

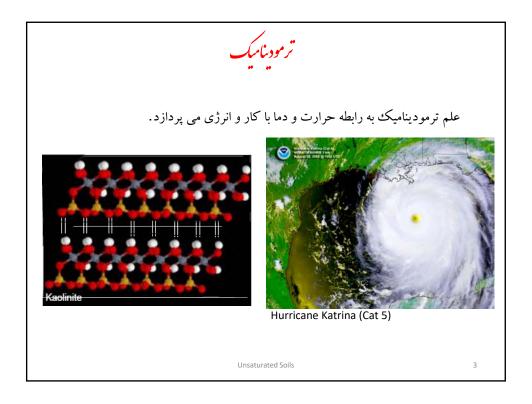
Unsaturated soils Thermodynamics

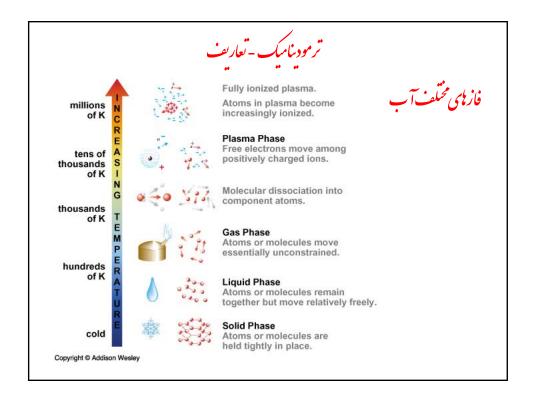
Hasan Ghasemzadeh

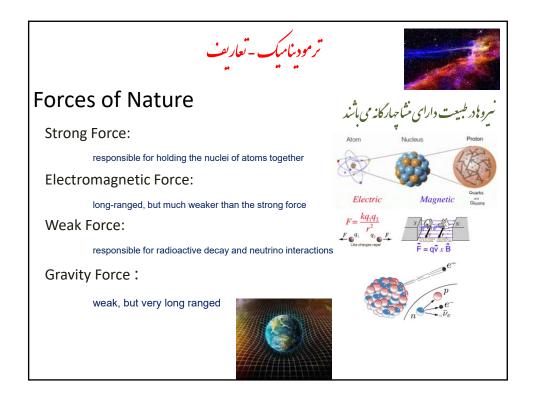
فهرست عناوین و فصول

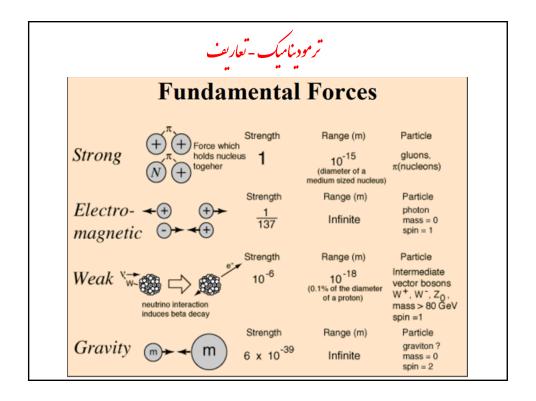
ا مقدمه – آشنایی – مختصری از ترمودینامیک
 ۲ – رفتار فازهای مختلف درخاک غیر اشباع
 ۳ – اندازه گیری در خاک غیر اشباع
 ۲ – نتایج آزمایشات خاک غیر اشباع
 ۵ – تنش موثر و کرنش
 ۶ – تئوری های خاک غیر اشباع
 ۷ – جریان در خاک غیر اشباع
 ۸ – کاربرد خاک غیر اشباع در مهندسی

