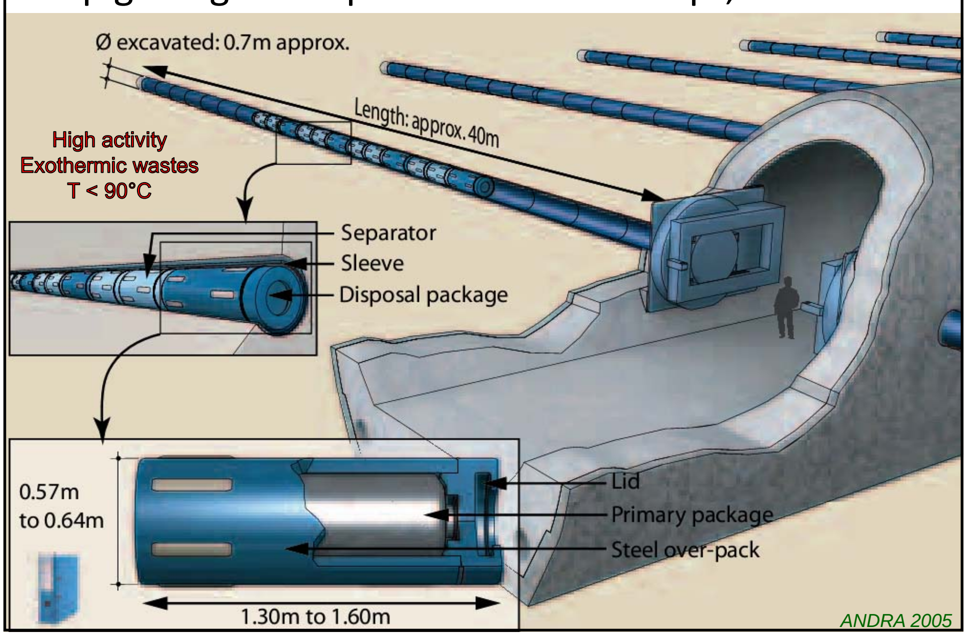
مکانیک خاک غیراشباع
انتقال حرارت

Heat Transfer

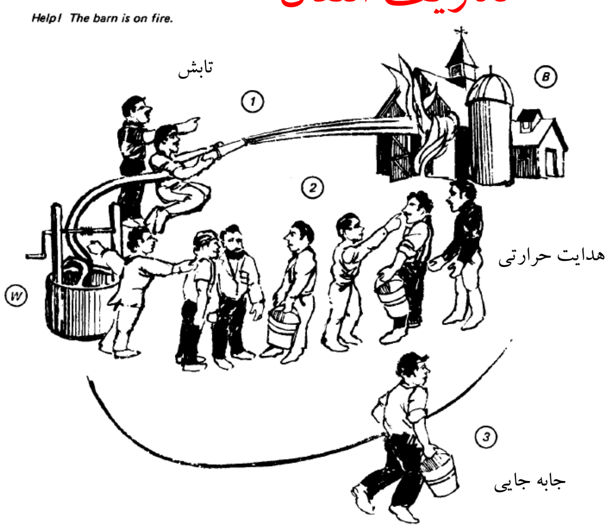
Hasan Ghasemzadeh

https://wp.kntu.ac.ir/ghasemzadeh/

Deep geological disposal: French concept, - 490m



تعاریف انتقال



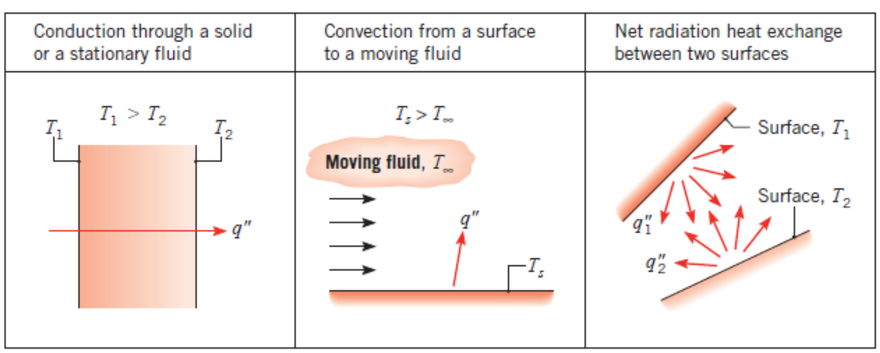
مودهای انتقال

هدایت حرارتی
جابه جایی
تابش

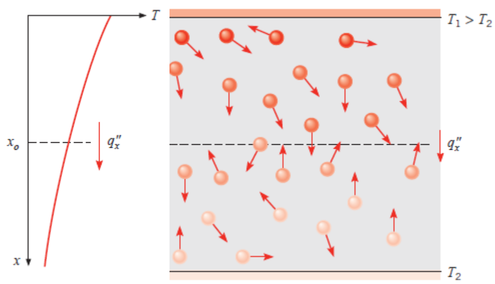
تعاریف انتقال

انتقال حرارت:

جابه‌جایی انرژی گرمایی به دلیل اختلاف دمای بین نقاط مختلف می‌باشد



هدایت حرارتی



$$q_x = -k \frac{dT}{dx}$$

kضریب هدایت گرمایی

$\mathrm{W/m \cdot {}^\circ K}$

$\mathrm{J/m \cdot s \cdot {}^\circ K}$

$\mathrm{Btu/h \cdot ft \cdot {}^\circ F}$

$$\vec{q} = -k \nabla T$$

شار حرارتی $q\left(\mathrm{W/m^2}\right)$

Conduction heat transfe: Diffusion of energy due to molecular activity

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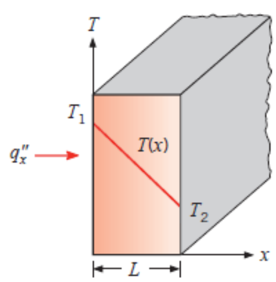
هدایت حرارتی



هدایت حرارتی

$$q_x = -k \frac{dT}{dx}$$

شار حرارتی $q_x\left(\mathrm{W/m^2}\right)$

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

$$q_x = -k \frac{T_2 - T_1}{L}$$

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هدایت حرارتی

ضریب هدایت حرارتی
در خاک غیر اشباع

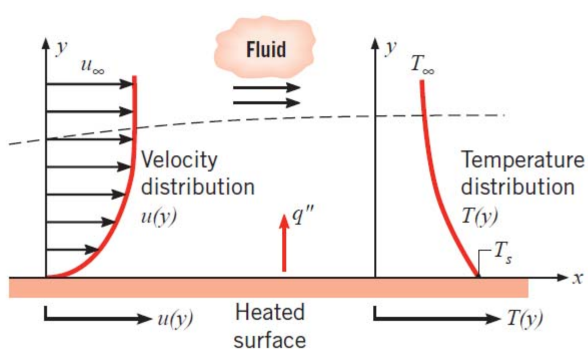
$$\lambda = (1-n)\lambda_s + \theta\lambda_w + (n-\theta)\lambda_v$$

$$\lambda = \frac{F_s\lambda_s(1-n) + F_w\lambda_w nS_w + F_a\lambda_a n(1-S_w)}{F_s(1-n) + F_w nS_w + F_a n(1-S_w)}$$

λ=thermal conductivity of the soil; λ_s=thermal conductivity of solids, typically around λ_s=6 (W/m°C); λ_w=thermal conductivity of water, typically around λ_w=0.57 (W/m°C); $\lambda_a=\lambda_{da}+\lambda_{va}$, where λ_{da}=thermal conductivity of dry air, typically ~λ_{da}=0.025 W/m°C and λ_{va}=thermal conductivity of water vapor, assumed as $\lambda_{va}=(0.0736)S_w$ (W/m°C); $F_{a,s}=1/3\Sigma_{i=1}^{3}[1+(\lambda_{a,s}/\lambda_w-1)g_i]^{-1}$; $F_w=1$ (water assumed as the continuum medium); $g_{1,2}=0.015+(0.333-0.015)S_w$ (assuming spherical particles); and $g_3=1-g_1-g_2$.

