} \int_0^t I_{in} dt \right) + \left( \frac{1}{C_s} + \frac{1}{C_b} + \frac{R_i}{R_t C_s} \right) I_{in} - \frac{1}{R_t C_s} V_{C_b}(0) + \frac{1}{R_d} I_{in} \frac{dC_b}{dt} \quad (5)$$

In this work, we assume that  $C_b$  is constant, hence  $\frac{dC_b}{dt} = 0$ . Moreover,  $I_{in}$  is chosen as a sinusoidal function i.e.  $I_{in} = 30 \sin(60t)$ . The first simulation is to validate the evaluation of derivative of a function using sliding mode concept. For simplicity, denote  $a_1 = \left(\frac{1}{C_s} + \frac{1}{C_b} + \frac{R_t}{R_t C_s}\right)$ ,  $a_2 = \frac{1}{R_t C_s}$  and  $a_3 = \frac{1}{R_t C_s} V_{Cb}(0)$ . For this simulation the assigned values for the parameters are  $a_1 = 60$ ,  $a_2 = 20$  and  $a_3 = 40$ . That is, we have assumed  $V_{Cb}(0) = 2$  V and  $R_t = 1193.6 \Omega$  (These values are truly based on random assumption). Then  $\frac{dV_o}{dt}$  can be calculated from Eq. (5) shown in Fig. 2(a).

In the next step, the validation of estimation technique is performed. In this simulation,  $\frac{dV_o}{dt}$  is obtained from the sliding mode method (Eq. 5). Then convergence of the parameters to the actual value is checked. In the simulation, the estimated values are made to pass through a low-pass filter block so as to eliminate the high frequency components in the calculation.

From the results in figures 2(b), 2(c), and 2(d), we see that the asymptotic parameter estimations are  $a_1 = 21.023$ ,  $a_2 = 60.0$  and  $a_3 = 39.456$ . Thus we find that there is an error of around 5% in the values of the estimation. This error is mainly due to 4.5% error in the computation of  $\frac{dV_o}{dt}$ . These estimation are used to compute the value of  $R_t = 1194.24 \Omega$ ,  $C_s = 0.01674$  F and  $V_{Cb}(0) = 2$  V giving a respective error of 0.048%, 0.0% and 0.01%. Hence, we see that the errors significantly get reduced in the estimation of  $R_t$  and  $C_s$ . Consequently, the proposed estimation technique is quite accurate in the estimation of battery parameters such as SOC (related to  $C_b$ ) for sinusoidal current input.

## 4 Conclusions

With the rising importance of the lead-acid batteries, both in the automotive industry and the energy sector, it is of critical importance to develop more accurate models of the battery. This paper presents an adaptive model of the battery where the inherent battery parameters are estimated using only the input-output relations of the battery. The proposed estimation technique was employed for the estimation of model parameters. The estimation technique was implemented for sinusoidal current input. From the estimation results, the proposed technique accurately estimates the battery model parameters with an error of just 0.05 of the cases. Hence this method can be employed in battery monitoring algorithms for the online estimation of state of charge and predicted time to run for many applications.

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