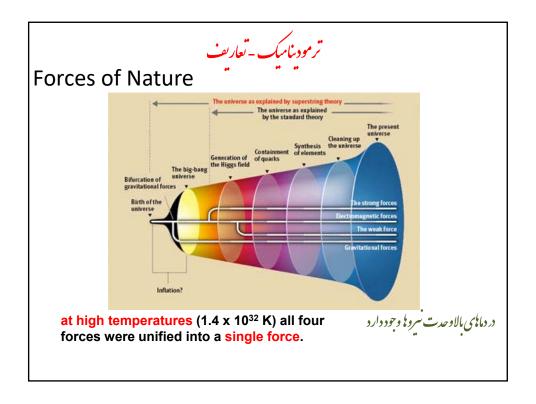
Unsaturated Soils

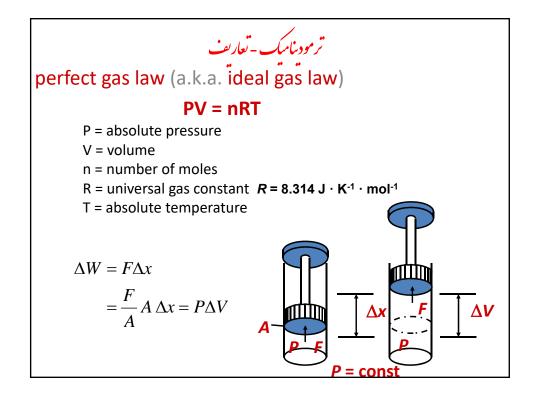












ترموديناميك - تعاريف

Internal Energy, U, sum of potential energy and kinetic energy

Enthalpy, H, The amount of energy in a system capable of doing mechanical work

Entropy, S, a measure of the randomness of a system. a measure of the unavailability of a system's energy to do work

State function a property of a system that depends only on the current state of the system, not on the way in which the system got to that state.

Pressure, P, Temperature, T, Volume, V, Internal energy, *E*, enthalpy, *H* and entropy, S, are the state functions and path independent.

Internal energy U(S,V)

Enthalpy H(S,p) = U + PVEntropy dS = dq/T



Thermodynamic free energy, the total amount of energy in a physical system which can be converted to do work, in particular:

Helmholtz free energy, the amount of thermodynamic energy in a system which can be converted into work at constant temperature

Gibbs free energy, the amount of thermodynamic energy in a system which can be converted into work at a constant temperature and pressure

Helmholtz free energy A(T,V) = U - TSGibbs free energy G(T,p) = U + PV - TS

T = Temperature in Kelvins

ترمودنامیک- جداول

Table 3.3 Excess Free Energies, Enthalpies, and Entropies of Hexane (apolar), Benzene (monopolar), Diethylether (monopolar), and Ethanol (bipolar) in the Ideal Gas Phase, in Hexadecane, and in Water at Infinite Dilution.^a All Data at 25°C. Reference: Pure Liquid Organic Compound.

Phase Compound (i)	$G_{i\mathrm{phase}}^{\mathrm{E}}$ (kJ·mol·1)	=	H ^E _{i phase} (kJ⋅mol ⁻¹)	-	$T S_{t \text{ phase}}^{\text{E}}$ (kJ·mol ⁻¹)	$S_{i\mathrm{phase}}^{\mathrm{E}}$ $(\mathbf{J}\cdot\mathrm{mol}^{-1}\mathbf{K}^{-1})$
Gas Phase	Maria II					
Hexane	4.0	=	31.6	-	27.6	92.6
Benzene	5.3	=	33.9	-	28.6	96.0
Diethylether	0.8	=	27.1	_	26.3	88.2
Ethanol	6.3	=	42.6	-	36.3	122.0
Hexadecane						
Hexane	-0.2	=	0.6	_	0.8	2.7
Benzene	0.4	=	3.5	-	3.1	9.7
Diethylether	0.0	=	1.9	_	1.9	6.4
Ethanol	8.8	=	26.3	-	17.5	58.7
Water						
Hexane	32.3	=	-0.4	+	32.7	-109.7
Benzene	19.4	=	2.2	+	17.2	-58.4
Diethylether	12.0	=	-19.7	+	31.7	-106.3
Ethanol	3.2	=	-10.0	+	13.2	-44.3

ترمود نامیک - تعاریف

Reversibility is the ability to run a process back and forth infinitely without losses.

Reversible Process E.g. Perfect Pendulum

Irreversible Process E.g. Dropping a ball of clay-

All Real processes

Impossible Process E.g. Decrease in Entropy

Adiabatic process no heat is transferred to or from the system Isothermal Process: A change in state occurring at constant temperature.