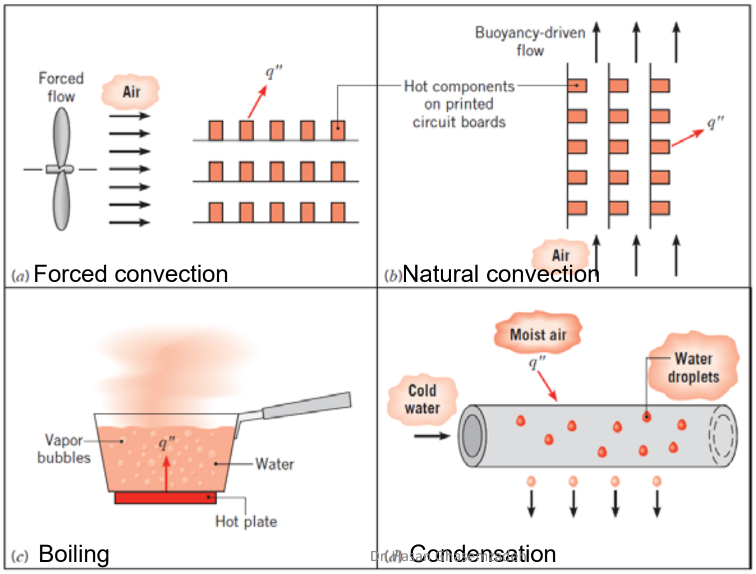
de Vries 1963

جابه‌جایی حرارت



The convection heat transfer :
Energy transfer due to *random molecular motion* (*diffusion*)
Plus
Bulk energy transfer, or *macroscopic, motion* of the fluid

جابه‌جایی حرارت



جابه‌جایی حرارت

$$q = h\left(T_s - T_\infty\right) \qquad T_s > T_\infty$$

convection heat transfer coefficient

Process	h $(\mathrm{W/m^2 \cdot K})$
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

جابه‌جایی حرارت

$$q = h\left(T_s - T_\infty\right) \qquad T_s > T_\infty$$

ظرفیت حرارتی
در خاک غیر اشباع

$$h = h_s(1-n) + h_w n S_w$$

de Vries 1963

heat capacity of saturated soil

$h_s = 2.235 \times 10^6 \ J/m^3 {}^\circ C$

$h_s = 4.154 \times 10^6 \ J/m^3 {}^\circ C \qquad at\, 35^\circ C$

C_T : specific heat capacity of unsaturated mixture

$$C_T = (1-n)\rho_s C_{ps} + \theta\rho_w C_{pw} + (n-\theta)\rho_v C_{pv} + (n-\theta)\rho_g C_{pg} + \theta C M_c C_{pc}$$

C_{ps}, C_{pw}, C_{pv}, C_{pg} and C_{pc} are the specific heat capacity of solid, liquid, vapour, gas and solute

M_c is the molar mass of solute

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انتقال به روش تابش

تابش تمام اجسام به صورت مداوم توسط فرایندی از تابش الکترومغناطیسی از خود انرژی ساطع می‌کنند

نوع	طول موج، λ
اشعه های کیهانی	$0.3\,\text{pm}>$
اشعه های گاما	$0.3-100$ pm
اشعه های X	$0.01-30$ pm
نور ماورابنفش	$3-400\,\text{nm}$
نور مرئی	$0.4-0.7\,\mu m$
تابش نزدیک به مادون قرمز	$0.7-0.30\,\mu m$
تابش دور از مادون قرمز	$30-1000\,\mu m$
امواج میلیمتری	$1-10\,\text{mm}$
میکروویو	$10-300\,\text{mm}$
امواج رادیویی و تلویزیونی کوتاه	$300\,\text{mm}-100\,\text{m}$
امواج رادیویی بلند	$100\,\text{m}-30\,\text{km}$

تابش حرارتی $0.1-1000\,\mu m$

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انتقال به روش تابش

تابش

Stefan Boltzmann law

$$E_b = \sigma T_s^{\,4}$$

For ideal radiator or *blackbody*

T_s *absolute temperature*

$\sigma = 5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}$ *Stefan Boltzmann constant*

For real surface

$$E = \varepsilon\sigma T_s^{\,4}$$

ε *Emissivity* of the surface

$$0 \le \varepsilon \le 1$$

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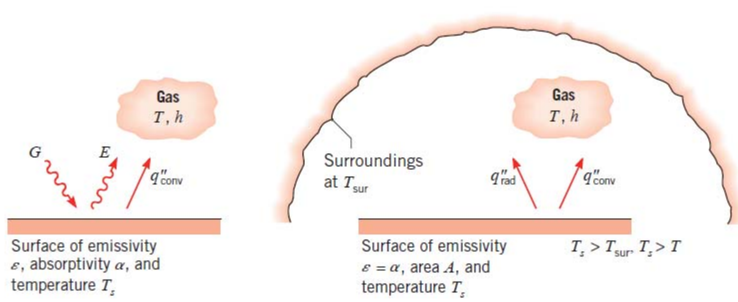
انتقال به روش تابش

تابش



$G_{abs} = \alpha G$ α *absorptivity*

اگر یک شار گرمایی به یک صفحه نیمه شفاف که سیاه نیست برخورد کند

$$q''_{rad} = \varepsilon E_b - \alpha G = \varepsilon\sigma(T_s^{\,4} - T_{sur}^{\,4})$$

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انتقال به روش تابش

تابش

$$q_{rad} = h_r A (T_s - T_{sur})$$

heat exchange

$$h_r = \varepsilon\sigma(T_s + T_{sur})(T_s^{\ 2} + T_{sur}^{\ 2})$$

linearized the radiation rate equation,
making the heat rate proportional to a temperature difference

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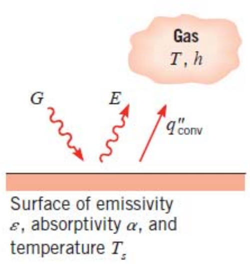
انتقال حرارت

تابش و همرفت همزمان



$$q = q_{conv} + q_{rad} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^{\ 4} - T_{sur}^{\ 4})$$

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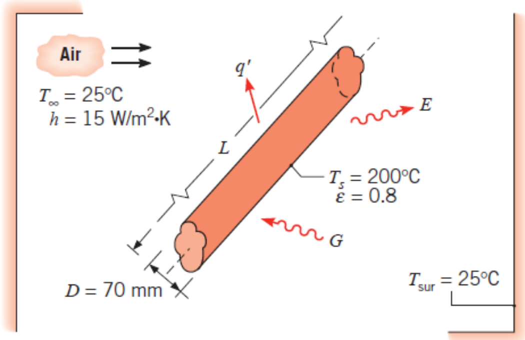
انتقال حرارت

مثال



Known: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures

Find:
1. Surface emissive power and irradiation.
2. Pipe heat loss per unit length, .

Assumptions:
1. Steady-state conditions.
2. Radiation exchange between the pipe and the room is between a small surface and a much larger enclosure.
3. The surface emissivity and absorptivity are equal.

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انتقال حرارت

مثال

Analysis:

$$E = \varepsilon\sigma T_s^4 = 0.8(5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(473\ \mathrm{K})^4 = 2270\ \mathrm{W/m^2}$$

$$G = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\,(298\ \mathrm{K})^4 = 447\ \mathrm{W/m^2}$$

Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls.

$$q = q_{conv} + q_{rad}$$

$$A = \pi DL$$

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma(T_s^4 - T_{sur}^4)$$

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انتقال حرارت

مثال

Analysis:

The heat loss per unit length of pipe is then

$$q' = \frac{q}{L} = 15\ \mathrm{W/m^2 \cdot K}(\pi \times 0.07\ \mathrm{m})(200 - 25)^\circ\mathrm{C}$$
$$+\ 0.8(\pi \times 0.07\ \mathrm{m})\ 5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}\ (473^4 - 298^4)\ \mathrm{K^4}$$
$$q' = 577\ \mathrm{W/m} + 421\ \mathrm{W/m} = 998\ \mathrm{W/m}$$

The net rate of radiation heat transfer from the pipe

$$q'_{\mathrm{rad}} = \pi D\ (E - \alpha G)$$
$$q'_{\mathrm{rad}} = \pi \times 0.07\ \mathrm{m}\ (2270 - 0.8 \times 447)\ \mathrm{W/m^2} = 421\ \mathrm{W/m}$$