First Law of Thermodynamics

Energy can neither be created nor destroyed

For adiabatic process

$$0 = C_{v}dT + PdV$$

• Definitions:

ترمودنامك-قوانين

• coefficient of isobaric thermal expansion

$$\alpha_P = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_P$$

· isothermal compressibility

$$\beta_T = -\frac{1}{V} \left(\frac{\partial V}{\partial P} \right)_T$$

• isentropic compressibility

$$lpha_P = rac{1}{V} \left(rac{\partial V}{\partial T}
ight)_P$$
 هنرب انبياط تجمی $eta_T = -rac{1}{V} \left(rac{\partial V}{\partial P}
ight)_T$ هابت تراکم پذری بمدا $eta_S = -rac{1}{V} \left(rac{\partial V}{\partial P}
ight)_S$ مناب آراکم پذری آدیا آب

$$c_{p} \equiv \left(\frac{\partial Q_{rev}}{\partial T}\right)_{p} = T \left(\frac{\partial S}{\partial T}\right)$$

• heat capacity at constant pressure
$$c_{p} \equiv \left(\frac{\partial Q_{rev}}{\partial T}\right)_{p} = T\left(\frac{\partial S}{\partial T}\right)_{p}$$
• heat capacity at constant volume
$$c_{v} \equiv \left(\frac{\partial Q_{rev}}{\partial T}\right)_{v} = T\left(\frac{\partial S}{\partial T}\right)_{v}$$



Second Law of Thermodynamics

Naturally occurring processes are directional & irreversible

The entropy of the universe does not change for reversible processes and increases for spontaneous processes.

Reversible (ideal) $\Delta S_{univ} = \Delta S_{system} + \Delta S_{surroundings} = 0$

Irreversible (real, spontaneous) $\Delta S_{univ} = \Delta S_{system} + \Delta S_{surroundings} > 0$

First and second Law combination

$$dU = TdS - PdV$$



Third Law of Thermodynamics

As a system approaches absolute zero of temperature, all processes cease and the entropy of the system approaches a minimum value

$$U \rightarrow U_{\min} \Rightarrow S \rightarrow 0$$

0

$$T \to 0 \Rightarrow S \to S_{\min}$$

a temperature of absolute zero is not possible



Chemical Thermodynamics
Thermodynamics of Solutions

- Phases: Part of a system that is chemically and physically homogeneous, bounded by a distinct interface with other phases and physically separable from other phases.
- Components: Smallest number of chemical entities necessary to describe the composition of every phase in the system.
- Solutions: Homogeneous mixture of two or more chemical components in which their concentrations may be freely varied within certain limits.

Unsaturated Soils

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Chemical Thermodynamics

Mole Fractions

$$x_A \equiv \frac{n_A}{\sum n} = \frac{n_A}{(n_A + n_B + n_c + \dots)}$$

where x_A is called the "mole fraction" of component A in some phase.

If the same component is used in more than one phase, Then we can define the mole fraction of component A in phase i as χ_A^i

For a simple binary system, $x_A + x_B = 1$

Unsaturated Soils



Chemical Thermodynamics

Raoult's law(1888)

when a dilute liquid solution of a volatile solvent and a nonelectrolyte solute is equilibrated with a gas phase:

$$p_A = x_A p_A^*$$

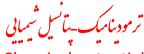
 p_A partial pressure of the solvent in the gas phase

 p_A^* saturation vapor pressure of the pure solvent (star stands for pure material)

 X_A mole fraction of component A in the solution

Unsaturated Soils

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Chemical potential

chemical potential, μ , the partial molar Gibbs free energy variation.

$$\mu_{A} \equiv \left(\frac{\partial G_{A}}{\partial x_{A}}\right)_{P,T,x_{B},x_{C}} \text{ direction of reaction}$$

standard state a reference state from which to calculate differences in chemical potential (25°C and 1 bar).

Fugacity is a measure of chemical potential $f = \exp(\mu/k_B T)$

Boltzmann constant
$$K_B = 1.38 \times 10^{-23} \text{ J} \cdot \text{K}^{-1}$$

For gases, ideal behavior is assumed, so a compound's fugacity is equal to its partial pressure: $f_{ig} = p$

As a result, fugacity is given in units of pressure, often Pa.