بقای انرژی

معادله بقای انرژی در خاک غیراشباع

$$\frac{\partial \varphi}{\partial t} + \mathrm{div}Q = 0$$

φ : the volumetric bulk heat content of medium $\varphi = C_T(T - T_0) + (n - \theta)\rho_v h_{fg}$

h_{fg} : the latent heat of vaporization

C_T : specific heat capacity of unsaturated mixture

$$C_T = (1-n)\rho_s C_{ps} + \theta\rho_w C_{pw} + (n-\theta)\rho_v C_{pv} + (n-\theta)\rho_g C_{pg} + \theta C M_c C_{pc}$$

C_{ps}, C_{pw}, C_{pv}, C_{pg} and C_{pc} are the specific heat capacity of solid, liquid, vapour, gas and solute

M_c is the molar mass of solute

Q : the heat flow

$$Q = -\lambda gradT + \left[C_{Pw}\rho_w U + C_{pv}\rho_w V + C_{pg}\rho_g V_g + C_{Pc}(q_c + q_{diff})\right](T - T_0) + \rho_w h_{fg} V + \rho_v V_g h_{fg}$$

جریان گرما درخاک

جریان گرما درخاک متاثر از هدایت و همرفت بوده و با تابش بطور ناچیز مرتبط است

هدایت حرارتی جسم جامد خاک بسیار بیشتر از آب موجود در خاک و هوا می باشد(هدایت حرارتی خاک برابر با ۰/۱۵-۱/۵و هدایت حرارتی آب ۰/۳و برای هوا ۰/۰۱۴
$BTu/hr/ft^2/°F/ft$

انتقال یا جابجایی حرارت تنها در صورت وجود نرخ بالای جریان آب یا هوا (مانند جریان در سنگریزه ها و مصالح ماسه ای درشت دانه) معمولا مهم می شود

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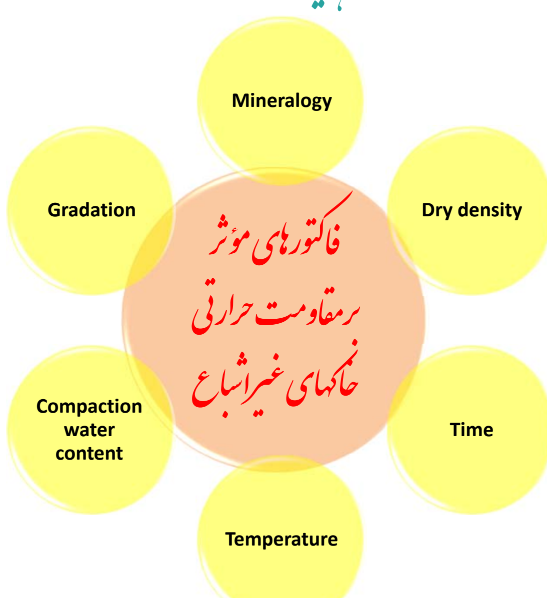
هدایت گرمائی



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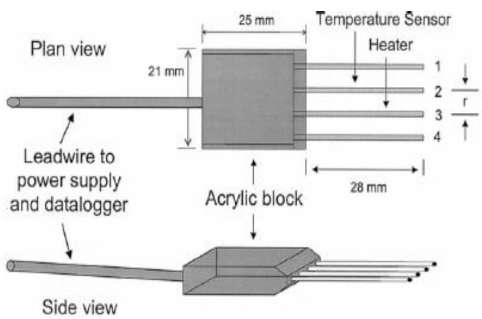
اندازه گیری خواص حرارتی

برای ارزیابی خواص حرارتی خاکها ، از آنالیز تغییرات دمائی خاکها در عمق می توان استفاده نمود

معمولا روش کاربردی، اندازه گیری در محل با استفاده از سیستم پروب گرمائی است.

دستگاه مورد استفاده ، یک لوله نمونه گیری چند سوزنه (چهار حسگر موازی جریان جاری) است.

از حسگرها برای اندازه گیری خواص حرارتی خاک و نیز محتوای آب خاک استفاده می‌شود



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اندازه گیری خواص حرارتی

اندازه گیری مقاومت حرارتی خاک در آزمایشگاه با استفاده از سوزن حرارتی آزمایشگاهی :
یک سوزن یا پروب حرارتی به عنوان منبع گرما با ورودی Q در واحد طول با مقاومت ثابت در یک محیط همگن نامحدود خاک با دمای یکنواخت

مدل ریاضی

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial T}{\partial r}\right)$$

T دمای جسم خاک

t مدت زمان گرمایش

r فاصله شعاعی از منبع گرماست

α ثابت پخش شدگی حرارتی

نرخ تغییرات دما، ΔT ، در فاصله زمانی

$$\Delta T = \left(\frac{Q}{4\pi k}\right)\mathrm{Log}_e\left(\frac{t_2}{t_1}\right)$$

شیب قسمت مستقیم گراف دما در مقابل لگاریتم زمان: $\frac{Q}{4\pi k}$

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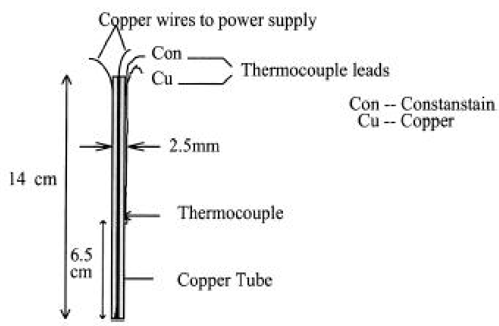
اندازه گیری خواص حرارتی

خواص میانگین جسم که عبارتست از هدایت حرارتی ، در این عبارت وارد شده و دیگر عبارتها در این معادله مقادیر قابل اندازه گیری توسط پروب حرارتی است

این روش برای تخمین مقدار مقاومت حرارتی نمونه های خاک خشک و مرطوب مورد استفاده قرار می گیرد



مزایای روش :

پروب کوچک و قابل حمل

ساخت و اجرای نسبتاً ارزان

آزمایشات در مدت زمان کوتاهی

اپراتور نیاز به مهارت و آموزش کم دارد

نیاز به محاسبات بسیار پیچیده نیست

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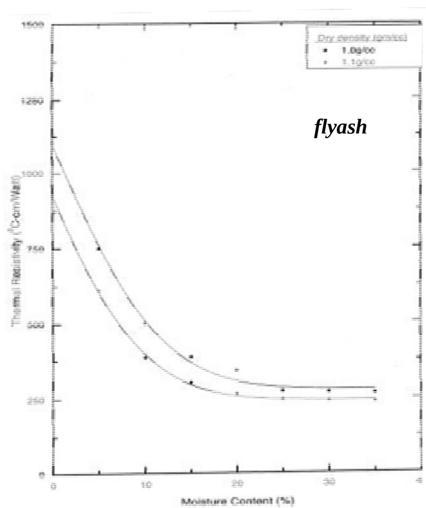
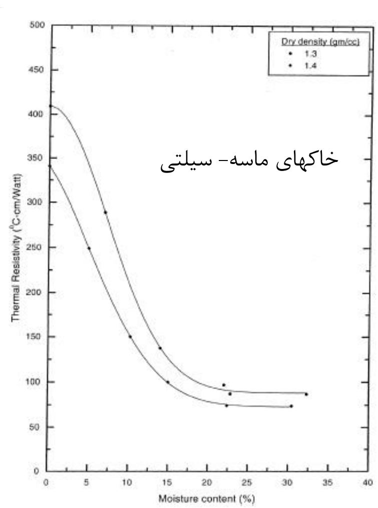
اندازه گیری خواص حرارتی