Pressure (and therefore f) is easy to measure, unlike μ



Partial Molar Gibbs Free Energy

$$dG = VdP - SdT + \sum_{i} \mu_{i} dx_{i}$$

$$\sum_{i} \mu_{i} dx_{i} = \mu_{A} dx_{A} + \mu_{B} dx_{B} + \mu_{C} dx_{C} + \dots + \mu_{n} dx_{n}$$

This equation demonstrates that changes in Gibbs free energy are dependent on:

- changes in the chemical potential, μ , through the concentration of the components expressed as mole fractions of the various phases in the system
- changes in molar volume of the system through dP
- changes in molar entropy of the system through dT
- Chemical potential is analogous to gravitational or electrical potentials: the most stable state is the one where the overall potential is lowest.
- At equilibrium the chemical potentials for any specific component in ALL phases must be equal.

Unsaturated Soils



For pure solid (s) or liquid (L), the fugacity is:
$$f_{il}^* = \gamma_{il} \, p_{il}^* \qquad \qquad f_{is}^* = \gamma_{is} \, p_{is}^*$$

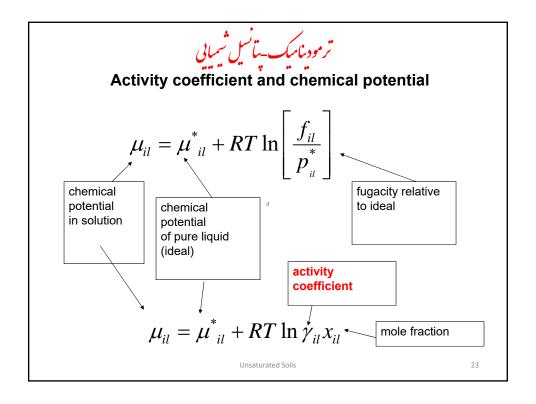
Where γ describes the non ideal behavior resulting from moleculemolecule interactions. For pure solid or liquid, γ is assumed to equal one.

$$f_{il} = x_{il}(f_{il}^*) = x_{il}(\gamma_{il}p_{il}^*)$$

In a mixture (i.e. aqueous solution): $f_{il} = x_{il} (f_{il}^*) = x_{il} (\gamma_{il} \, p_{il}^*)$ aqueous solutions of organic chemicals are usually not ideal. $\gamma \neq 1$

Activity coefficients: γ_{ii}

	• 11			
solute =>	hexane	benzene	diethylether	ethanol
solvent ↓				
n-hexadecane	~1	~1	~1	35
CHCl ₃	1.8	0.8	0.3	4.5
ethanol	12	5.4	n.a.	1
water	460,000	2500	130	3.6



Chemical reaction

$$aA + bB + \dots \rightleftharpoons cC + dD + \dots$$
reactants products

Stoichiometry coefficient
$$aA + bB + \dots \rightleftharpoons cC + dD + \dots$$
activity coefficients

Equilibrium constant $K = \frac{[C]^c[D]^d}{[A]^a[B]^b}$
Unsaturated Soils 24



Phase transfer processes

Consider a chemical (A) equilibrating between air and water:

$$A_a \rightleftharpoons A_w$$

$$\mu_{Aa} = \mu_{AL}^* + RT \ln \gamma_{Aa} x_{Aa}$$
$$\mu_{Aw} = \mu_{AL}^* + RT \ln \gamma_{Aw} x_{Aw}$$

Note: still using pure liquid as reference state

At equilibrium, μ is equal in the two phases

Unsaturated Soils

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After some rearranging you get:

$$\ln \frac{x_{Aa}}{x_{Aw}} = -\frac{RT \ln \gamma_{Aa} - RT \ln \gamma_{Aw}}{RT}$$

$$\frac{x_{Aa}}{x_{Aw}} = e^{-\frac{RT\ln\gamma_{Aa} - RT\ln\gamma_{Aw}}{RT}}$$

$$K'_{Aaw} = e^{-\frac{\Delta G}{RT}} = e^{-\frac{\Delta H - T\Delta S}{RT}}$$

entropic and enthalpic terms

Unsaturated Soils

Chemical reaction

$$aA + bB + ... \rightarrow mM + nN + ...$$

$$\Delta G_r = \Delta G_r^0 + RT \ln \left(\frac{[M]^m [N]^n \dots}{[A]^a [B]^b \dots} \right)$$

Standard free energies $\Delta G_r^0 = \sum G_{products}^0 - \sum G_{reactants}^0$ of formation

At thermodynamic equilibrium $\Delta G_r = 0$ and so

$$\Delta G_r^0 = -RT \ln K$$
 ΔG° can be looked up in tables,

Where K = equilibrium constant

Unsaturated Soils

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Equilibrium is related to kinetics

$$A \overset{k_1}{\underset{k_{-1}}{\rightleftharpoons}} B$$
 Simple reversible reaction

$$-\frac{dA}{dt} = k_1[A] - k_{-1}[B] = \frac{dB}{dt}$$

At equilibrium, there is no change in concentrations:

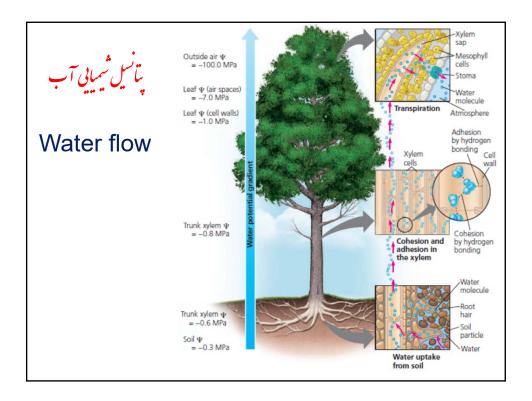
$$-\frac{dA}{dt} = \frac{dB}{dt} = 0 \qquad \text{thus} \qquad k_1[A] = k_{-1}[B]$$

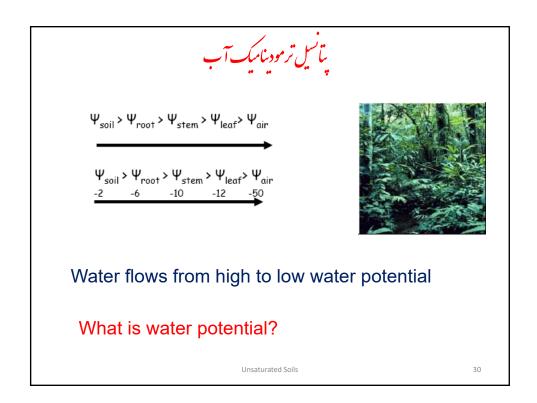
$$\frac{[B]}{[A]} = \frac{k_1}{k_{-1}} = K_{eq}$$

 $\frac{[B]}{[A]} = \frac{k_1}{k_{-1}} = K_{eq}$ Example: sorption of PCBs to organic matter. K_{eq} is large, implying that sorption is fast and desorption is slow



PCBs: polychlorinated biphenyls





پانسل ترمودینامیک آب

مطابق تعریف کمیسیون انجمن بین المللی علوم خاک (۱۹۶۳) پتانسیل کل : مقدار کاری که باید روی مقدارواحدآب خالص در استخر آب صورت پذیرد تا در شرایط بازگشت پذیر و همدما آب به یک نقطه مشخص از خاک منتقل شود. استخر در تراز مشخص و با دمای و فشار گاز مساوی با نقطه مزبور در خاک می باشد.

توجه : آب خالص ماده (H_2O) می باشد در صورتیکه آب در خاک محلولی است که مواد دیگری نیز در آن حل شده اند.

پانسل ترمودینامیک آب

در حالت کلی تر پتانسیل ترمودینامیکی کل آب بصورت زیر می تواند تعریف شود: مقدار انرژی پتانسیل آب در خاک نسبت به آب خالص در شرایط مرجع

شرایط مرجع: استخرآب خالص با تراز و دما و فشار مشخص می باشد. پتانسیل ترمودینامیکی تمایل جابجایی آب تحت نیروهای تقلی و شیمیایی و فشارهای مکانیکی را نشان می دهد

$$\psi_t = \psi_m + \psi_o + \psi_g + \psi_p + \psi_T + \psi_v$$

 Ψ_t Total thermodynamics potential

 $\psi_{\it m}$ Matric potential

 ψ_o Osmotic potential

 ψ_g gravity potential

 ψ_{n} Pressure potential

 ψ_T Thermal potential

 ψ_v Humidity potential

$$\psi_t^w = \psi_m^w + \psi_o^w + \psi_g^w + \psi_p^w + \psi_T^w + \psi_v^w$$

Unsaturated Soils

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پانسل ترمودینامیک آب

مطابق تعریف کمیسیون انجمن بین المللی علوم خاک (۱۹۶۳)