تغییرات مقاومت حرارتی با محتوای حجمی آب در خاک برای چند نمونه از خاکها





در حالت کلی با افزایش تراکم خاک، مقاومت کاهش می یابد

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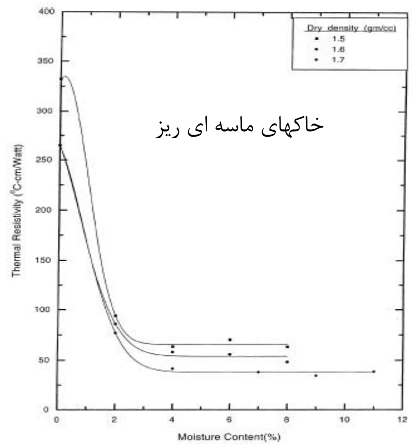
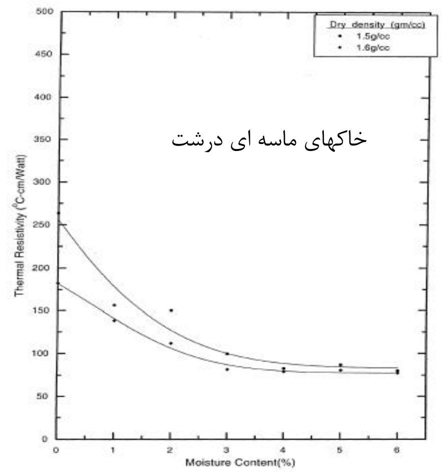
اندازه گیری خواص حرارتی

تغییرات مقاومت حرارتی با محتوای حجمی آب در خاک برای چند نمونه از خاکها





در حالت کلی برای یک تراکم مشخص از خاک ، با افزایش رطوبت خاک ، مقاومت کاهش می یابد

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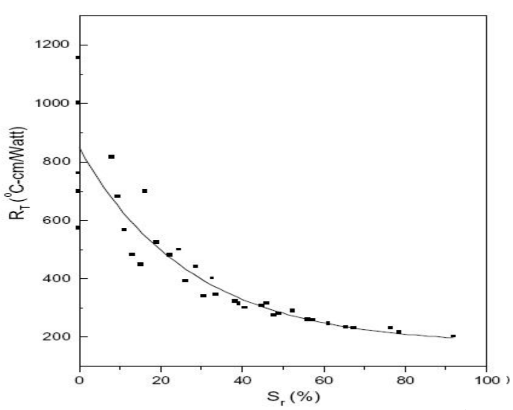
مقاومت حرارتی خاک

مقاومت حرارتی خاک برای انواع خاکها با درجات اشباع مختلف ، با استفاده از روابط کلی مقاومت حرارتی



تغییرات مقاومت حرارتی با درجه اشباع برای خاکهای ماسه- سیلتی

تغییرات مقاومت حرارتی با درجه اشباع برای خاکهای **Black Cotton**

نتیجه: هرچه درجه اشباع خاک بیشتر ، مقاومت حرارتی کمتر

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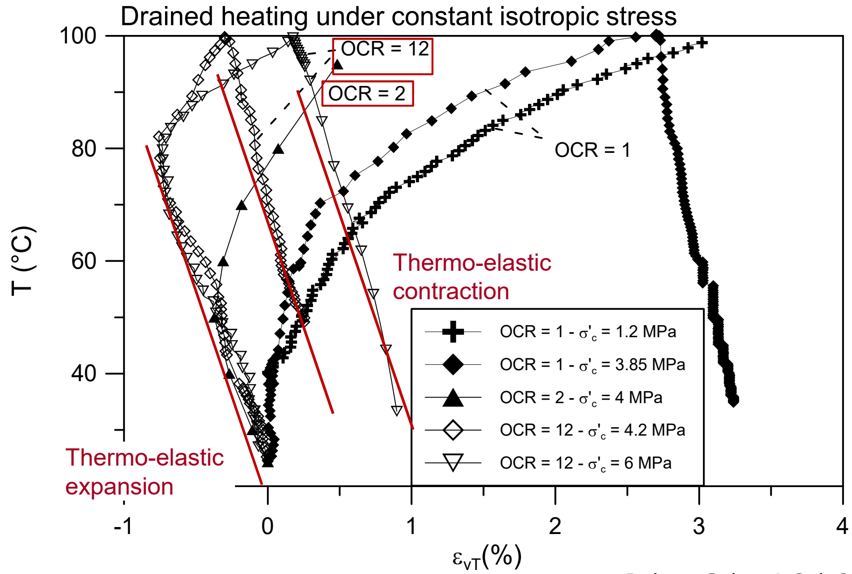
Boom Clay, thermal volumetric changes



Delage, Sultan & Cui, CGJ 2000

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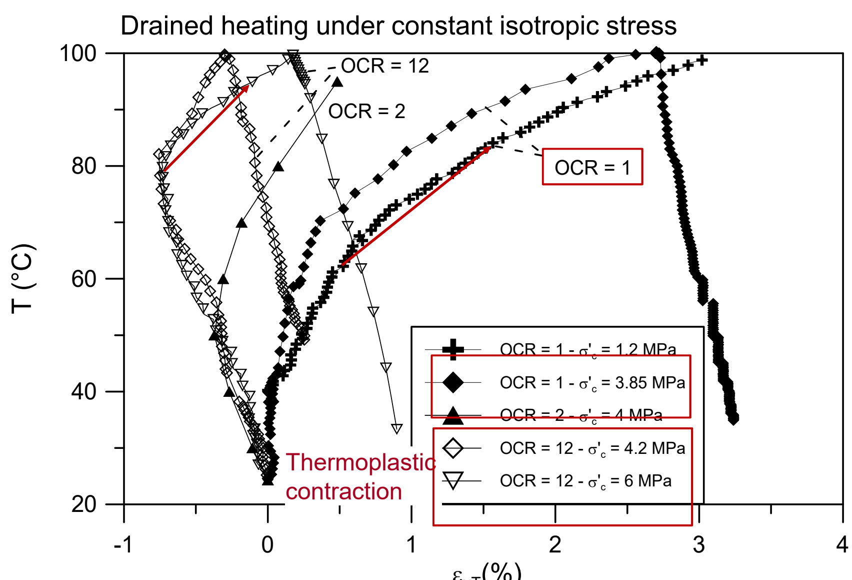
Boom Clay, thermal volumetric changes



Delage, Sultan & Cui, CGJ 2000

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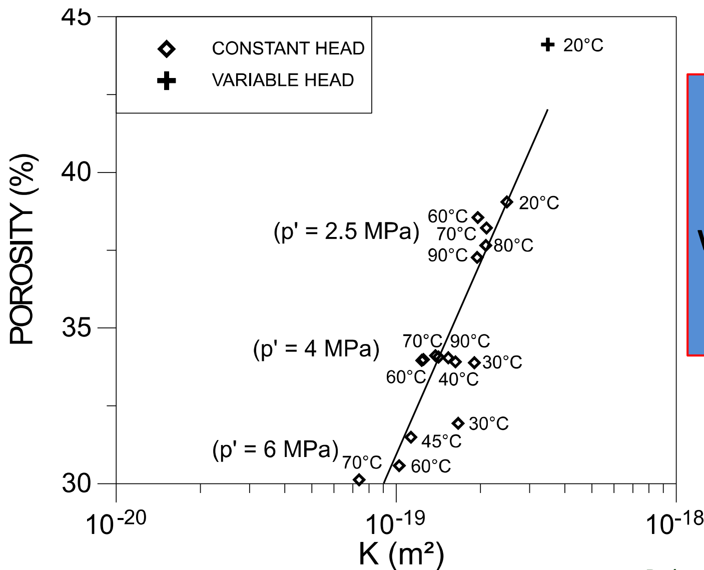
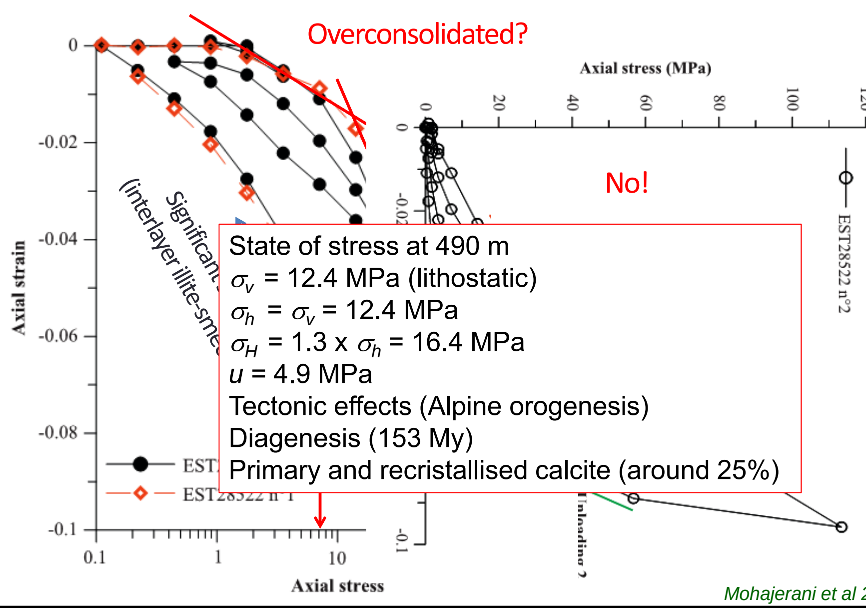
Boom Clay, effects of temperature on the intrinsic permeability