پتانسیل ماتریسی: مقدار کاری که باید روی حجم واحدآب خالص (آب به عنوان یک جزء) در استخر آب صورت پذیرد تا در شرایط بازگشت پذیر و همدما آب به یک نقطه مشخص از خاک منتقل شود. استخر در تراز و با دما و فشار گاز مساوی با نقطه مزبور در خاک می باشد.

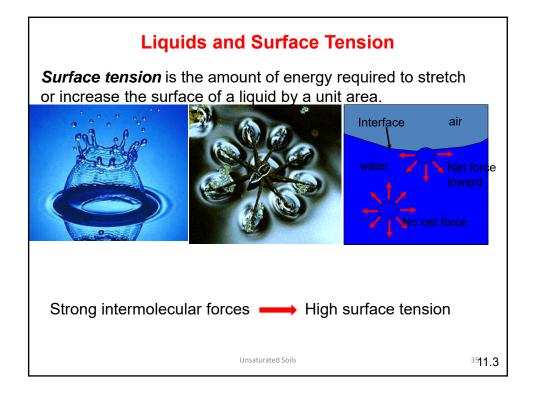
توجه: آب در استخر و درخاک دارای ترکیب یکسان می باشد.

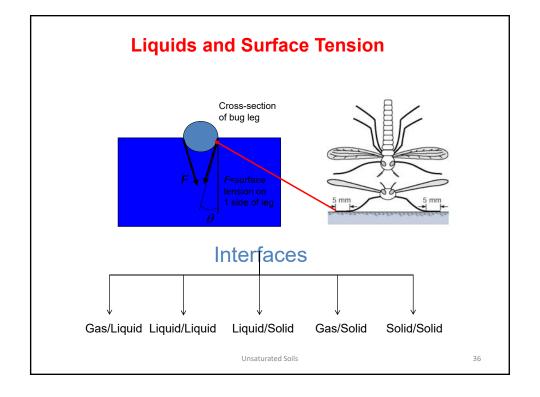
نیروهای موِثر نیروی مویینگی و نیروی جذب آب به دانه های خاک می باشد.

$$\psi_m = \psi_{cap} + \psi_{ads}$$

 ${\psi}_{\it cap}$ capillary potential

 $\Psi_{\it ads}$ Adsorption potential





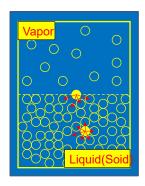
Dr. Hasan Ghasemzadeh

Liquids and Surface Tension

Interfacial tension γ_{ij} - force/unit length (N/m) arises from the imbalance of forces on molecules at liqid-gas interface

Interfacial energy = free energy necessary to increase by one unit area of contact between two different phases i, j

$$\Delta G/\Delta A = \gamma_{ij}$$



- Below surface, forces act equally in all directions
- At surface, some forces are missing, pulls molecules down and together, like membrane exerting tension on the surface
- If interface is curved, higher pressure will exist on concave side
- Pressure increase is balanced by surface tension, σ
- $\sigma = \gamma_{ij} = 0.073 \text{ N/m} (@ 20^{\circ}\text{C})$

Unsaturated Soils

Surface tension of var	ious liquids in <u>dyn</u> /cm	against air 1 dyn = 1
Mixture	%'s are by weight	
Liquid	Temperature °C	Surface tension, γ
Acetic acid	20	27.6
Acetic acid (40.1%) + Water	30	40.68
Acetic acid (10.0%) + Water	30	54.56
<u>Acetone</u>	20	23.7
Diethyl ether	20	17.0
<u>Ethanol</u>	20	22.27
Ethanol (40%) + Water	25	29.63
Ethanol (11.1%) + Water	25	46.03
<u>Glycerol</u>	20	63
<u>n-Hexane</u>	20	18.4
Hydrochloric acid 17.7M aqueous solution	20	65.95
<u>Isopropanol</u>	20	21.7
Mercury	15	487
<u>Methanol</u>	20	22.6
<u>n-Octane</u>	20	21.8
Sodium chloride 6.0M aqueous solution	20	82.55
Sucrose (55%) + water	20	76.45
<u>Water</u>	0	75.64
<u>Water</u>	25	71.97
<u>Water</u>	50	67.91
Water	Insaturated Soils	58.85

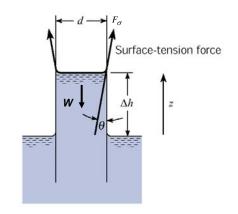
Capillary Rise

$$F_{\sigma,z} - W = 0$$

$$\sigma\pi d\cos\theta - \gamma(\Delta h)(\frac{\pi}{4}d^2) = 0$$

$$\Delta h = \frac{4\sigma\cos\theta}{\gamma\,d}$$

$$d = 2r \quad \Delta h = \frac{2\sigma\cos\theta}{\gamma r}$$



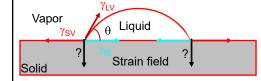
Unsaturated Soils

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Young equation

(homogeneous, smooth, rigid solids)



Horizontal balance of forces: Young equation

 $\gamma_{SV} = \gamma_{SL} + \gamma_{LV} \cos \theta$

 $\begin{array}{l} dG^S = (dG^S)_{SV} + (dG^S)_{SL} + (dG^S)_{LV} \\ dG^S = dA \left(\gamma_{SL} - \gamma_{SV}\right) + dA \, \gamma_{LV} \cos \left(\Theta'\right) \\ At \ equilibrium: \\ dG^S / dA = 0 \Rightarrow \end{array}$

 γ_{SV} - γ_{SL} = γ_{LV} cos θ

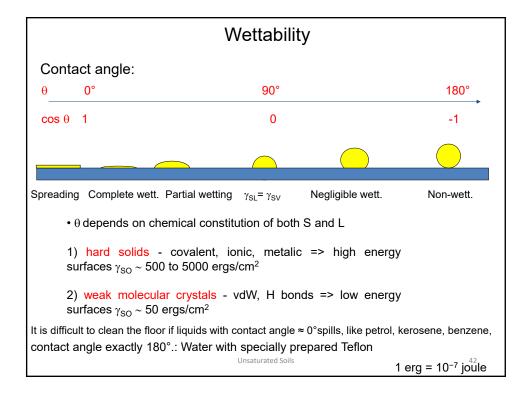
Vertical balance of forces: $\gamma_{LV} \sin \theta$ = ? Hard solids - no visible effect but the stress is there

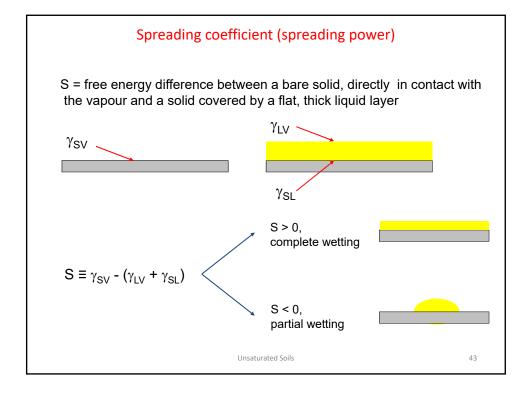
If γ_{SV} - γ_{SL} > γ_{LV} $cos\theta$ then cos θ = 1