Changes only due to changes in water viscosity with temperature

Delage, Sultan & Cui, CGJ 2000

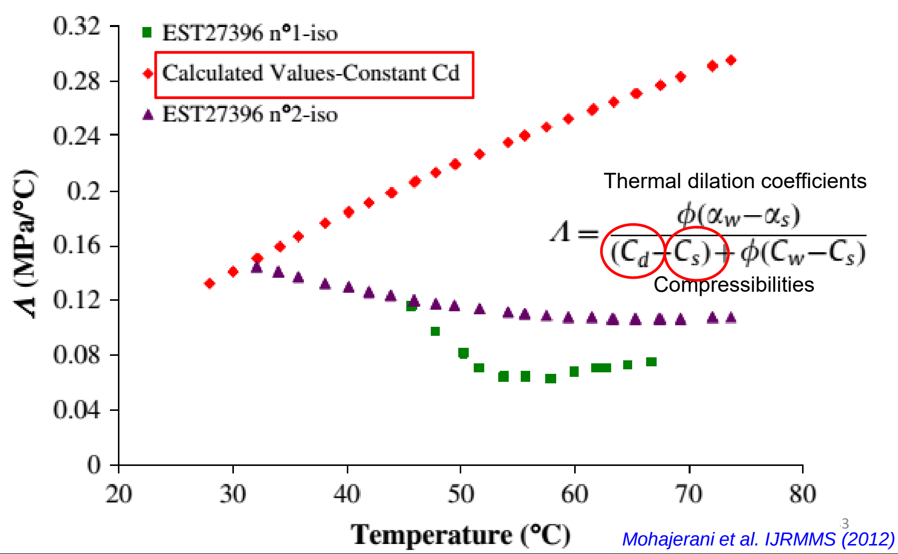
Compression behaviour (oedometer)



State of stress at 490 m
σ_v = 12.4 MPa (lithostatic)
σ_h = σ_v = 12.4 MPa
σ_H = 1.3 x σ_h = 16.4 MPa
u = 4.9 MPa
Tectonic effects (Alpine orogenesis)
Diagenesis (153 My)
Primary and recristallised calcite (around 25%)

Mohajerani et al 2011

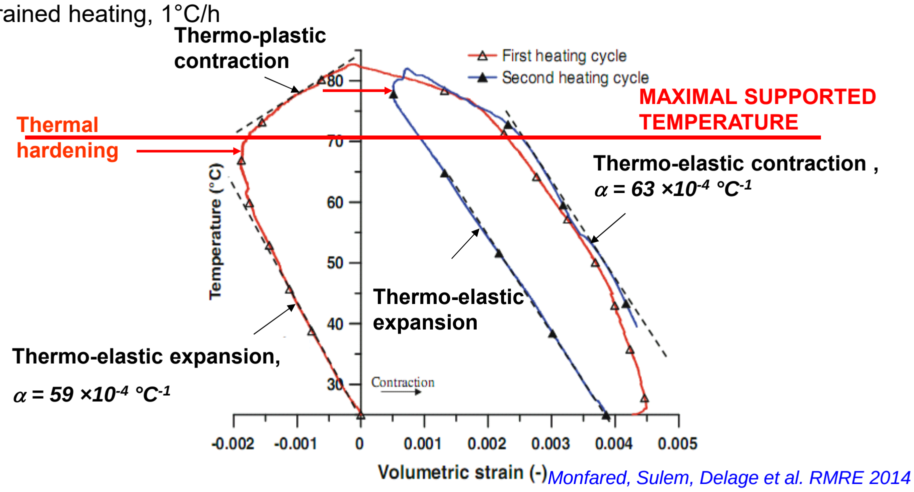
Thermal pressurization coefficient Λ, COx claystone



Mohajerani et al. IJRMMS (2012)

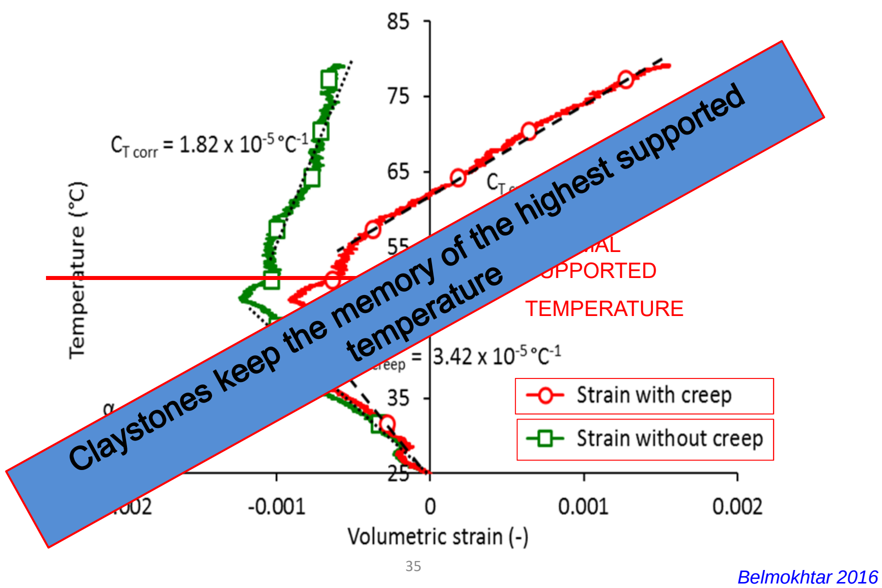
Thermal hardening, Opalinus clay

In situ stresses Opalinus
Confinement = 4.1 MPa
Pore pressure = 2.2 MPa
Drained heating, 1°C/h



Monfared, Sulem, Delage et al. RMRE 2014

Thermal hardening, Callovo-Oxfordian claystone



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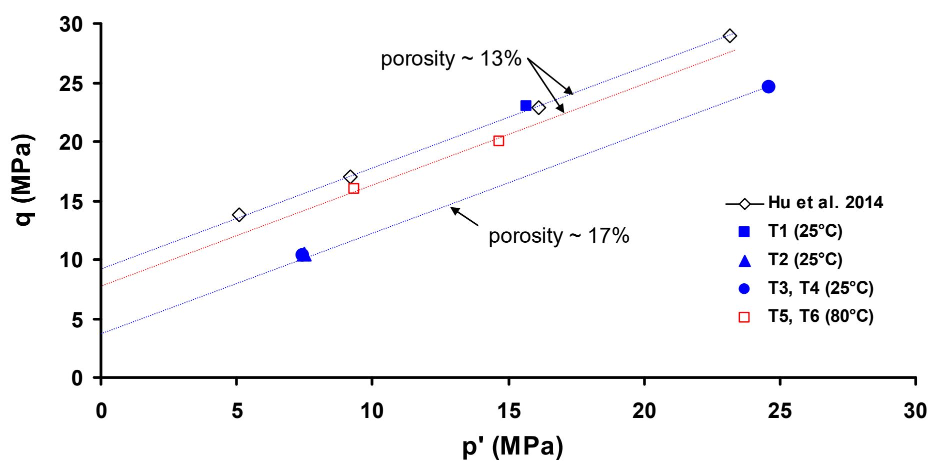
Belmokhtar 2016

Failure criterion (temperature), COx



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Menaceur et al. IJRMMS 2015

Heat transfer example

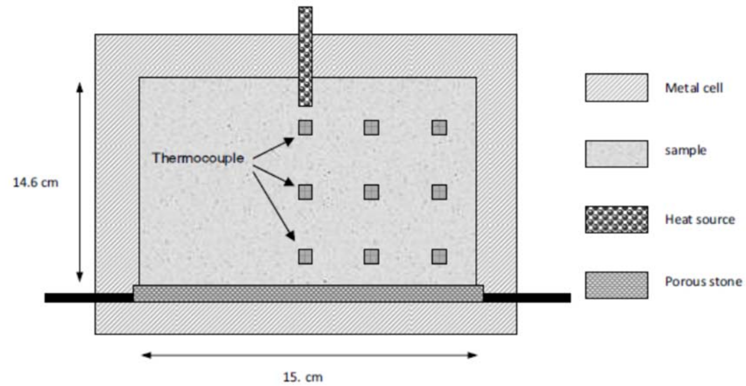

Fig. 2 Schematic diagram of heat transfer cell (Villar et al., 1993)

Heat transfer example

Table 1 Physical characteristics of the medium

Properties	Unit	Values
Initial degree of saturation	S_r (Cm^3/Cm^3)	0.5
Porosity	n (Cm^3/Cm^3)	0.73
Density of grain	ρ (kg/m^3)	2780
Calorific capacity of soil grain	C_{ps} (J/kg°C)	800
Calorific capacity of water	C_{pw} (J/kg°C)	4180
Calorific capacity of vapour	C_{pv} (J/kg°C)	1870
Calorific capacity of air	C_{pg} (J/kg°C)	1000
Thermal conductivity of soil grain	λ_s (W/m°C)	0.9
Thermal conductivity of water	λ_w (W/m°C)	0.6
Thermal conductivity of gas	λ_g (W/m°C)	0.0258
Latent heat of evaporation	h_{fg} (J/kg)	2.40 % 10^6

Heat transfer example

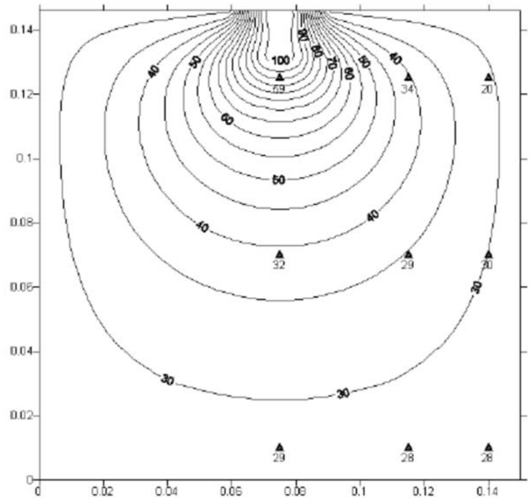

Fig. 3 Experimental (Villar et al., 1993) and calculated temperature

Heat transfer example

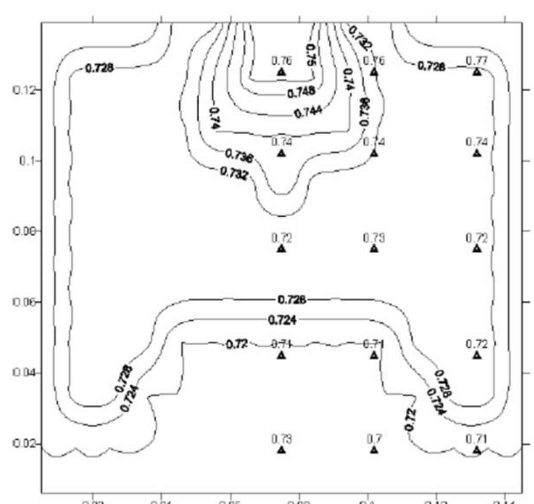

Fig. 5 Experimental (Villar et al., 1993) and calculated void ratio

Heat transfer example

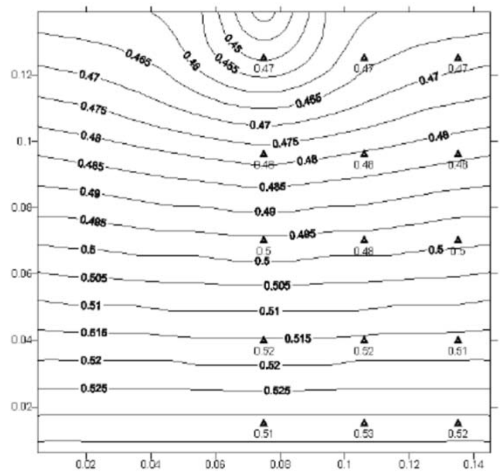

Fig. 4 Experimental (Villar et al., 1993) and calculated degree of saturation

Heat transfer example

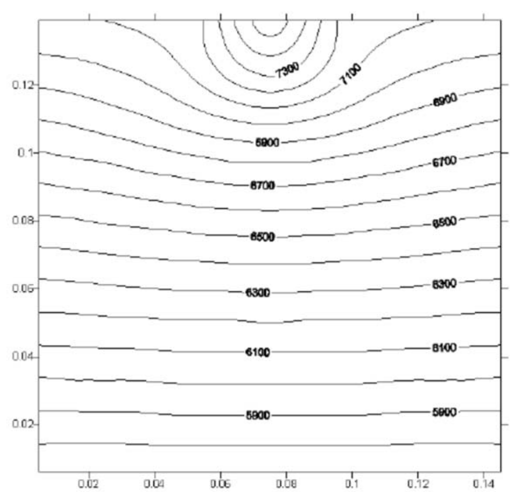

Fig. 6 Calculated suction in present study

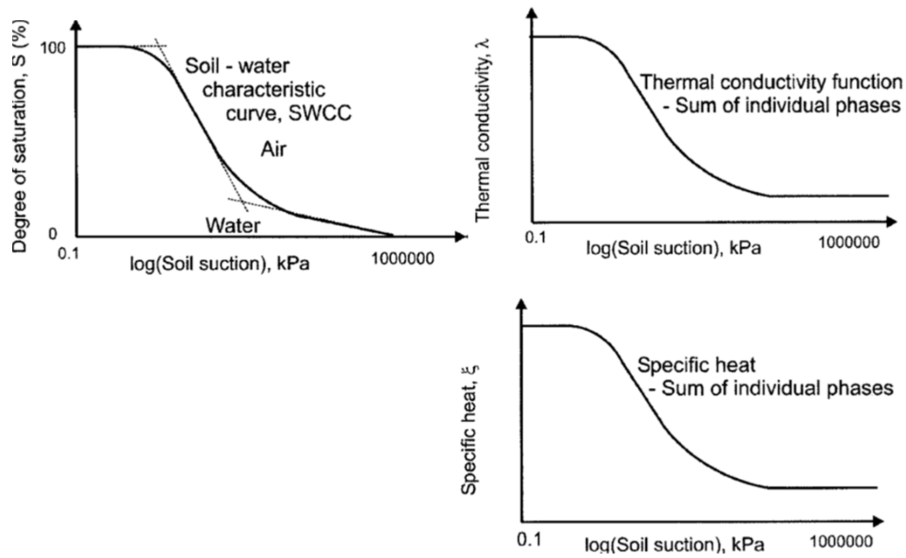

Table 7. Functions for Heat Capacity and Thermal Conductivity of an Unsaturated Soil

Reference	Equation	Description
de Vries (1963)	$\zeta=\zeta_s(1-n)+\zeta_w n S_w$ (heat capacity of air phase is neglected)	ζ=heat capacity of the soil; ζ_s=volumetric specific heat of solids, 2.235×10^6 [J/m^3°C]; and ζ_w=volumetric specific heat of water, 4.154×10^6 at 35°C [J/m^3°C].
de Vries (1963)	$\lambda=\frac{F_s\lambda_s(1-n)+F_w\lambda_w n S_w+F_a\lambda_a n(1-S_w)}{F_s(1-n)+F_w n S_w+F_a n(1-S_w)}$	λ=thermal conductivity of the soil; λ_s=thermal conductivity of solids, typically around $\lambda_s=6$ (W/m°C); λ_w=thermal conductivity of water, typically around $\lambda_w=0.57$ (W/m°C); $\lambda_a=\lambda_{da}+\lambda_{va}$, where λ_{da}=thermal conductivity of dry air, typically $\sim\lambda_{da}=0.025$ W/m°C and λ_{va}=thermal conductivity of water vapor, assumed as $\lambda_{va}=(0.0736)S_w$ (W/m°C); $F_{a,s}=1/3\Sigma_{i=1}^{3}[1+(\lambda_{a,s}/\lambda_w-1)g_i]^{-1}$; $F_w=1$ (water assumed as the continuum medium); $g_{1,2}=0.015+(0.333-0.015)S_w$ (assuming spherical particles); and $g_3=1-g_1-g_2$.