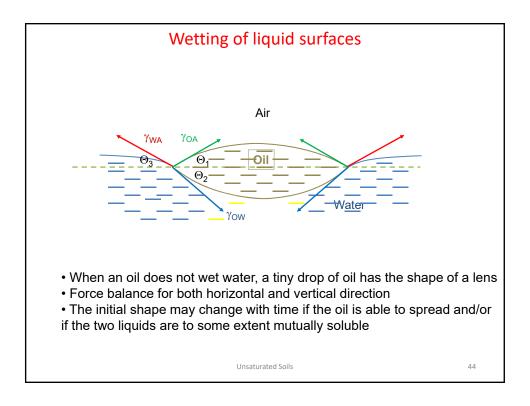
Unsaturated Soils

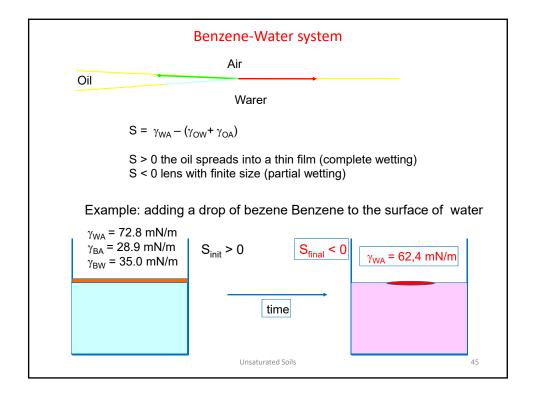
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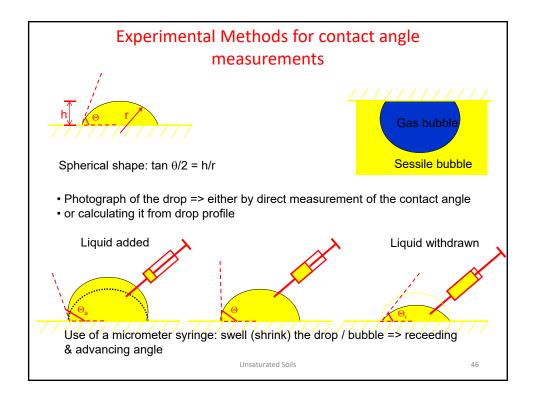
Some liquid-solid contact angles					
Liquid	Solid	Contact angle			
<u>water</u>					
soda-lime glass		0°			
lead glass					
fused quartz					
<u>ethanol</u>	soda-lime glass lead glass				
diethyl ether	fused quartz				
<u>carbon</u>	Tusca quartz				
<u>tetrachloride</u>					
<u>glycerol</u>					
acetic acid					
water	paraffin wax	107°			
<u>water</u>	silver	90°			
methyl iodide	soda-lime glass	29°			
	lead glass	30°			
	fused quartz	33°			
mercury	soda-lime glass	140°			

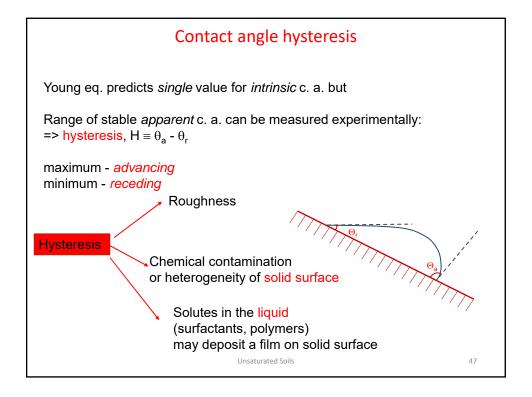


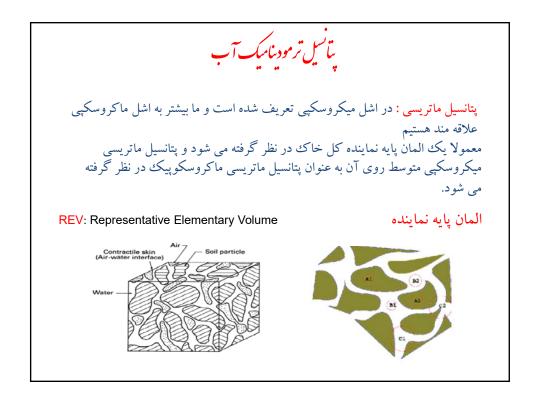












پانسل ترمودینامیک آب

بتانسیل اسمزی:

پ یا در خاک یک محلول می باشد که مواد غیر محلول نیز در آن وجود دارد این مواد بر روی کشش سطحی آب و همچنین بر روی نیروهای جذب مولکولهای آب به دانه های خاک اثر می گذارند

بنابر این تعریف پتانسیل اسمزی مانند پتانسیل ماتریسی بوده با این تفاوت که آب در خاک بصورت یک محلول (دارای چند جزء) می باشد. پتانسیل برای جزء آب است و برای محلول اگر پتانسیل بخواهیم باید پتانسیل تک اجزا را حساب کرده با هم چمع کنبم.