Note: n=soil porosity and S_w=water degree of saturation.

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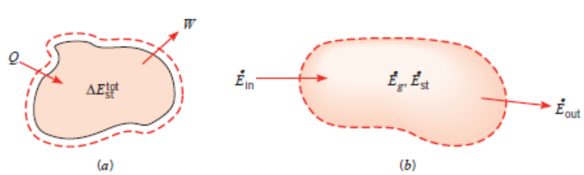
First Law of Thermodynamics

Conservation of energy

closed system *or* a region of fixed mass
open system or *control volume*

The increase in the amount of energy stored in a control volume must equal the amount of energy that enters the control volume, minus the amount of energy that leaves the control volume



closed system over a time interval — a control volume at an instant

$$dQ = dU + dw \qquad \Delta E_{st}^{tot} = Q - W$$

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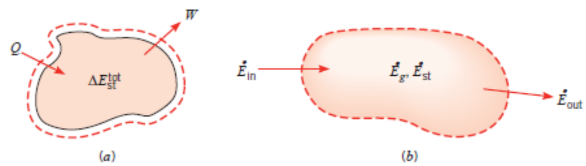
First Law of Thermodynamics

Conservation of energy



$$\Delta E_{st}^{tot} = E_{in} - E_{out} + E_g$$

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

نرخ تولید انرژی $\dot{E}_g$

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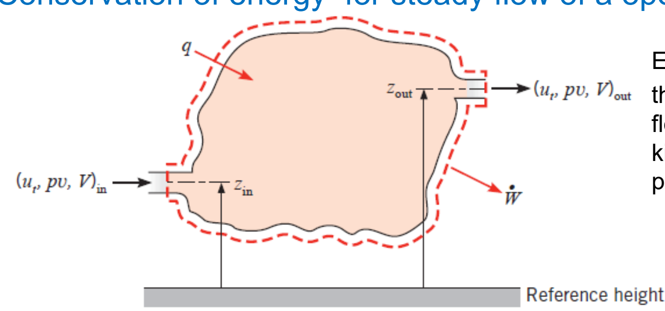
First Law of Thermodynamics

Conservation of energy for steady flow of a open system



Energy per unit mass
thermal,
flow work,
kinetic,
potential

$$\dot{m}(u_t + pv + 1/2V^2 + gz)_{in} - \dot{m}(u_t + pv + 1/2V^2 + gz)_{out} + q - \dot{W} = 0$$

$\dot{m}$ mass flow rate

$i = u_t + pv$ Enthalpy per unit mass

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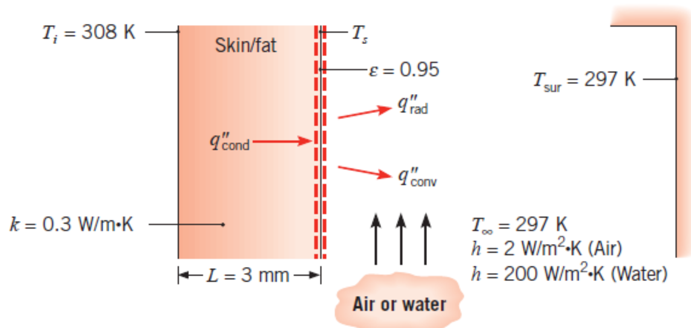
Example person is in air and water

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, emissivity, and surface area. Ambient conditions.



Find: Skin surface temperature and heat loss rate for the person in air and the person in water.

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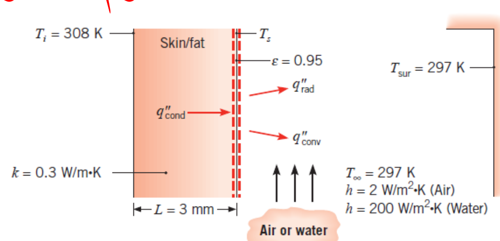
Example



Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat layer.
3. Thermal conductivity is uniform.
4. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
5. Liquid water is opaque to thermal radiation.
6. Bathing suit has no effect on heat loss from body.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

$$q''_{cond} - q''_{conv} + q''_{rad} = 0$$

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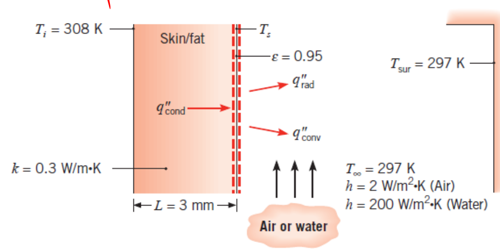
Example



$$E_{in} - E_{out} = 0$$

$$q''_{cond} - q''_{conv} + q''_{rad} = 0$$

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{sur}^4)$$

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + h_r(T_s - T_{sur})$$

$$T_s = \frac{\frac{kT_i}{L} + (h + h_r)T_\infty}{\frac{k}{L} + (h + h_r)}$$

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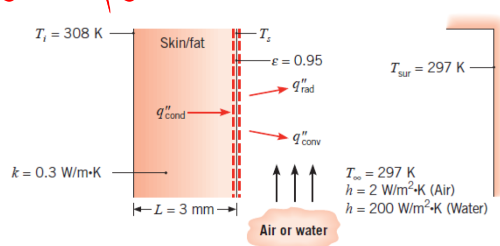
Example



person is in air

$$h_r = \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2) = 5.9\ W/m^2\,^\circ K$$

$$T_s = \frac{\frac{0.3\text{ W/m}\cdot\text{K} \times 308\text{ K}}{3 \times 10^{-3}\text{ m}} + (2 + 5.9)\text{ W/m}^2\cdot\text{K} \times 297\text{ K}}{\frac{0.3\text{ W/m}\cdot\text{K}}{3 \times 10^{-3}\text{ m}} + (2 + 5.9)\text{ W/m}^2\cdot\text{K}} = 307.2\text{ K}$$

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\text{ W/m}\cdot\text{K} \times 1.8\text{ m}^2 \times \frac{(308 - 307.2)\text{ K}}{3 \times 10^{-3}\text{ m}} = 146\text{ W} = 37 + 109$$

Radiation is not negligible

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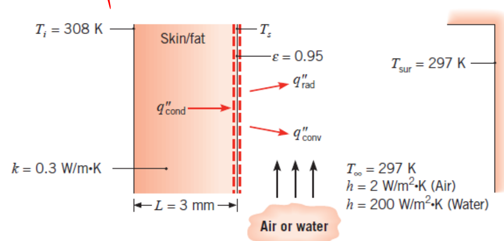
Example



person is in water

liquid water is opaque to thermal radiation, heat loss from the skin surface is by convection only. $h_r = 0$

$$T_s = \frac{\dfrac{0.3\ \text{W/m}\cdot\text{K} \times 308\ \text{K}}{3\times10^{-3}\ \text{m}} + 200\ \text{W/m}^2\cdot\text{K}\times 297\ \text{K}}{\dfrac{0.3\ \text{W/m}\cdot\text{K}}{3\times10^{-3}\ \text{m}} + 200\ \text{W/m}^2\cdot\text{K}} = 300.7\ \text{K}$$

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\ \text{W/m}\cdot\text{K} \times 1.8\ \text{m}^2 \times \frac{(308-300.7)\ \text{K}}{3\times10^{-3}\ \text{m}} = 1320\ \text{W}$$

typical rate of metabolic heat generation is 100 W

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Second Law of Thermodynamics

naturally occurring processes are directional & irreversible

The efficiency of a heat engine is defined as the fraction of heat transferred into the system that is converted to work

$$\eta = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

$$\eta_c = 1 - \frac{T_c}{T_h}$$

Carnot efficiency is the maximum possible efficiency that any heat engine can achieve operating between those two temperatures

T_c and T_h are the absolute temperatures of the low- and high-temperature reservoirs

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Second Law of Thermodynamics

$$\eta_m = 1 - \frac{Q_{out}}{Q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_{c,i}}{T_{h,i}}$$

Modified efficiency for (irreversible) heat transfer processes

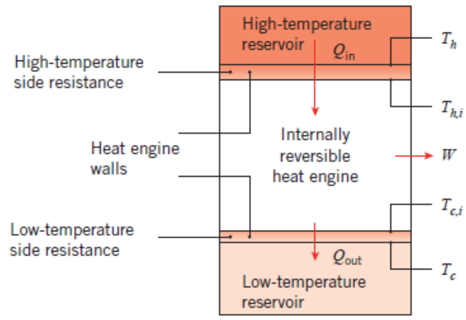
$$q_{in} = (T_h - T_{h,i})/R_{t,h}$$

$$q_{out} = (T_{c,i} - T_c)/R_{t,c}$$

$R_{t,h}$, $R_{t,c}$ Thermal resistance

$$T_{h,i} = T_h - q_{in} R_{t,h}$$

$$T_{c,i} = T_c + q_{out} R_{t,c} = T_c + q_{in}(1 - \eta_m) R_{t,c}$$



Internally reversible heat engine exchanging heat with high- and low-temperature reservoirs through thermal resistances

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Second Law of Thermodynamics

$$\eta_m = 1 - \frac{T_c}{T_h - q_{in} R_{tot}} \qquad R_{tot} = R_{t,h} + R_{t,c}$$

$$R_{t,h} = R_{t,c} = 0 \text{ or } q_{in} = 0 \rightarrow \eta_m = \eta_c$$

For realistic case $R_{tot} \neq 0 \rightarrow \eta_m < \eta_c$

$T_h = T_c + q_{in} R_{tot} \rightarrow \eta_m = 0$ no power could be produced even though the Carnot efficiency is nonzero

power output $\dot{W} = q_{in} \eta_m = q_{in} \left[1 - \frac{T_c}{T_h - q_{in} R_{tot}} \right]$

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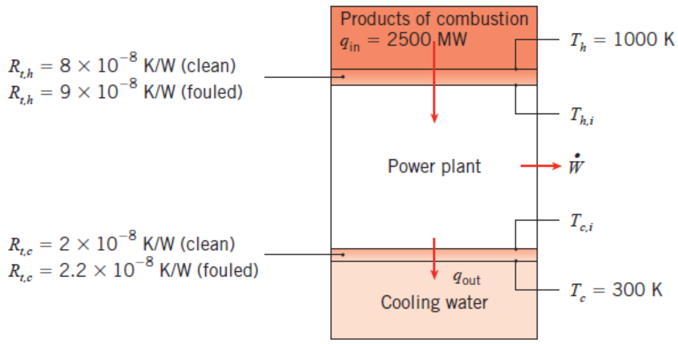
Example large steam power plant

Known: Source and sink temperatures and heat input rate for an internally reversible heat engine. Thermal resistances separating heat engine from source and sink under clean and fouled conditions.



Find:

1. Efficiency and power output for clean conditions.
2. Efficiency and power output under fouled conditions.

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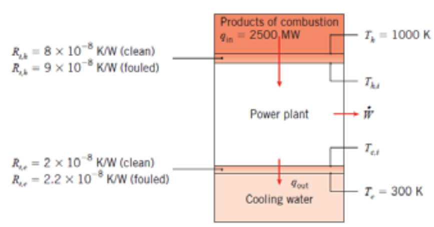
Example large steam power plant



for clean conditions

$$R_{tot} = R_{t,h} + R_{t,c} = 8 \times 10^{-8}\ \text{K/W} + 2 \times 10^{-8}\ \text{K/W} = 1.0 \times 10^{-7}\ \text{K/W}$$

$$\eta_m = 1 - \frac{T_c}{T_h - q_{in}R_{tot}} = 1 - \frac{300\ \text{K}}{1000\ \text{K} - 2500 \times 10^6\ \text{W} \times 1.0 \times 10^{-7}\ \text{K/W}} = 0.60 = 60\%$$

$$\dot{W} = q_{in}\eta_m = 2500\ \text{MW} \times 0.60 = 1500\ \text{MW}$$

Under fouled conditions

$$\eta_m = 0.583 = 58.3\% \text{ and } \dot{W} = 1460\ \text{MW}$$

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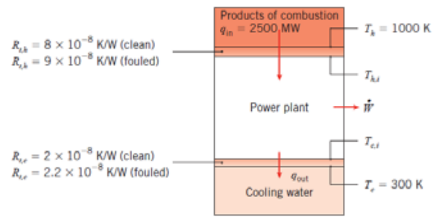
Example large steam power plant



Comments:

1. The actual efficiency and power output of a power plant would be much less than the foregoing values, since there would be other irreversibilities internal to the power plant

2. The Carnot efficiency

$$\eta_C = 1 - T_c/T_h = 1 - 300\text{ K}/1000\text{ K} = 70\%$$

$$\dot{W} = q_{in}\eta_C = 2500\text{ MW} \times 0.70 = 1750\text{ MW}$$

3. Fouling reduces the power output of the plant by 40 MW and electricity at a price of \$0.08/kWh

C =40,000 kW * \$0.08/kWh * 24 h/day = \$76,800/day daily lost revenue

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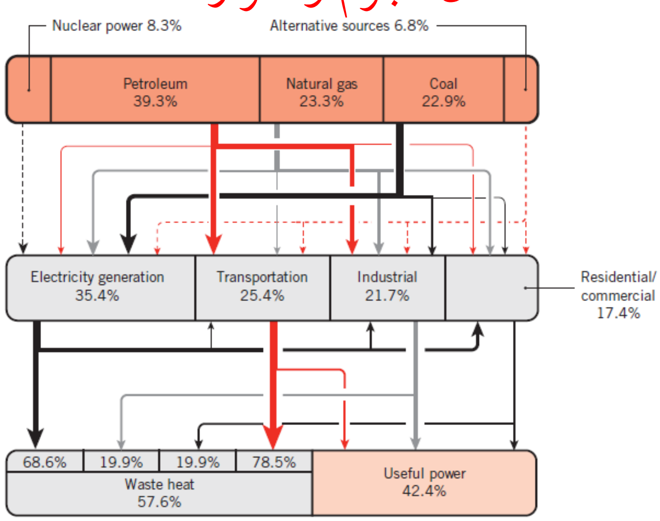

Energy production and consumption

United States in 2007

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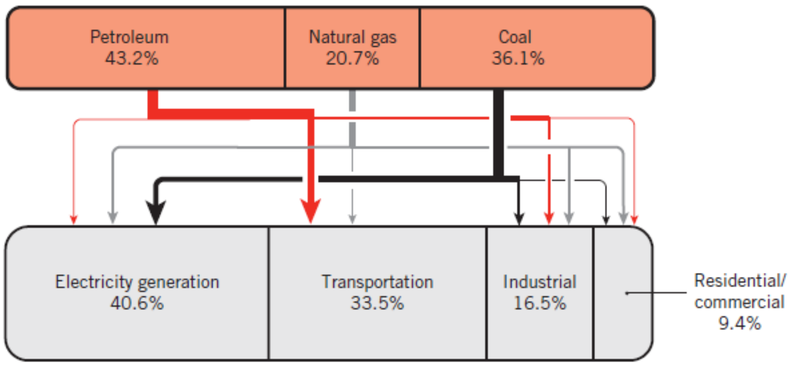

Carbon dioxide by source of fossil fuel and end-use application United States in 2007

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- Summary of heat transfer processes

Mode	Mechanism(s)	Rate Equation
Conduction	Diffusion of energy due to random molecular motion	$q''_x(\text{W/m}^2) = -k\dfrac{dT}{dx}$
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(\text{W/m}^2) = h(T_s - T_\infty)$
Radiation	Energy transfer by electromagnetic waves	$q''(\text{W/m}^2) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q(\text{W}) = h_r A(T_s - T_{sur})$

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